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J.A. Phillips, J.E. Thorsett, and S.R. Kulkarni

THE DISTRIBUTION OF COMETS AROUND STARS

SCOTT TREMAINE
Canadian Institute for Theoretical Astrophysics
McLennan Labs, University of Toronto
60 St. George St., Toronto M5S 1A7, Canada

ABSTRACT Stars surrounded by planetary systems that formed from disks are likely to have comets as well. Many of the properties of comet clouds around such stars can be deduced from simple dynamical arguments.

INTRODUCTION

Most plausible formation scenarios for the planets around PSR1257+12 imply that they formed, like the terrestrial planets, by accretion from a disk of planetesimals.

Since planet formation is unlikely to be 100% efficient, there should be some planetesimals left in the disk after formation is complete. Residual planetesimals from the outer regions of our solar system ($r \gtrsim 10$ AU) are identified with comets, so I will call residual planetesimals "comets" even though their mass and composition may well differ from those of solar system comets ("asteroids" would be an equally good name).

Comets remaining in the disk after planet formation have two possible dynamical fates. Comets on near-circular orbits that are well-separated from the planets will survive on these orbits with little or no dynamical evolution. Examples of such objects in the solar system include the asteroids, and comets in a hypothetical Kuiper belt beyond Neptune.

On the other hand, orbits that approach too close to a planet are chaotic. The most important class of chaotic orbits is planet-crossing orbits, which are low-inclination orbits with periastron less than and apastron greater than the planet's semi-major axis. Comets on such orbits eventually collide with the planet or the central star or are ejected from the planetary system. In the solar system, it is believed that this ejection process has led to the formation of the Oort cloud, which contains 10^{11}–10^{12} comets at typical semi-major axes of $(2-3)\times 10^4$ AU.

The aim of this paper is to apply our understanding of the formation of the Oort cloud to deduce the properties of Oort-type comet clouds around other stars. Most of the ideas are well-known in the solar system context (Oort 1950, Hills 1981, Duncan et al. 1987, Fernández and Ip 1991); only the application to other planetary systems is new.

In this paper, an Oort-type cloud is defined to be a roughly spherical distribution of bound comets with typical semi-major axis a_f, formed by ejection of comets from a planetary system with characteristic size $a_p \ll a_f$.

FORMATION OF OORT-TYPE COMET CLOUDS

We consider a system containing a central star of mass $M_\star$, a single planet of mass M_p on a circular orbit of radius a_p, and a number of comets on planet-crossing orbits.

Usually, ejection occurs by a gradual random walk or diffusion of the comet orbit towards escape energy (as is also the case for escape of stars from star clusters). This process leads to highly eccentric orbits, since the comet's semi-major axis a becomes large while its periastron q remains comparable to a_p. The comet receives a kick from the planet's gravity each time it passes through periastron; the orbit is chaotic because the kick depends on the phase of the planet in its orbit.

We describe the comet's energy by the variable $x \equiv 1/a$, and let the diffusion coefficient $D_x = \langle(\Delta x)^2\rangle^{1/2}$ be the rms change in x per periastron passage arising from perturbations by the planet (D_x is independent of a so long as $a \gg a_p$, since all such orbits look like parabolas near periastron). Numerical integrations of highly eccentric, low-inclination, planet-crossing orbits yield (Fernández 1981, Duncan et al. 1987)

$$D_x \simeq \frac{10}{a_p}\frac{M_p}{M_\star}. \tag{1}$$

The characteristic diffusion time, in which the energy x changes by of order itself, is then

$$t_{\rm diff} \equiv P\frac{x^2}{D_x^2}, \tag{2}$$

where $P = 2\pi a^{3/2}/(GM_\star)^{1/2}$ is the orbital period. Thus

$$t_{\rm diff}(x) = 1.1 \times 10^9\ {\rm y} \left(\frac{M_\star}{M_\odot}\right)^{3/2} \left(\frac{M_p}{M_\oplus}\right)^{-2} \left(\frac{x}{1\ {\rm AU}^{-1}}\right)^{1/2} \left(\frac{a_p}{1\ {\rm AU}}\right)^2. \tag{3}$$

