

A PESSIMIST'S VIEW OF GALACTIC STRUCTURE

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Our conference title is taken from the movie “Back to the Future”, which describes the adventures of a time-traveller who is trapped in 1955. To set the conference in historical context, I would like to review briefly the progress we have made since that time towards understanding the major components and fundamental parameters of the Galaxy. The year 1955 is particularly appropriate because the first 21-cm neutral hydrogen surveys of the Galaxy had just been completed. As my title implies, I will focus on areas in which progress has been slow, since these areas are slighted by most speakers.

First consider the distance to the Galactic center, R_0 , and the speed of the Local Standard of Rest, v_{LSR} . The first 21-cm surveys used $R_0 = 8.2$ kpc, $v_{\text{LSR}} = 216 \text{ km s}^{-1}$, following van de Hulst *et al.* (1954). In 1964, following recommendations by Schmidt (1965), the IAU adopted the standard values $R_0 = 10$ kpc, $v_{\text{LSR}} = 250 \text{ km s}^{-1}$. Most subsequent determinations of both quantities have been smaller than Schmidt’s values, and a review by Kerr and Lynden-Bell (1986) led to revised IAU standards, $R_0 = 8.5$ kpc and $v_{\text{LSR}} = 220 \text{ km s}^{-1}$. In a more recent review, Reid (1989) concluded that $R_0 = 7.7 \pm 0.7$ kpc (an analysis of globular cluster distances by Racine and Harris 1989 yields almost the same answer); recent estimates of v_{LSR} range from $248 \pm 16 \text{ km s}^{-1}$ (Schechter *et al.* 1988) to $200 \pm 10 \text{ km s}^{-1}$ (Merrifield, 1992 and this volume).

Thus neither R_0 nor v_{LSR} is known to better than $\pm 20\%$ at the 2σ level (and the uncertainty is even larger if—as is usually the case—there are unrecognized systematic errors); moreover, the best estimates and likely uncertainty of both parameters have not substantially changed since 1955.

Another fundamental parameter is the total mass density in the solar neighborhood, ρ_0 (the Oort limit), which is determined from the dynamics of disk stars in the direction normal to the Galactic plane (Kapteyn 1922). Comparison of ρ_0 to the total density of known stars and gas (about $0.10 M_{\odot} \text{ pc}^{-3}$) could reveal the presence of dark matter in the Galactic disk. The first reliable estimate was made by Oort (1932), who found $\rho_0 = 0.09 M_{\odot} \text{ pc}^{-3}$; later, Oort (1960) revised his estimate to $\rho_0 = 0.15 M_{\odot} \text{ pc}^{-3}$ with an uncertainty of $\pm 10\%$. For comparison, recent estimates include $0.09 \pm 0.03 M_{\odot} \text{ pc}^{-3}$ (Kuijken 1991a; see also Fuchs & Wielen, this volume) and $0.26^{+0.19}_{-0.12} M_{\odot} \text{ pc}^{-3}$ (Bahcall *et al.*

1992). Clearly we have not made much progress in converging to an accurate estimate of ρ_0 in the past generation.

The nature and origin of spiral structure were not understood in 1955. Since then, there has been considerable theoretical progress on a related issue, the stability and response of self-gravitating gaseous and stellar disks. Analytical studies, mostly using the WKB approximation (Lin & Shu 1964) or Hill's equations (Goldreich & Lynden-Bell 1965, Julian & Toomre 1966), have been complemented by N -body simulations and numerical calculations of linearized normal modes, to yield a secure understanding of many of the principal features of disk dynamics. However, the relation between disk dynamics and spiral structure remains obscure (see Toomre 1977 for a review), and as Kerr has stressed in his introductory talk, many basic questions about the spiral pattern in our own Galaxy remain unanswered: Is the pattern global or local, i.e. is there a well-defined "grand design" pattern or just a chaotic superposition of spiral patches? Is there a well-defined pattern speed—that is, will the same pattern persist for several Galactic years—and if so what is it? Is the pattern trailing or is there a leading component? What drives the spiral pattern—a central bar? a recent encounter with a satellite galaxy? local gravitational instability? or an unstable normal mode?

