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The Dynamical Evidence for Dark Matter

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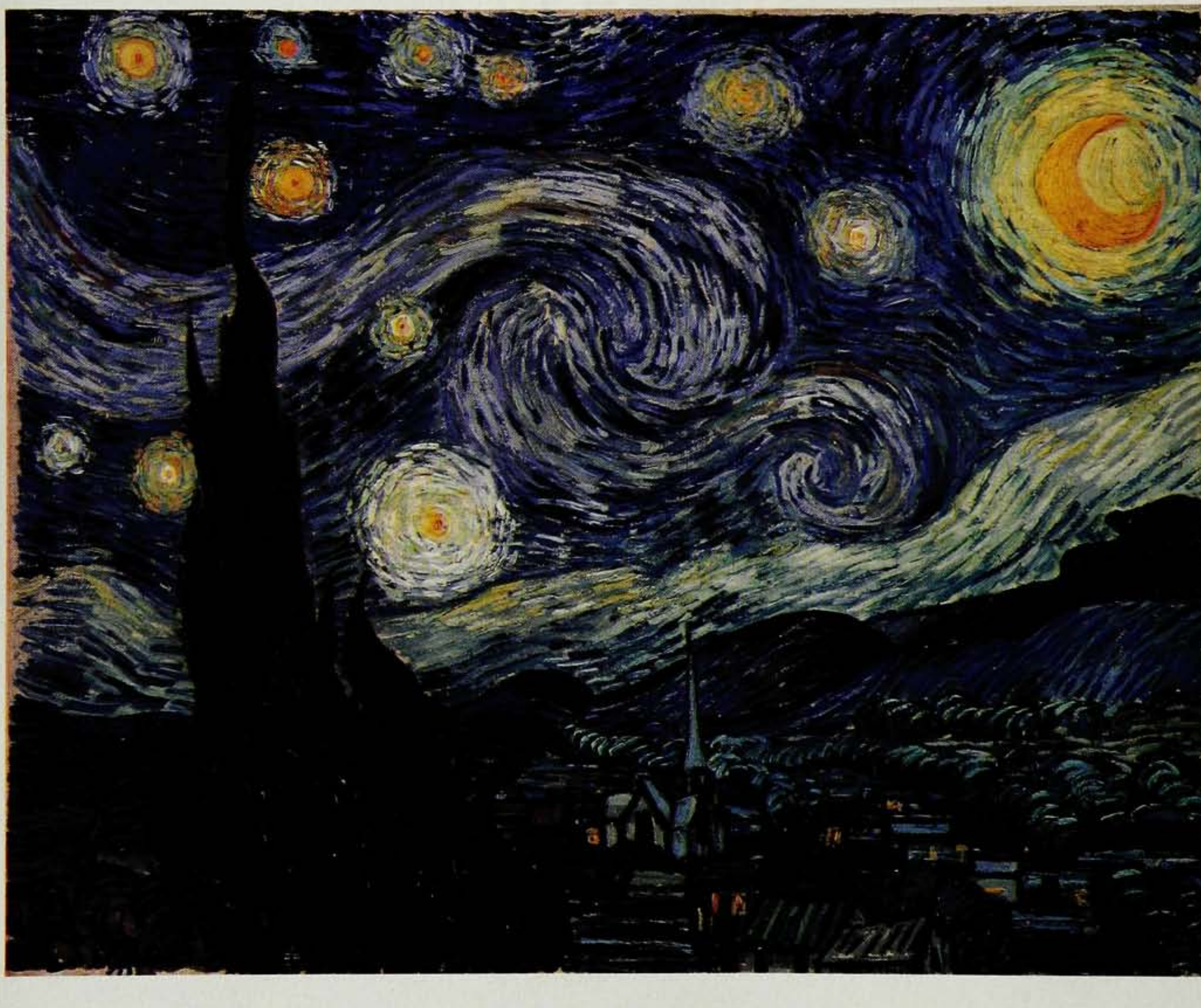
THE DYNAMICAL EVIDENCE FOR DARK MATTER



OIL ON CANVAS, 29×36 1/4". COLLECTION, THE MUSEUM OF MODERN ART, NEW YORK. ACQUIRED THROUGH THE LILLIE P. BLISS BEQUEST

'The Starry Night,' by Vincent van Gogh. The 1889 oil painting suggests how the night sky might look if all of the mass in the universe were luminous. Observations of galaxy dynamics and modern theories of galaxy formation imply that the visible components of galaxies, composed mostly of stars, lie at the centers of vast halos of dark matter that may be 30 or more times larger than the visible galaxy. In most models of galaxy formation, the halos are comparable in size to the distance between galaxies. The halos form as a result of the gravitational instability of small density fluctuations in the early universe; the star-forming gas collects at the minima of the halo potential wells. Infall of outlying material into existing halos and mergers of small halos with larger ones continue at the present time. If the halos were visible to the naked eye, there would be well over 1000 nearby galaxies with halo diameters larger than the full Moon. **Figure 1**

Studies of the dynamics of galaxies show that at least 90% of the mass in the universe is in some invisible, unknown form.