The diffusion rate speeds up ($t_{\rm diff}$ decreases) as the comet energy increases (x decreases). Initially the comets are on orbits similar to that of the planet, so $x \approx a_p^{-1}$. Thus substantial orbit evolution occurs if and only if

$$t_{\rm ev} \equiv t_{\rm diff}(x = a_p^{-1}) \lesssim t_\star, \tag{4}$$

where $t_\star$ is the age of the planetary system. Thus a necessary condition for the formation of an Oort-type comet cloud is

$$\frac{M_p}{M_\oplus} \gtrsim \left(\frac{M_\star}{M_\odot}\right)^{3/4} \left(\frac{t_\star}{10^9\ {\rm y}}\right)^{-1/2} \left(\frac{a_p}{1\ {\rm AU}}\right)^{3/4}. \tag{5}$$

In an isolated system, most comets on planet-crossing orbits would escape in a few times $t_{\rm ev}$. However in practice, once the orbit becomes large, the torque from the Galactic tide changes the orbital angular momentum and thus the periastron distance q. Once q is a few times a_p, planetary perturbations become ineffective, and the random walk of the comet's energy ceases: planet-induced diffusion in energy at fixed angular momentum is replaced by Galaxy-induced evolution in angular momentum at fixed energy. For highly eccentric orbits, the specific angular momentum $L = (2GM_\star q)^{1/2} \approx (2GM_\star a_\mathrm{p})^{1/2}$, and the torque per unit mass is $dL/dt = 5\pi k G\rho a^2$ where ρ is the local Galactic mass density and k is a geometrical factor that varies between 0 and 1 depending on the orientation of the orbit (Heisler and Tremaine 1986). For a typical orientation, the time required for the Galactic tide to change the angular momentum by of order itself is then (Duncan et al. 1987)

$$t_\mathrm{tide} = 1\times10^{15}\ \mathrm{y}\left(\frac{M_\star}{M_\odot}\right)^{1/2}\left(\frac{\rho}{0.15M_\odot\ \mathrm{pc}^{-3}}\right)^{-1}\left(\frac{q}{1\ \mathrm{AU}}\right)^{1/2}\left(\frac{x}{1\ \mathrm{AU}^{-1}}\right)^{2}. \quad (6)$$

The density $0.15M_\odot\ \mathrm{pc}^{-3}$ is a compromise between the values obtained for the solar neighborhood by Kuijken (1991) and Bahcall et al. (1992).

Tidal torques freeze the comet out of the random walk in energy at the semi-major axis a_f where $t_\mathrm{tide}(q = a_\mathrm{p}) = t_\mathrm{diff}$, that is,

$$a_\mathrm{f} = 1\times10^{4}\ \mathrm{AU}\left(\frac{M_\star}{M_\odot}\right)^{-2/3}\left(\frac{\rho}{0.15M_\odot\ \mathrm{pc}^{-3}}\right)^{-2/3}\left(\frac{M_\mathrm{p}}{M_\oplus}\right)^{4/3}\left(\frac{a_\mathrm{p}}{1\ \mathrm{AU}}\right)^{-1}. \quad (7)$$

Many comets escape on their next orbit if $x = 1/a \lesssim D_x$. Thus freezing is an effective barrier to the escape process only if

$$1/a_\mathrm{f} \gtrsim D_x; \quad (8)$$

this condition implies that many of the comets escape unless

$$\frac{M_\mathrm{p}}{M_\oplus} \lesssim 1.7\left(\frac{M_\star}{M_\odot}\right)^{5/7}\left(\frac{\rho}{0.15M_\odot\ \mathrm{pc}^{-3}}\right)^{2/7}\left(\frac{a_\mathrm{p}}{1\ \mathrm{AU}}\right)^{6/7}. \quad (9)$$

The frozen comets form an extended disk in the plane of the planets, which is steadily thickened by perturbations from passing stars and the Galactic tide. The timescale to convert the disk into a roughly spherical cloud is simply t_tide, evaluated at $q = a = a_\mathrm{f}$. The condition that this timescale is less than the age $t_\star$ can be shown to be the same as (5). Thus comet clouds formed by this process should be approximately spherical.