The most profound change in our picture of the Galaxy since 1955 has been the introduction of massive halos by Ostriker *et al.* (1974), who argued that an unseen dark halo may extend to many times the radius of the visible disk, with mass increasing roughly in proportion to radius, so that the halo contains virtually the whole mass of the Galaxy. The visible stars and gas constitute a small cesspool at the center, containing only a few percent of the total mass. The evidence cited for the halo included the Local Group timing argument, the dynamics of satellite galaxies, and the rotation curve; in addition, there are strong reasons to believe that such halos form naturally in a range of cosmological models (Gunn 1977). Almost twenty years later, the observational and theoretical evidence for dark halos surrounding our own and other galaxies is far stronger, but our understanding of the distribution of mass in our halo is not greatly improved over that of Ostriker *et al.*

The preceding paragraphs offer a bleak picture of progress in understanding Galactic structure over the last few decades. Obviously, this picture is far from correct. There has in fact been tremendous progress, particularly through surveys at wavelengths that were inaccessible in 1955 (such as the infrared surveys from COBE described here by Hauser). Nevertheless, it is striking that estimates of many of the basic parameters of the Galaxy—the solar radius, the rotation speed of the Local Standard of Rest, the Oort limit, the properties of the spiral structure—are not much better now than they were in 1955. Given the dramatic improvements in the quality and quantity of the data, and the many new types of data available, why can we not do better? *Why is understanding Galactic structure so hard?*

I think this is an interesting question, and the answer could help to direct observational and theoretical research efforts in the future. In the rest of this summary, I will try to identify some of the reasons why accurate models of the Galaxy have been so slow to arrive.

The Galaxy is very responsive Hot stellar systems, like the spheroid or halo of the Galaxy, do not support sound waves: small-scale disturbances are strongly Landau damped. This well-known result led many of us to believe that all normal modes in hot stellar systems were Landau damped, so that such systems would settle rapidly to a steady state once galaxy formation was complete. This expectation was shown to be false by Mathur (1990), who demonstrated that self-gravitating spherical stellar systems can support discrete large-scale oscillations that do not Landau damp. Mathur's work provides theoretical support for observations of long-lived oscillations in a number of N -body simulations of stellar systems (Hénon 1968, Miller *et al.* 1982, Smith & Miller, this volume).

The Galactic disk is even more responsive than the spheroid. Leading spiral disturbances can be amplified by factors of 10 or more as they shear into trailing disturbances, even in disks that are safely stable to axisymmetric perturbations in the sense that Toomre's thermometer Q exceeds the critical value for stability by 50% or so (Goldreich & Lynden-Bell 1965, Julian & Toomre 1966). One consequence of this "swing amplification" process (Toomre 1981) is that concentrated lumps in the disk, such as giant molecular clouds, are surrounded by a trailing wake of disk stars, whose mass may exceed the mass of the original lump by an order of magnitude or more. A second is that large parts of the combined star-gas disk may quiver on the threshold of instability: gas cooling continually reduces Q and turns up the gain on the swing amplifier, until the heating induced by the amplification of small disturbances and resulting star formation stirs up the gas, increases Q , and quenches the amplifier.

Not only is the disk sensitive to gravitational disturbances, but the disk environment is a noisy one, containing almost 10^4 giant molecular clouds with masses $10^{5.5} M_{\odot}$ or larger (plus their associated wakes). Note that the ratio of the mass of one such cloud to the total mass of the disk is about the same as the ratio of the mass of the Earth to the mass of the Sun; thus the Galactic disk is as dynamically noisy as a solar system containing 10^4 planets like the Earth!

Non-axisymmetric distortions Ever since the work of Shapley, Lindblad and Oort in the 1910s and 1920s, astronomers have understood that the Galaxy is approximately axisymmetric. Nevertheless, non-axisymmetric distortions—warps, bars, and spiral structure, to name three—are common in other galaxies and likely to affect our own.