Scott Tremaine

Almost all of our information about the universe beyond Earth comes from photons—visible photons from stars, x-ray photons from hot plasmas, radio photons from the 21-cm hyperfine transition in hydrogen, microwave photons from the cosmic background radiation and so forth.

It would be folly to assume that all the matter in the universe emits detectable photons. Thus we should not be surprised if the mass of a galaxy or other astronomical system, as measured by its gravitational field, exceeds the sum of the masses of those of its components that shine brightly enough to be detected in our telescopes. The difference between this "luminous mass" and the total mass is ascribed to "dark matter"—matter whose existence is inferred solely from its gravitation.[1]

An early example of this reasoning was the prediction in 1846 of the existence and location of Neptune from unexplained residuals in the motion of Uranus. Another example from the solar system was the anomalous precession of Mercury's perihelion. A hypothetical planet ("Vulcan"), or else a ring of material, inside Mercury's orbit was invoked to explain this anomaly, but Einstein showed in 1916 that it was a consequence of general relativity rather than of dark matter. This is a cautionary reminder that dark matter may sometimes be explained away by revisions to the accepted laws of physics.

At present there is no significant dynamical evidence for dark matter in the solar system. On larger scales, however, the story is quite different. There is convincing evidence not just that dark matter is present but that most of the mass in galaxies is dark. The visible parts of galaxies, composed mainly of stars, are surrounded by extended halos of dark matter that may be a factor of 30 or more larger in both mass and size. Van Gogh's famous painting "The Starry Night" (figure 1) provides a surprisingly accurate view of what the dark halos might look like if they were visible. The average mass density of the dark matter could exceed the critical value needed to close the universe.

An equally remarkable conclusion, based on nucleosynthesis arguments, is that most of the dark matter—and hence most of the mass in the universe—is not composed of protons or neutrons. Thus the material that makes up the stars that we see and the everyday world that we know is only a minor pollutant in a sea of invisible material of unknown nature.

The solar neighborhood

The first natural place beyond the solar system to look for dark matter is the solar neighborhood—an imaginary volume centered on the Sun that is large enough to contain plenty of stars for statistical analyses but small enough compared with the size of the Galaxy that the bulk properties of the stellar distribution are constant within it.

The distance of the nearest star to the Sun is 1.3 parsecs (1 pc is 3.086×10^{13} km). About the smallest volume containing a statistically useful sample of stars is a Sun-centered sphere of radius 10 pc, in which there are 300 known stars.[2] An instructive exercise is to divide these into an inner sample of 61 stars within 5 pc and an outer sample of 239 stars between 5 and 10 pc from the Sun. The corresponding densities are 0.12 and 0.065 stars per cubic parsec. Since the density ought to be constant over such small distances, the drop in density by a factor of two from the inner to the outer sample implies that the outer sample is seriously incomplete. Thus even at the smallest interstellar distances, many of the stars are so faint that they have yet to be discovered, which is a hint that substantial dark mass might lurk in faint stars.

Most stars are in a state of thermal equilibrium, in which energy generated by hydrogen fusion is balanced by heat lost through thermal radiation. However, below a transition mass M_c of 0.08 times the mass $M_\odot$ of the Sun, stars cannot fuse hydrogen, as their electrons become degenerate before they are dense and hot enough for fusion to proceed. The luminosity of stars with $M<M_c$, usually called "brown dwarfs," is supplied by slow gravitational contraction rather than by fusion[3] and hence is much smaller than the luminosity of hydrogen-burning stars. (The luminosity drops by more than two orders of magnitude between $0.10M_\odot$ and $0.07M_\odot$.) Brown dwarfs are so faint that they would be very difficult to detect even if they were much more numerous than the brighter, hydrogen-burning stars. Hence they are a natural candidate for dark mass.

We can attempt to estimate the number density of brown dwarfs by extrapolating the number density of brighter stars. The plausible assumption in this extrapolation is that the transition mass M_c plays no special role in the physics of star formation: Since the transition mass involves nuclear physics and star formation probably does not, the rates of formation of stars above and below M_c should be similar. The luminosity $L(M)$ of a star of mass M can be computed from stellar structure theory and checked by observations of binary star orbits. Let $\phi(L)\,\mathrm{d}L$ be the number density of stars with luminosity in the range $[L, L+\mathrm{d}L]$, as determined from star catalogs. Then

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the number density $n(M)\,\mathrm{d}M$ of stars with mass in the range $[M, M+\mathrm{d}M]$ is given by

$$n(M) = \phi[L(M)]\,\frac{\mathrm{d}L(M)}{\mathrm{d}M}$$

Figure 2 shows the number density derived in this way. The density becomes quite uncertain as we approach the transition mass M_c, both because $\mathrm{d}L/\mathrm{d}M$ becomes very large and because measurement of $\phi(L)$ becomes harder and harder at low luminosities. The figure suggests that $n(M)$ is relatively flat for masses below M_c, which would imply that brown dwarfs contain a negligible fraction of the mass in the solar neighborhood. A sharp upturn in $n(M)$ below about $0.15M_\odot$ is not excluded by the data, although there is no reason to suppose that it is present.