The cloud cannot extend beyond the Roche surface of the star, set by the tidal field of the Galaxy. For the Sun the Roche surface has semi-axes 1.47 pc, 0.98 pc, and 0.68 pc, with the shortest axis normal to the Galactic plane (Antonov and Latyshev 1972, Heisler and Tremaine 1986); these values are for $\rho = 0.15M_\odot\ \mathrm{pc}^{-3}$. The size of the Roche surface can be roughly estimated by comparing the tidal force between two points separated by a vertical distance z, $4\pi G\rho z$, to the star's gravitational force at that distance, $GM_\star/z^2$. If we denote the distance at which these forces are equal as the Roche or tidal radius a_t, we have

$$a_\mathrm{t} = 1.7\times10^{5}\ \mathrm{AU}\left(\frac{M_\star}{M_\odot}\right)^{1/3}\left(\frac{\rho}{0.15M_\odot\ \mathrm{pc}^{-3}}\right)^{-1/3}. \quad (10)$$

Planets cannot survive outside the tidal radius, so

$$a_{\rm p} \lesssim a_{\rm t}. \tag{11}$$

A comet cloud can only form if the freezing semi-major axis is less than the tidal radius, $a_{\rm f} \lesssim a_{\rm t}$, which implies

$$\frac{M_{\rm p}}{M_\oplus} \lesssim 8 \left(\frac{M_\star}{M_\odot}\right)^{3/4} \left(\frac{\rho}{0.15 M_\odot \ {\rm pc}^{-3}}\right)^{1/4} \left(\frac{a_{\rm p}}{1 \ {\rm AU}}\right)^{3/4}, \tag{12}$$

and the formation process as described here only applies if the freezing semi-major axis exceeds the planet's semi-major axis, $a_{\rm f} \gtrsim a_{\rm p}$, which implies

$$\frac{M_{\rm p}}{M_\oplus} \gtrsim 10^{-3} \left(\frac{M_\star}{M_\odot}\right)^{1/2} \left(\frac{\rho}{0.15 M_\odot \ {\rm pc}^{-3}}\right)^{1/2} \left(\frac{a_{\rm p}}{1 \ {\rm AU}}\right)^{3/2}. \tag{13}$$

An additional constraint is that the comets must not impact the planet before they diffuse to the cloud (I am indebted to Al Harris for stressing the importance of this process). Comets initially on orbits with $a \approx a_{\rm p}$ make roughly $N = (x_{\rm p}/D_x)^2$ orbits before reaching the cloud. If their typical inclination is $\Delta\theta$ and the escape speed from the planet is less than the orbital speed so that gravitational focusing is negligible, the chance of an impact on each periastron passage is roughly $p = (R_{\rm p}/a_{\rm p})^2/\Delta\theta$, where $R_{\rm p}$ is the planetary radius. In order that most of the comets do not strike the planet we must have $Np \lesssim 1$, or

$$\frac{M_{\rm p}}{M_\oplus} \gtrsim 13 \left(\frac{M_\star}{M_\odot}\right)^{3/2} \left(\frac{a_{\rm p}}{1 \ {\rm AU}}\right)^{-3/2} \left(\frac{\rho_{\rm p}}{3 \ {\rm g \ cm}^{-3}}\right)^{-1/2} \left(\frac{\Delta\theta}{0.1 \ {\rm rad}}\right)^{-3/4}. \tag{14}$$

where $\rho_{\rm p}$ is the planet density.