Speakers at this meeting described several independent lines of evidence that yield a similar picture of a central bar in our Galaxy: (i) Blitz argued that $2.4\mu\text{m}$ balloon observations imply that the stars in the central kpc are arranged in a bar whose near side is at positive Galactic longitude (see also

Blitz & Spergel 1991b). (ii) This result is confirmed by the COBE/DIRBE near-infrared maps of the Galactic bulge described by Hauser and by Spiesman *et al.* (iii) As Weinberg reported, the distribution of variables in the IRAS point-source catalog is consistent with a bar with semi-major axis of about 5 kpc (see also Weinberg 1992). The position angle of the long axis of Weinberg's bar is $\theta = 36^\circ \pm 10^\circ$, where position angle is relative to the Galactic center-Sun axis and positive in the direction of Galactic rotation, so that $0 < \theta < 90^\circ$ implies that the near side is at positive longitude. (iv) Rohlfs and Kampmann's paper shows that HI terminal velocities indicate the presence of a bar with $\theta \simeq 45^\circ$, and semi-major axis 2–3 kpc; (v) Binney argued (following Binney *et al.* 1991) that the CO kinematics in the central kpc indicate the presence of a bar with $\theta \simeq 16^\circ \pm 2^\circ$, pattern speed $63 \text{ km s}^{-1} \text{ kpc}^{-1}$, and corotation radius 2.4 kpc. Sellwood confirmed that such a bar arises naturally in N -body simulations of an unstable disk.

The impressive agreement of all five groups on the quadrant containing the long axis suggests that most or all have identified the same real structure, although the parameters of the bar (length, strength, pattern speed, position angle) are still quite uncertain.

Several authors have invoked other plausible $m = 2$ distortions to explain kinematic features in the solar neighborhood or the outer Galaxy: (i) Blitz and Spergel (1991a; see also Spergel's review) argue that the longitude-velocity distribution of distant HI implies that the Local Standard of Rest is moving outward at 14 km s^{-1} ; this outward motion is induced by an $m = 2$ distortion from a triaxial spheroid with position angle $\theta = -45^\circ \pm 20^\circ$ and pattern speed $6 \text{ km s}^{-1} \text{ kpc}^{-1}$. (ii) Kalnajs (1991) analyzes the distribution of stellar velocities in the solar neighborhood and finds evidence for two distinct star streams, which he associates with crossing orbits near an outer Lindblad resonance; he deduces that the Galaxy contains an $m = 2$ disturbance with pattern speed $46 \text{ km s}^{-1} \text{ kpc}^{-1}$ and position angle $\theta = 56^\circ$, possibly due to a bar (though the parameters appear incompatible with those deduced by Binney *et al.*). (iii) Kuijken and I argued at this meeting that the axis ratio of the local velocity ellipsoid and apparent inconsistencies between outer rotation curves measured by different methods can be explained if the Sun is near the minor axis of an $m = 2$ distortion with near zero pattern speed, possibly arising from a triaxial dark halo. (iv) Most prominent "grand design" spiral patterns in other galaxies have two arms, and many authors have sought to explain features of local kinematics and the HI terminal velocity curve in terms of $m = 2$ spiral distortions.

Lopsided ($m = 1$) distortions are also common in many galaxies (Sellwood & Wilkinson 1992). One-armed spirals are seen in M31 and other galaxies (García-Gómez & Athanassoula 1991), although they are not as common as two-armed spirals; the HI distributions in the outer parts of galaxies are often lopsided (Baldwin *et al.* 1980); Kuijken (1991b) has suggested that a similar $m = 1$ distortion of the outer Galaxy could explain the asymmetries in the HI distri-

bution described by Blitz and Spørgel (1991a); in Magellanic irregular galaxies the photometric center is offset from the kinematic center (de Vaucouleurs & Freeman 1970).