Fortunately there is a dynamical method of measuring the total mass density in this region. The stars in the solar neighborhood belong to the Galactic disk, which has a radius of about 10 kpc but a thickness of only a few hundred parsecs. Because the disk is so thin, it can be approximated as an infinite slab. The gravitational potential of the slab is $U(z)$, where z is the distance perpendicular to the slab's midplane. The phase-space density $f(\mathbf{x},\mathbf{v},t)$ of stars of a given type obeys the collisionless Boltzmann equation, which expresses the conservation of phase-space density along a trajectory:[4]

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla_x f - \nabla_x U\cdot\nabla_v f = 0 \qquad (1)$$

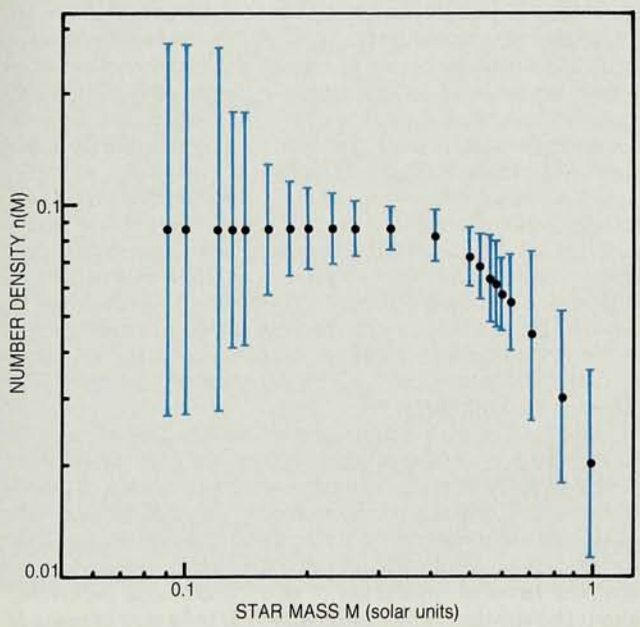
Here ∇_x and ∇_v denote gradients with respect to position and velocity. Assuming slab symmetry and a stationary distribution (that is, no dependence on x, y or t), we multiply the equation by v_z and integrate over velocity to get

$$\frac{\mathrm{d}}{\mathrm{d}z}\nu\sigma_z^2 = -\nu\frac{\mathrm{d}U}{\mathrm{d}z} \qquad (2)$$

where $\nu(z) = \int f(z,\mathbf{v})\mathrm{d}\mathbf{v}$ is the number density of stars and $\sigma_z{}^2(z) = \int v_z{}^2 f(z,\mathbf{v})\mathrm{d}\mathbf{v}/\nu(z)$ is their mean-square velocity in the z direction. Thus measurements of the number density and velocity dispersion of any given type of star as a function of height above the Galactic midplane determine the potential $U(z)$ through equation 2 and the mass density $\rho(z)$ through Poisson's equation. The method is difficult to apply in practice, mostly because statistical uncertainties in ν and $\sigma_z{}^2$ are amplified by the two differentiations needed to derive the density.

This argument was first used in 1922 by Jacobus C. Kapteyn, who deduced that the total density in the solar neighborhood was no more than a factor of 2 or so larger than the density in visible stars. Modern estimates have not substantially changed this conclusion: Two recent studies found[5] that the ratio of the total density to the density in known objects (stars and gas) was 1.0 ± 0.3 and $2.6^{+1.9}_{-1.2}$ ($1-\sigma$ limits). Thus neither the extrapolation of the number density of stars per unit mass $n(M)$ nor the dynamical estimates strongly suggest that there is substantial dark matter in the solar neighborhood, although a dark matter density similar to the density in known objects is not excluded.

A simple way to parameterize the ratio of dark to luminous matter is the mass-to-light ratio Υ, which is usually measured in solar units: $\Upsilon_\odot = 1$ solar mass/1 solar luminosity, or about 0.5 in cgs units. Here "light" usually means light in the visible part of the spectrum. This is the natural wavelength range to use, since stars are the largest known contributor to the mass in galaxies and they are most easily studied in visible light. In the likely case that there is no dark matter in the solar neighborhood, its mass-to-light ratio is about $1.5\Upsilon_\odot$—close to unity since the Sun is an average star—but the larger of the dynamical estimates above allows a value up to $7\Upsilon_\odot$.