Comets are ejected from the cloud by gravitational perturbations from passing stars and other objects. The half-life of a comet orbiting a star in a region with kinematics similar to the solar neighborhood is (Weinberg et al. 1987, Figure 7a)

$$t_{1/2} = 10^{10} \ {\rm y} \left(\frac{M_\star}{M_\odot}\right) \left(\frac{10^4 \ {\rm AU}}{a}\right) \left(\frac{\rho}{0.15 M_\odot \ {\rm pc}^{-3}}\right)^{-1}. \tag{15}$$

This result neglects the uncertain contribution from molecular clouds, and is strictly valid only for $a \ll a_{\rm t}$. Replacing a by the cloud radius $a_{\rm f}$ and requiring that $t_{1/2}$ exceed the age of the system $t_\star$, we obtain the constraint

$$\frac{M_{\rm p}}{M_\oplus} \lesssim 6 \left(\frac{M_\star}{M_\odot}\right)^{5/4} \left(\frac{t_\star}{10^9 \ {\rm y}}\right)^{-3/4} \left(\frac{\rho}{0.15 M_\odot \ {\rm pc}^{-3}}\right)^{-1/4} \left(\frac{a_{\rm p}}{1 \ {\rm AU}}\right)^{3/4}. \tag{16}$$

For Population II (halo) stars this constraint must be modified because the relative velocities are higher than in the disk. In this case, the appropriate constraint is strongly dependent on the unknown composition of the dark halo; if the dark matter is in objects of stellar mass or less, the constraint is much weaker than the constraint (16) for disk stars.

APPLICATIONS

The solar system

Figure I shows the constraints (5), (9), (11), (12), (13), (14), and (16) for parameters appropriate for the solar system: $M_\star = 1M_\odot$, $t_\star = 4.5 \times 10^9$ y, and $\rho = 0.15M_\odot\ \mathrm{pc}^{-3}$. The positions of the eight major planets are also plotted, and the region of parameter space allowed by the constraints is stippled. The figure exhibits some well-known and some new results:

- Comets crossing the orbits of Jupiter and Saturn cannot create an Oort-type cloud: the planets are so massive that most comets are ejected. Thus Oort cloud comets must come from the Uranus-Neptune region.
- The typical semi-major axis of the comet cloud formed by Uranus or Neptune is $a_f \approx 10^4$ AU, close to the semi-major axes of new comets (the observed semi-major axes of new comets are somewhat larger, $(2-3) \times 10^4$ AU, because comets with smaller semi-major axes are not visible from Earth until after they have been perturbed by Jupiter).
- Venus or Earth could not efficiently form an Oort-type cloud, even if the massive outer planets were not present: most comets on Venus- or Earth-crossing orbits collide with these planets before reaching the cloud.
- At any planetary semi-major axis a_p there is only a narrow range (less than a factor 5) of planetary masses that are large enough to create an Oort-type cloud within the age of the solar system, yet small enough not to eject most comets. Within this allowed range, the typical size of the comet cloud, a_f, is almost independent of a_p. Outside this range, cloud formation can still occur, but with low efficiency.

PSR1257+12

Figure II shows the same constraints for parameters appropriate to PSR1257+12: $M_\star = 1.4M_\odot$, $t_\star = 8 \times 10^8$ y, and planetary semi-major axes and masses $a_{p1} = 0.36$ AU, $a_{p2} = 0.47$ AU, $M_{p1} = 3.4M_\oplus$, $M_{p2} = 2.8M_\oplus$ (for $\sin i = 1$; if $\sin i < 1$ the masses are larger).

The constraints on the planets that can efficiently form an Oort-type cloud are similar to those in the solar system. However, the allowed range of parameter space is even smaller, mainly because more massive planets are needed to create the cloud within the shorter age of the system.

Comets crossing the orbits of the planets P1 and P2 would mostly be ejected or collide with the planets. Those that did manage to reach the cloud would be found at semi-major axes of a few times 10^4 AU.

HALO STARS

Figure III shows constraints on the formation of a comet cloud for a typical halo star, $M_\star = 0.5M_\odot$, $t_\star = 10^{10}$ y, $\rho = 0.01M_\odot\ \mathrm{pc}^{-3}$. The constraints are qualitatively similar to those for the solar system, but the planets that are most effective in producing a cloud are about a factor of three smaller at a given semi-major axis.