The distribution of CO near the Galactic center is lopsided, with roughly three times as much gas in the north as the south. Binney *et al.* (1991) interpret this displacement as an effect of the perspective from which we view an $m = 2$ barlike distortion; however, an $m = 1$ distortion is an alternative explanation. Several of the other lines of evidence presented at this meeting for a central bar, including asymmetries in the infrared photometry and the distribution of IRAS variables, could equally well be evidence for an $m = 1$ distortion. A possible weakness of this interpretation is that a rotating $m = 1$ distortion in the inner Galaxy would normally induce a non-zero radial velocity between the Galactic center and the Local Standard of Rest, but observations show that this velocity is very near zero (see Blitz & Spørgel 1991a and Kuijken & Tremaine 1991 for two estimates).

There are also theoretical reasons to be interested in lopsided distortions. The most unstable mode in stellar Mestel disks is lopsided (Zang 1976), as is the most unstable mode in models of the Galaxy that have no massive halo (Sellwood 1985); some lopsided modes are only weakly damped in spherical stellar systems (Weinberg 1991a); a lopsided mode is the dominant instability in massive gas disks orbiting a central point mass (Adams *et al.* 1989, Shu *et al.* 1990); and lopsided distortions appear in N -body simulations (see, for example, Smith and Miller, this volume). Finally, I note that as our understanding of stellar dynamics has increased, models of stellar systems have lost their artificial symmetries: from the spherical models investigated by Eddington and Jeans early in this century, we progressed to axisymmetric models in the 1960s and early 1970s (Lynden-Bell 1962, Gott 1973), then to triaxial models in the 1980s (Schwarzschild 1979, de Zeeuw 1985). However, the triaxial models still contain symmetries: they are invariant under reflection in the three principal planes (the D_{2h} point group, Landau & Lifshitz 1977). These symmetries are less artificial than spherical symmetry or axisymmetry, since they appear to be spontaneously generated in violent relaxation: N -body experiments suggest that initial states with no symmetries collapse to form equilibrium systems with D_{2h} symmetry. Nevertheless, it seems worthwhile to investigate models of stellar systems that contain fewer symmetries.

There can also be distortions normal to the Galactic plane; examples include the well-known warp in the outer Galaxy, and discrete vertical oscillation modes of the disk (Weinberg 1991b).

Non-axisymmetric distortions complicate attempts to model the structure of the Galaxy. There are at least five known or likely sources of distortion (central bar, triaxial spheroid, triaxial halo, spiral structure, warp)—not to mention possible $m = 1$ distortions or local inhomogeneities—and each is described by at least three parameters (amplitude, pattern speed, phase)—not to mention the

parameters needed to describe the radial dependence. Unless one or two of the distortions dominate, it may prove difficult to disentangle their effects.

A particularly pernicious sort of distortion is one whose symmetry axis coincides with the line between the Sun and the Galactic center. Such distortions are symmetric under reflection of positive and negative Galactic longitude, and hence are difficult to detect, but Kuijken and I have argued here that they can have a drastic effect on measurements of the rotation curve, the speed of the Local Standard of Rest, and other kinematic parameters.

Incomplete mixing The Galaxy is usually assumed to be in a steady state but this state is approached only gradually, by phase mixing. When mixing is not complete, even large samples of stars may not provide representative samples for analyzing Galactic structure.

The presence of moving groups of stars in the solar neighborhood (Eggen 1965) implies that the disk is not well-mixed, at least for A-type and earlier stars, a conclusion consistent with simple estimates of the clumpiness of star formation and the mixing rate (e.g. Kuijken & Tremaine 1991).