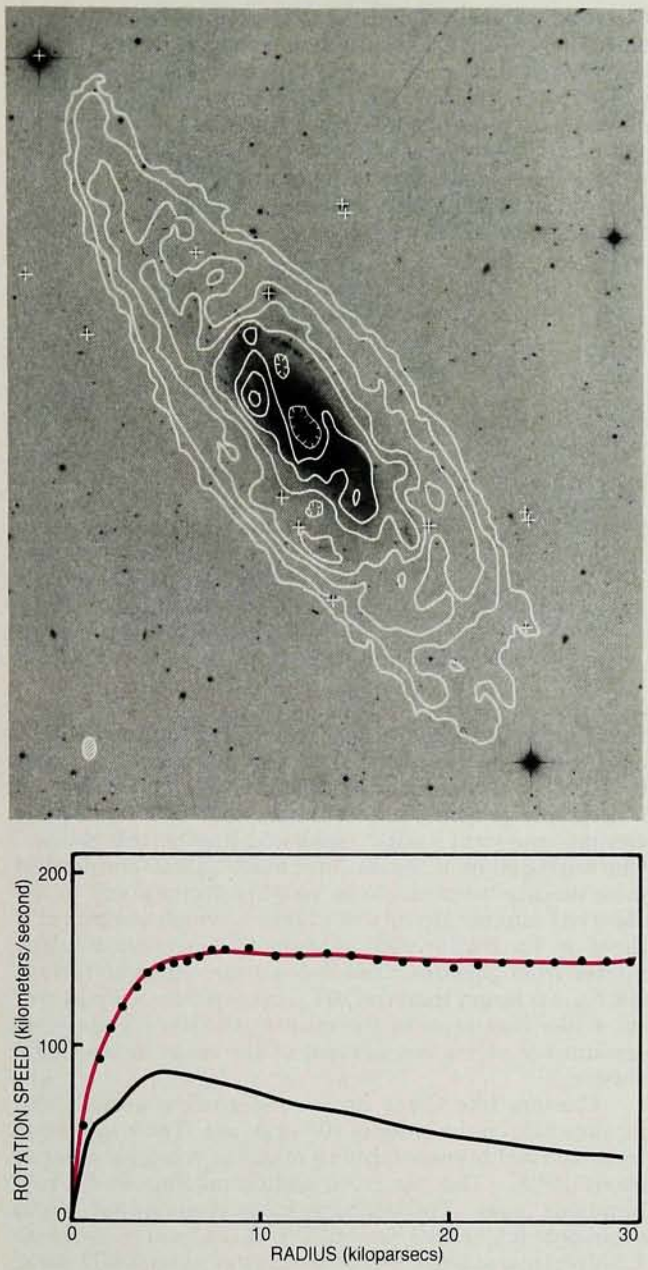


Number density of stars in the solar neighborhood as a function of mass. The black points denote a smooth fit to the data, but other curves lying within the error bars are also consistent with the data. Units are stars per cubic parsec per solar mass. (Adapted from P. Kroupa, C. A. Tout, G. Gilmore, Mon. Not. R. Astron. Soc. **244**, 76, 1990.) **Figure 2**

Galaxy rotation curves

The stars in most galaxies lie mainly in a thin disk and travel on nearly circular orbits around the galactic center. The circular speed v_c at a given radius R can be determined from the Doppler shift of spectral lines in either the integrated starlight or the interstellar gas that rotates with the stars.

To look for dark matter in a galaxy, we compare the observed centripetal acceleration $v_c{}^2/R$ with the calculated gravitational acceleration due to the luminous mass,



Disk galaxy and its rotation curve. Top: An optical image of the spiral galaxy NGC 3198 with a superimposed contour map of the column density of hydrogen gas. The hydrogen is detectable out to much larger radii than the stars, and so provides a better probe of the mass distribution at large radii. The shaded ellipse at lower left indicates the resolution of the hydrogen observations. Bottom: The rotation speed of the gas as a function of radius (black points), together with the circular speed derived from the assumption that all of the mass is in the visible stars and gas (black curve). The red curve shows the best-fit circular speed derived by assuming that the galaxy also contains a spherical dark halo with a density that follows the empirical law $\rho \propto 1/(1 + r^2/r_c^{\,2})$, where r_c is the core radius. (Adapted from T. S. van Albada *et al.*, Astrophys. J. **295**, 305, 1985, and S. M. Kent, Astron. J. **93**, 816, 1987.) **Figure 3**