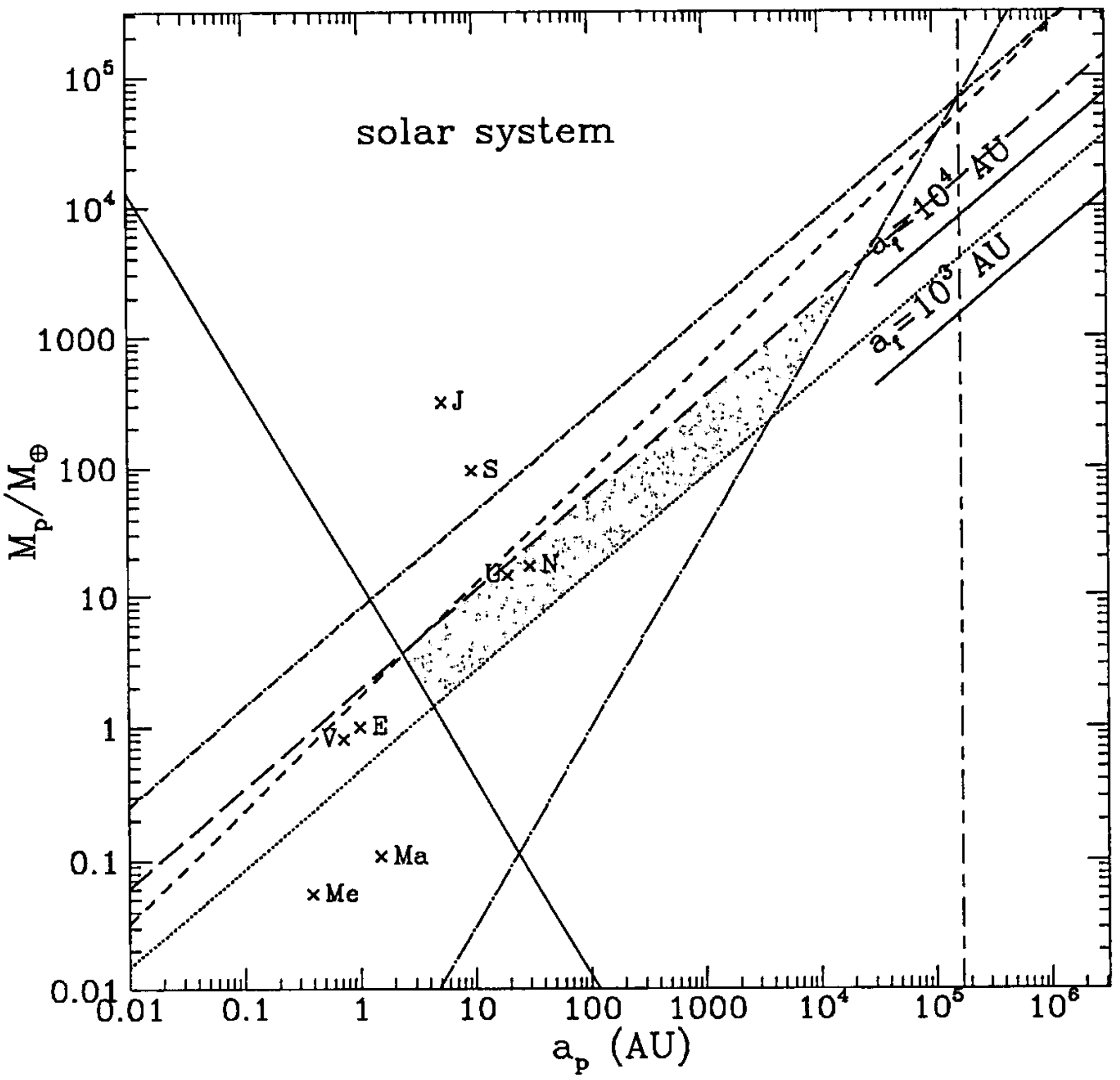

FIGURE I A plot of planet mass against planet semi-major axis showing the regions that are able to create an Oort-type cloud efficiently, for a star of mass $1M_\odot$ and age 4.5×10^9 y that is located in the solar neighborhood. The constraints plotted are: the diffusion time must be less than the age (eq. 5, dotted line); the freezing energy must exceed the rms energy change per orbit (eq. 9, short-dash line); the planet's semi-major axis must be less than the tidal radius (eq. 11, vertical short dash-long dash line); the freezing semi-major axis must be less than the tidal radius (eq. 12, dot-short dash line); the freezing semi-major axis must exceed the planet's semi-major axis (eq. 13, dot-long dash line); the comets must not impact the planet (eq. 14, solid line); the comets must not be ejected by passing stars (eq. 16, long-dash line). Lines have also been marked showing the location of freezing semi-major axes of 10^3 and 10^4 AU; although the lines have been truncated for clarity, they can be extrapolated across the whole figure.