Next consider mixing in the spheroid. We assume that the spheroid is composed of stars from N lumps (protogalaxies? globular clusters?), each of mass m and radius Δr , that were tidally disrupted at the time of spheroid formation, $t = 0$. The lumps would be disrupted at orbital radius $r \approx \Delta r(M/m)^{1/3}$, where M is the mass of the Galaxy inside r . The stars from each disrupted lump spread out into a tidal stream, which we assume has typical radius comparable to the tidal disruption radius r . After time t the length of the stream is $L \approx r(d\Omega/dr)\Delta r t \approx \Delta r\Omega t$, where Ω is the orbital angular speed of the stars in the stream, while its cross-sectional area would be $A \approx \Delta r^2$. The filling factor—the fractional volume containing one or more tidal streams—is then $1 - \exp(-f_3)$, where

$$f_3 \approx N \frac{AL}{r^3} \approx \frac{Nm}{M} \Omega t = \frac{M_s}{M} \Omega t,$$

and M_s is the mass in the spheroid. A well-mixed system should have $f_3 \gg 1$; note that f_3 is independent of the lump mass m . If some lumps are disrupted later than $t = 0$ (because of late infall or dynamical friction), then f_3 would be even smaller. A related statistic is the fraction of the sky containing one or more streams seen in projection, $1 - \exp(-f_2)$ where

$$f_2 \approx N \frac{L\Delta r}{r^2} \approx \frac{M_s}{M} \left(\frac{M}{m}\right)^{1/3} \Omega t \approx f_3 \left(\frac{M}{m}\right)^{1/3}.$$

For the spheroid at the solar radius, we may take $M_s/M \approx 0.01$, $\Omega t \approx 100$, so $f_3 \approx 1$ independent of the lump mass m ; in other words, at a given spatial location the spheroid is likely to contain stars from at most a few lumps. Given this result it is not surprising that the phase-space distribution of spheroid stars in the solar neighborhood appears clumpy (Doinidis & Beers 1989, Norris & Ryan

1989), or that there are variations between the kinematical properties of various samples of spheroid stars. Oort (1965) already suggested that Eggen's moving groups of spheroid stars (which however are of doubtful statistical significance) might arise from disrupted globular clusters.

We may also apply this argument to stars in the distant halo, say at $r > 50$ kpc. Here $\Omega t \approx 20$, and M_*/M may be taken to be the ratio of the mass in stars to the total mass, which is poorly known—since few stars are seen at this distance—but certainly small. Thus $f_3 \ll 1$, and it is likely that $f_2 \ll 1$ as well.

Let us then imagine how the Galaxy would look if we had a special telescope that was only sensitive to stars at $r > 50$ kpc. We would not see a smooth distribution of stars randomly dotted around the celestial sphere; rather, the stars would be concentrated in randomly oriented streaks of various lengths, as well as a few isolated galaxies such as the Magellanic Clouds that have not (yet) disrupted. The Magellanic Stream of HI is presumably one such streak. At some locations in the sky the streaks might fold back on themselves as seen from our perspective near the Galactic center; in such cases there would be a strong local enhancement of the density, which might be mistaken by us for an isolated low surface-brightness galaxy. It is possible that the Draco and Ursa Minor dwarf “galaxies” are a mirage of this sort; this radical hypothesis eliminates the need for dark matter in these systems and is consistent with the clumpy distribution of stars in Ursa Minor, which would not be expected in an equilibrium stellar system (Olszewski & Aaronson 1985).

Insufficient statistics Even if the Galaxy were well-mixed, fluctuations due to Poisson statistics in samples of limited size can obscure the effects we seek. Suppose, for example, we want to measure some velocity parameter from the globular cluster system (velocity of the LSR, mean rotation velocity of the clusters, etc.). There are about 115 clusters with known radial velocities and the system has a dispersion of 110 km s^{-1} (Thomas 1989). Thus the 2σ limits on any mean velocity will at best be $\pm 20 \text{ km s}^{-1}$, and smaller if the sample is restricted to metal-rich or metal-poor clusters (the two types have different kinematics), or if some clusters contribute less weight to the answer (for example, clusters near the Galactic poles do not contribute to estimates of rotation speed). Typical samples of nearby metal-weak stars yield comparable statistics.