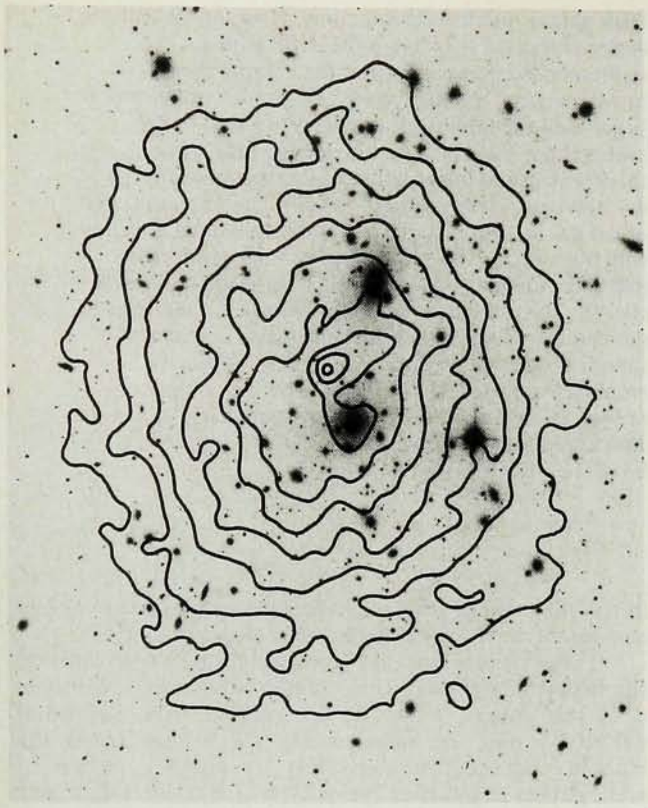
which is mostly in stars. Figure 3 shows the observed rotation curve $v_c(R)$ in the disk galaxy NGC 3198, along with the circular speed derived from the assumptions that the disk surface brightness is proportional to the surface density and that there is no dark mass. To obtain this particular curve, the mass-to-light ratio of the disk was chosen to be as large as possible. (With any larger value, the predicted speed would exceed the observed speed in the inner parts.) Even with this extreme assumption, the predicted speed is more than a factor of 3 lower than the observed speed at the outermost measured point. (At larger radii the density of interstellar gas is too low to permit measurement of the velocity.) This implies that the calculated gravitational field from the disk is too small by a factor of 10 to account for the observed rotation.

We conclude that stars and other luminous mass make up less than 10% of the total mass in that galaxy. The remaining 90% or more is dark matter. Most of the dark matter must be located at radii larger than that of the stars; otherwise the rotation speed would exhibit Keplerian behavior—that is, $v_c(R) = (GM/R)^{1/2} \propto R^{-1/2}$—in the outer parts, whereas in fact the speed is more or less constant over the outer two-thirds of the galaxy.

These results can also be considered a lower limit on the mass-to-light ratio. The limit depends on the distance d to the galaxy, which is determined from its radial velocity v and the relation $v = H_0 d$, where H_0 is the Hubble constant. (More precisely, $v = H_0 d + v_p$, where v_p is the galaxy's "peculiar" velocity, which is typically under 500 km/sec in magnitude; a galaxy's peculiar velocity is defined in this equation as the difference between its actual velocity and the Hubble velocity.) We shall use $H_0 = 75\ \mathrm{km\ sec^{-1}\ Mpc^{-1}}$, although respectable estimates[6] of H_0 range from below 50 to above 100. With this value for the Hubble constant, the distance of NGC 3198 is 9.2 Mpc and its mass-to-light ratio Υ is at least $40\Upsilon_\odot$, which is about a factor of 20 larger than the mass-to-light ratio in the solar neighborhood. Similar flat rotation curves and mass-to-light ratios are found in most disk galaxies, including our own.[7]

The shapes of rotation curves suggest that the dark matter is distributed in extended halos that surround the visible stars. A simple empirical model for the halo density is a spherical distribution $\rho(r) = \rho_0/(1 + r^2/r_c^{\,2})$, where the core radius r_c and central density ρ_0 are fitting parameters. The rotation curve fits produced by this model are quite good, as figure 3 shows. The corresponding dark mass within radius r is proportional to r for $r \gg r_c$; evidently this growth must stop at some sufficiently large radius r_{max}, since otherwise the mass of the galaxy would be infinite, but the rotation curves imply only that r_{max} must lie near or beyond the last measured points on the rotation curve. Less accurate than rotation-curve analysis, such methods as measurement of the relative velocities of galaxy pairs or the kinematics of satellite galaxies[8] suggest that r_{max} is 100 kpc or even larger. Thus we reach two remarkable conclusions: The total mass and extent of ordinary galaxies are almost completely unknown, and between 90% and 99% of the mass in galaxies is dark.

Before about 1970, measurements of rotation curves were restricted to the inner parts of galaxies. It was natural for observers to extrapolate the rotation curves assuming Keplerian behavior beyond the last measured point, since most of the light from the galaxy was contained well within that point. This extrapolation gave a direct—but spurious—estimate of the total mass of the galaxy. In retrospect, it is remarkable that the dangers of this extrapolation were not more clearly recognized. By the early 1970s, high-resolution observations of interstel-



Cluster of galaxies. Superimposed on the optical image of the Coma cluster is a contour map of the x-ray surface brightness measured by the Einstein satellite. The distance between the two most prominent galaxies is 7 minutes of arc, or 190 kiloparsecs. (Courtesy of William Forman and Christine Jones, Harvard–Smithsonian Center for Astrophysics, Cambridge, Mass.) **Figure 4**

lar gas began to provide flat rotation curves that clearly showed that the mass was not contained solely in the visible disk stars.[7] By now, rotation curves of dozens of galaxies provide convincing evidence that most of the mass in disk galaxies is dark.

Clusters of galaxies

Galaxies are not distributed uniformly throughout the universe but instead have a rich hierarchy of structure ranging from binary galaxies through groups containing a few galaxies to clusters containing thousands of galaxies. One of the largest nearby clusters is the Coma cluster, shown in figure 4.