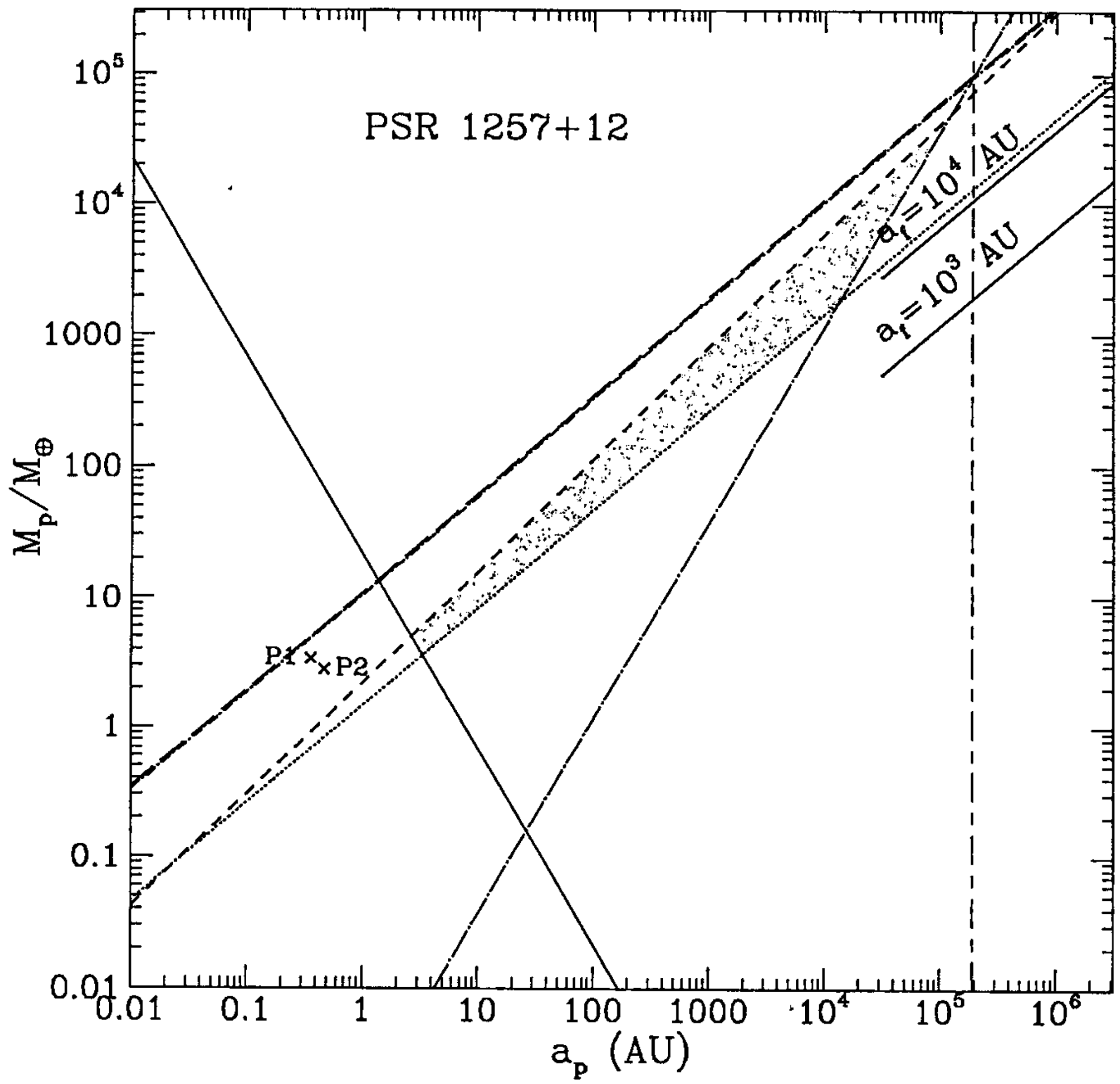

FIGURE II This plot is similar to Figure I, except the parameters are chosen to match those of the pulsar: a $1.4M_\odot$ star of age 0.8×10^9 y, located in the solar neighborhood.