The Galaxy is not a relaxed system Are galaxies important archaeological sites, which retain a record of their formation and history? The answer is not obvious: in a different context, the study of isolated stars tells us very little about star formation, because of the Vogt-Russell theorem (the structure of a non-rotating star is uniquely determined by its mass and chemical composition). Similarly, there may be relaxation processes that operate or have operated in galaxies to erase all memory of the initial conditions and the formation history; such processes might include violent relaxation in halos, late infall of gas, or

angular momentum transport by bars and spiral structure.

The weight of evidence now suggests, however, that galaxies are not relaxed, so that many of their properties depend strongly (and stochastically) on their individual histories. For example, dark halos are not spherical, but instead have a range of triaxial shapes determined by the primordial fluctuation spectrum (Dubinski & Carlberg 1991); warps may reflect the excitation of a particular normal mode early in the history of the galaxy (see Sparke's review) or reorientation of the spin axis of the disk in response to continuing infall (E. Ostriker & Binney 1989, Binney 1992); the distribution of gas near the center and nuclear activity may be strongly time-variable, and so on. A particularly important stochastic process is merging, which has affected the star-formation history and morphology of many nearby galaxies (Schweizer 1990). Tóth and J. Ostriker (1992; see also J. Ostriker, this volume) have argued that the present thickness of the disk implies that no more than a few percent of the mass inside the solar radius can have been accreted in recent mergers. This merger rate is substantially smaller than expected in standard (cold dark matter, $\Omega = 1$) cosmological models (Carlberg & Couchman 1989), but an alternative possibility is that much of the vertical energy imparted to the disk in the merger is swept out by bending waves.

Unrelaxed galaxies are expected to be more diverse and harder to understand than relaxed galaxies would be, but the study of such systems offers insights into details of cosmology and galaxy formation that would not otherwise be accessible. Another consequence of the dependence of present structure on past history is that large segments of galactic structure, as a field distinct from galaxy formation, are likely to wither away. More and more, it has become impossible to address issues of galactic structure except in the context of galaxy formation.

The importance of astrometry Much of our understanding of galactic structure rests, directly or indirectly, on distances and proper motions of nearby stars and clusters. This is a shaky foundation: there are only about 1000 stars with trigonometric parallaxes accurate to within 15% (Gliese *et al.* 1986), while Oort's B -constant, which measures the mean proper motion of nearby stars in an inertial frame, is only known to within 25% at the $1\text{-}\sigma$ level (Kerr & Lynden-Bell 1986). This situation will improve dramatically with the release of data (in about 1996) from the European astrometric satellite Hipparcos, launched in 1989 (Perryman 1989). Hipparcos is expected to improve the number of stars with useful parallaxes by more than an order of magnitude, in addition to improving the parallax accuracy and greatly reducing systematic errors. A short list of potential applications of Hipparcos data includes studying fine structure in the HR diagram, Galactic kinematics using proper motions of Cepheids, the distance to the Hyades, the bright end of the HR diagram, internal kinematics of clusters and associations, stellar masses, proper motions of globular clusters, structure in the phase-space distribution of spheroid stars, the Oort constants,

the Oort limit, and the velocity ellipsoid and its spatial variation. Even more accurate results could be obtained from a possible Hipparcos II mission, for which the present mission would provide first-epoch positions. Ground-based optical and radio astrometric techniques are also advancing rapidly (Monet 1988); a future milestone for radio astrometry will be measurement of the distance to the Galactic center from the trigonometric parallax of Sgr A*.

I think that the improved accuracy of the Hipparcos survey of the solar neighborhood will “trickle down” to larger scales and thereby resolve many of the puzzles and inconsistencies in our current understanding of Galactic structure.

This is an incomplete listing of some of the particular features of the Galaxy that make it difficult to study. Psychiatrists tell us that admitting our problems is the first step to solving them, and, for similar reasons, thinking further about the fundamental obstacles to progress in studying Galactic structure is likely to be a therapeutic and useful exercise.

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