The central regions of clusters of galaxies are the largest equilibrium structures in the universe and hence are natural sites to prospect for dark mass. The phase-space density $f(\mathbf{x},\mathbf{v},t)$ of cluster galaxies obeys the collisionless Boltzmann equation (equation 1), which can be analyzed by assuming spherical symmetry (the substantial ellipticity of many clusters does not strongly affect the results), no time dependence and an isotropic velocity distribution. Then multiplying equation 1 by the radial velocity v_r and integrating over velocity space yields a result reminiscent of the hydrostatic equilibrium equation,

$$\frac{\mathrm{d}(\nu\sigma^2)}{\mathrm{d}r} = -\nu\frac{\mathrm{d}U}{\mathrm{d}r} \tag{3}$$

Here $\nu(r)$, defined as $\int f(r,v)\mathrm{d}\mathbf{v}$, is the number density of galaxies at radius r; $\sigma^2(r)$, defined as $\frac{1}{3}\int v^2 f(r,v)\mathrm{d}\mathbf{v}/\nu(r)$, is their mean-square velocity in one dimension; and $U(r)$ is the gravitational potential.

Observations yield the projected number-density distribution and the velocity dispersion along the line of sight, and these determine $\nu(r)$ and $\sigma(r)$. The potential $U(r)$ then follows from equation 3, and we can estimate the mass $M(r)$ contained within radius r through $GM(r)/r^2 = \mathrm{d}U/\mathrm{d}r$.

This analysis shows that the mass contained within $r = 1.3$ Mpc of the center of the Coma cluster is about $8\times10^{14}M_\odot$. Our assumption that the velocity distribution is isotropic is somewhat arbitrary, but other plausible assumptions yield similar results, at least at this radius.[9] One selling point of the isotropic model is that the derived mass density turns out to be roughly proportional to the observed number density of galaxies, which is natural if there is no cluster-wide process that segregates dark matter from galaxies. The derived mass-to-light ratio is $300\Upsilon_\odot$, far larger than the $10\Upsilon_\odot$ expected for a mixture of stars like that seen in the cluster galaxies. Thus stars account for only a few percent of the mass in the Coma cluster.

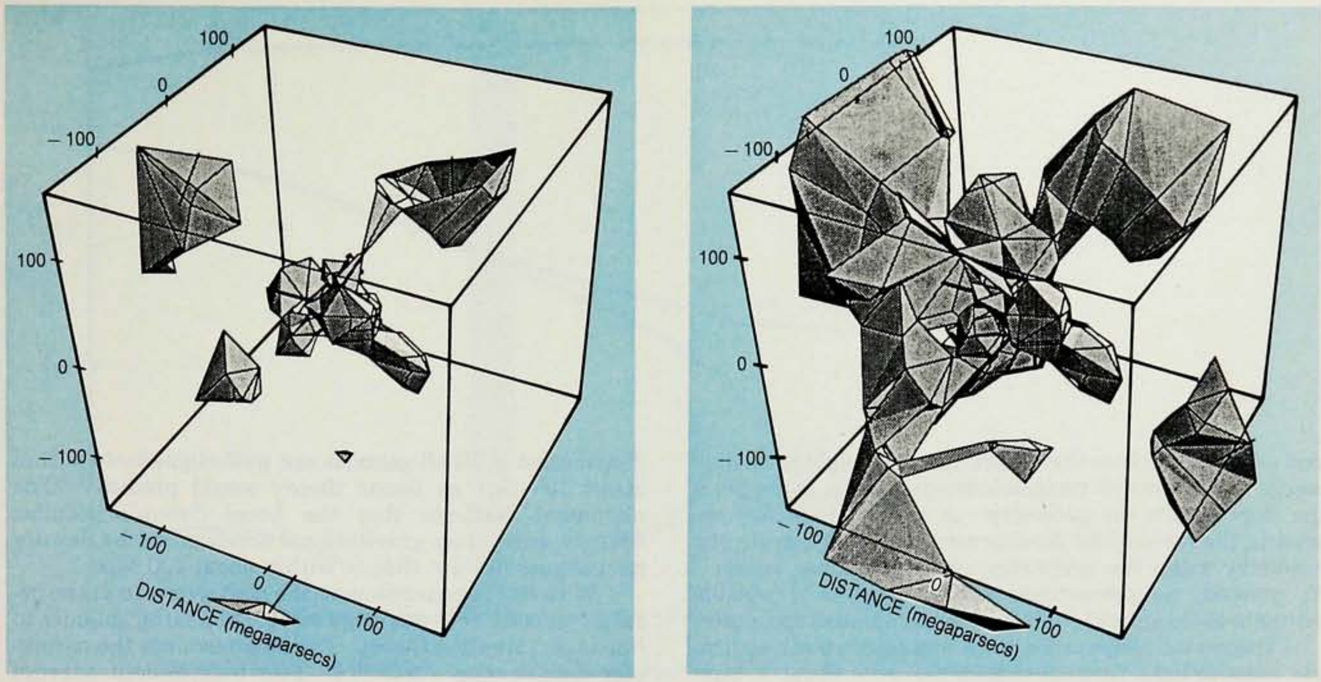
Clusters like Coma are strong x-ray sources,[9] with luminosities on the order of 10^{44} ergs/sec. The x rays arise from thermal bremsstrahlung in gas at a temperature of about 10^8 K. The gas is an additional source of x-ray-luminous mass. The total gas mass is somewhat model dependent but cannot exceed 20% of the total mass inside 1.3 Mpc; thus at least 80% of the cluster mass is still dark.