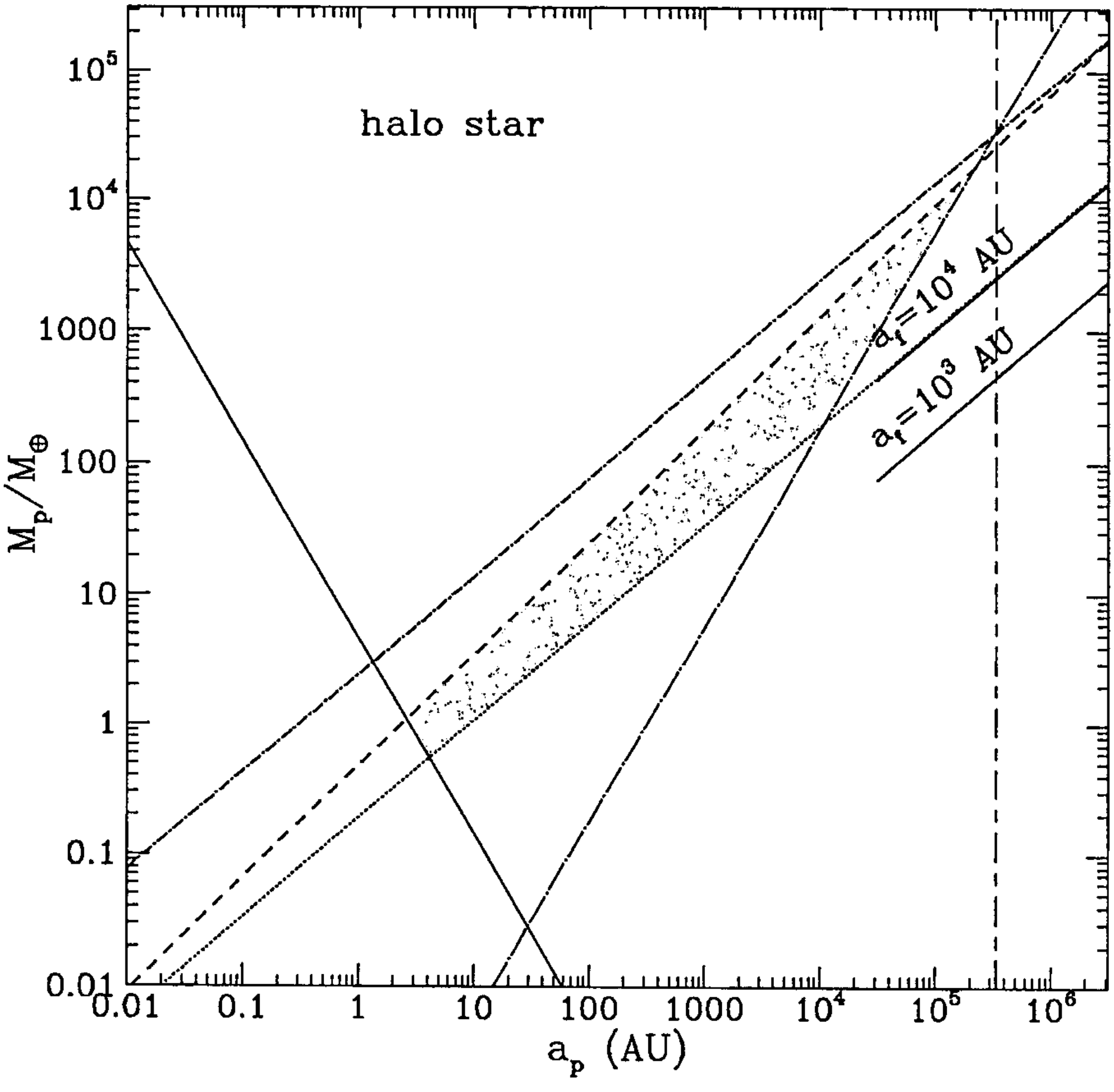

FIGURE III This plot is similar to Figure 1, except the parameters are chosen to represent those of halo stars rather than the solar system: a $0.5 M_\odot$ star with an age of 10^{10} y, and a local density of $0.01 M_\odot$ pc^{-3}. The constraint that comets are not ejected by passing stars or other objects is not plotted, since the effects of such stars are weak and the number density and mass of the objects composing the halo are not known.

SUMMARY

When planets form from a disk of planetesimals, there is likely to be a residual population of small bodies after planet formation is complete. It is natural to call these residual planetesimals "comets" or "asteroids", although their mass and composition may differ from their namesakes in our own planetary system.

Comets are likely to be found in one of the following configurations:

- A disk or belt of comets on regular (low-inclination, low-eccentricity) orbits that are well-separated from the planets (e.g. the asteroid belt).
- A cloud of comets scattered to large distances by planetary perturbations (e.g. the Oort cloud).

A variety of other configurations is possible. For example, a disk of comets on planet-crossing orbits can be present, if the planetary masses are small enough that the lifetime of such orbits is longer than the stellar lifetime. Another possibility is a spherical distribution of intermingled comets and planets, which can occur if the planets form at large enough radius that their orbits are isotropized by tidal forces after planet formation is complete.

If the configuration of the planets is known, then many of the properties of the comet distribution, in particular of any Oort-type cloud, can be predicted by the same dynamical arguments that successfully explain the structure of the solar system cloud. This paper presents a preliminary outline of the likely properties of Oort-type clouds around stars other than the Sun.

The theory presented here ignores many important issues, such as: the effect of two or more planets (note that comets are injected into the Oort cloud mainly by Uranus and Neptune but removed mainly by Jupiter and Saturn); survival of the comets in the stellar radiation field; the rate of collisions with the star; perturbations by passing molecular clouds; tidal shocks as halo stars pass through the disk; loss of comets when external perturbations bring the periastron back into the planetary system; and so forth.

The stippled areas in the figures show parts of parameter space in which formation of an Oort-type cloud is efficient, that is, in which most comets initially on low-inclination planet-crossing orbits will now be found in the cloud. Clouds may still be present even if the parameters of the star and planets lie outside the stippled region, but in this case cloud formation is inefficient.

The following conclusions indicate the results that can be derived from this type of analysis:

- Stars similar to the Sun can only generate an Oort-type cloud efficiently if they contain planets at semi-major axis $a_{\rm p} \gtrsim 3$ AU, whose masses lie within a factor of two or so of the relation $M_{\rm p}/M_\oplus = (a_{\rm p}/1\ {\rm AU})^{3/4}$. Thus Oort-type clouds are not expected to be found around every star, and may in fact be rather rare.
- The typical semi-major axis in an Oort-type cloud around a solar-type star will be $a_{\rm f} \approx 10^4$ AU.
- Halo stars require planets that are smaller by about a factor of 3 at a given semi-major axis, and produce a somewhat larger cloud, $a_{\rm f} \approx 2 \times 10^4$ AU.

- From equations (5), (9), and (11) we may deduce that no Oort-type cloud can be formed if $t_\star \lesssim 3 \times 10^7$ y$(\rho/0.15 M_\odot\ \mathrm{pc}^{-3})$. Hence young, massive stars ($M \gtrsim 7M_\odot$) do not have Oort-type comet clouds.

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