The x-ray observations can also be used to check our estimate of the cluster mass. The gas in the central parts of the cluster is in hydrostatic equilibrium, which implies that

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\rho_\mathrm{g}\frac{\mathrm{d}U}{\mathrm{d}r} \tag{4}$$

where $\rho_\mathrm{g}(r)$ and $p(r)$ are the gas density and pressure. Imaging observations by the Einstein satellite (figure 4) and spectral observations by the Tenma and Exosat satellites can be fit to models for the temperature and density distribution of the gas. These can be combined with equation 4 and the ideal-gas equation to yield the potential gradient $\mathrm{d}U/\mathrm{d}r$ and hence the mass distribution. The mass contained within 1.3 Mpc is found to be 5–$10\times10^{14}M_\odot$, close to the value obtained from the galaxy kinematics.[9]

The dark mass in clusters cannot be attached to extended galaxy halos, as these will be shorn off by tidal forces to form a smooth dark matter distribution spread throughout the cluster, through which the truncated galaxies swim.



Density of galaxies around the Sun. The shaded volumes denote regions in which the fractional enhancement in the number density of galaxies, $\delta_n = [n(\mathbf{x})/\bar{n}] - 1$, exceeds $+0.5$ (left panel) and $+0.3$ (right panel). All distances are in megaparsecs. (Adapted from ref. 20.) **Figure 5**

Fritz Zwicky first pointed out the presence of dark matter in clusters of galaxies in 1933. His original paper was based on only seven galaxy radial velocities and a distance to Coma that was too small by at least a factor of 5. Fortunately, distance errors scale out of his calculation, and his conclusion remains unchanged and by now seems inescapable: Almost all of the mass in the Coma cluster is dark. Studies of other galaxy groups and clusters yield similar results, as do measurements of the distortion and splitting of images of distant galaxies by the gravitational fields of clusters.[10]

Dynamics on larger scales

In the standard Friedmann–Robertson–Walker cosmological model, the universe is homogeneous and isotropic on sufficiently large scales.[11] Of course there are small-scale irregularities such as stars, galaxies and clusters, but if the FRW approximation is correct there must be some distance $r_\star$ such that the mean density $\bar{\rho}$ and all other local properties are approximately the same in every cube of side $r_\star$, wherever it may be in the universe. Surveys suggest that $r_\star$ is about 50 Mpc, or only about 1% of the size of the visible universe, which is of order $c/H_0 = 4000$ Mpc, and so the FRW approximation is reasonable.

A useful measure of the cosmological significance of dark matter in FRW models is the density parameter Ω, defined as the ratio of the mean density of the universe to the critical density: $\Omega = \bar{\rho}/\rho_c$, where $\rho_c = 3H_0^{\,2}/8\pi G = 1.06\times10^{-29}\,\mathrm{g/cm^3}$. The density parameter is the ratio of the kinetic energy of the Hubble expansion to the absolute value of the gravitational potential energy; thus an expanding universe with $\Omega<1$ will expand forever, whereas one with $\Omega>1$ must eventually collapse. (I assume the cosmological constant[12] is zero.) The geometry of the universe is also determined by Ω: If $\Omega>1$ the universe is closed and the geometry is spherical, whereas if $\Omega<1$ the universe is open and the geometry is hyperboloidal.

The mean mass density $\bar{\rho}$ is the product of the mean luminosity density $\bar{\epsilon}$ and the mean mass-to-light ratio $\bar{\Upsilon}$. Galaxy surveys[13] show $\bar{\epsilon} = 1.3\times10^8\,L_\odot/\mathrm{Mpc}^3$ to within a factor of 2, which implies

$$\Omega \equiv \bar{\Upsilon}/\Upsilon_c \qquad (5)$$

where $\Upsilon_c = 1200\,\Upsilon_\odot$. Thus, for example, if $\bar{\Upsilon}$ is equal to the mass-to-light ratio of the Coma cluster—$300\Upsilon_\odot$—then Ω is 0.25 and the universe is open.

Many cosmologists believe that $\Omega = 1$ to high accuracy. Reasons for this belief include the inflation hypothesis,[14] which resolves several traditional problems with FRW cosmology and predicts $\Omega = 1$, and the temporal Copernican principle. (To understand that principle, suppose, for example, that $\Omega\approx0.25$ and the universe is open; then in an FRW model Ω is very near unity at early times and very near zero at late times. There is no obvious reason why we should be living during the special epoch at which Ω first peels away from unity.) A further advantage of $\Omega = 1$ is that infall of distant bound material surrounding a protogalaxy provides a natural explanation for the origin of extended dark halos.

There is also a worrisome argument against $\Omega = 1$, based on a comparison of the age of an FRW model with stellar ages. Stellar evolution models show that the oldest stars are $t_\star = 15\pm3$ billion years old.[15] The age of an FRW universe is $t_0 = f(\Omega)/H_0$, where $f(\Omega) = 1$ for $\Omega = 0$ and $\frac{2}{3}$ for $\Omega = 1$. If $\Omega = 1$, then for consistency ($t_\star < t_0$) we must have $H_0 < 45\pm9\ \mathrm{km\,sec^{-1}\,Mpc^{-1}}$, far smaller than our preferred value of 75 and near the lowest values obtained by any of the methods of determining the Hubble constant.[6] The significance of this argument is difficult to assess, however, without more secure limits on the Hubble constant and without a compelling alternative to the standard FRW model.

Because the geometry of the universe depends on Ω, the mean density $\bar{\rho}$ can in principle be determined from geometrical measurements, such as the dependence on distance of the brightness or the number density of galaxies. However, the uncertain effects of galaxy evolution—galaxies fade as their stars age, and brighten as they merge with nearby companions—generally overwhelm the dependence on geometry. A more promising approach, though still far from practical, is to constrain the geometry using the properties of gravitational lenses.[10] At present, no geometrical approach gives a reliable estimate of Ω, and so I shall focus on dynamical estimates.

The measurements we have discussed so far sample the mass-to-light ratio on scales not over about 1 Mpc, whereas equation 5 requires the average mass-to-light ratio on scales exceeding $r_\star$, which is about 50 Mpc. The two may be different: If, for example, stars and galaxies form preferentially in high-density regions such as clusters of galaxies, then the average mass-to-light ratio may be substantially larger than the mass-to-light ratio in the central parts of clusters like Coma. In one simple empirical model for such "biased" galaxy formation,[16] fluctuations in the number density $n(\mathbf{x})$ of galaxies are proportional to fluctuations in the mass density $\rho(\mathbf{x})$:

$$\delta_n(\mathbf{x}) = b\delta_\rho(\mathbf{x}) \qquad (6)$$

where $\delta_n = n(\mathbf{x})/\bar{n} - 1$, $\delta_\rho = \rho(\mathbf{x})/\bar{\rho} - 1$ and the constant $b \gtrsim 1$ is called the bias factor.

The determination of mass-to-light ratios on scales of order $r_\star$ or greater requires methods different from those applicable to galaxies or clusters. Structures of this size are still just starting the process of gravitational collapse, and their evolution is described by linear perturbations to an FRW model. One simple prediction of linear theory is that the peculiar velocity $\mathbf{v}_\mathrm{p}$ is directly proportional to the peculiar gravitational acceleration $\mathbf{g}$ arising from the density fluctuations $\delta\rho(\mathbf{x})$, which in turn, if equation 6 applies, can be determined from the fluctuations $\delta n(\mathbf{x})$ in the number density of galaxies.[17]

This prediction can be checked against the peculiar velocity of our own Galaxy, or more properly the peculiar velocity of the center of mass of the Local Group of galaxies. The Local Group includes our own Galaxy plus its bound companion M31 at a distance of 0.7 Mpc. The peculiar velocity of the Local Group, determined from the motion of the Sun relative to the cosmic background radiation,[17] is 600 ± 27 km/sec toward Galactic longitude 268° and latitude 27°.

The best available sample for determining the number-density fluctuations $\delta n(\mathbf{x})$ is the set of galaxies detected by the Infrared Astronomical Satellite. The IRAS survey is unaffected by dust obscuration, covers almost the whole sky, has well-calibrated flux limits and samples a sufficiently large volume of the universe—to distances beyond 100 Mpc. Figure 5 shows an estimate of $\delta n(\mathbf{x})$ from the IRAS survey.

It turns out that the Local Group's peculiar velocity $\mathbf{v}_\mathrm{p}$ and the peculiar acceleration $\mathbf{g}$ determined from the distribution of IRAS galaxies are well aligned—to within about 10°—just as linear theory would predict.[18] The alignment confirms that the Local Group's peculiar velocity arose from gravitational acceleration by density fluctuations nearby, that is, within about 100 Mpc.

With this encouragement, the next step is to fit the peculiar-velocity field of a large sample of nearby galaxies to linear perturbation theory. The fit determines the parameter combination $\lambda \equiv \Omega^{0.6}/b$. Two independent surveys yielded $\lambda = 0.95 \pm 0.20$ and 0.89 ± 0.16, consistent with $\Omega = 1$ if the bias factor b is near unity on large scales.[19,20]

Thus analysis of the phase-space distribution of galaxies out to about 100 Mpc supports the density parameter $\Omega = 1$ that is favored on theoretical grounds, so long as the galaxy density traces the mass density on large scales.

What is the dark matter?

Although my main goal in this article is to describe the evidence for dark matter, I will briefly summarize what little we know about its nature.

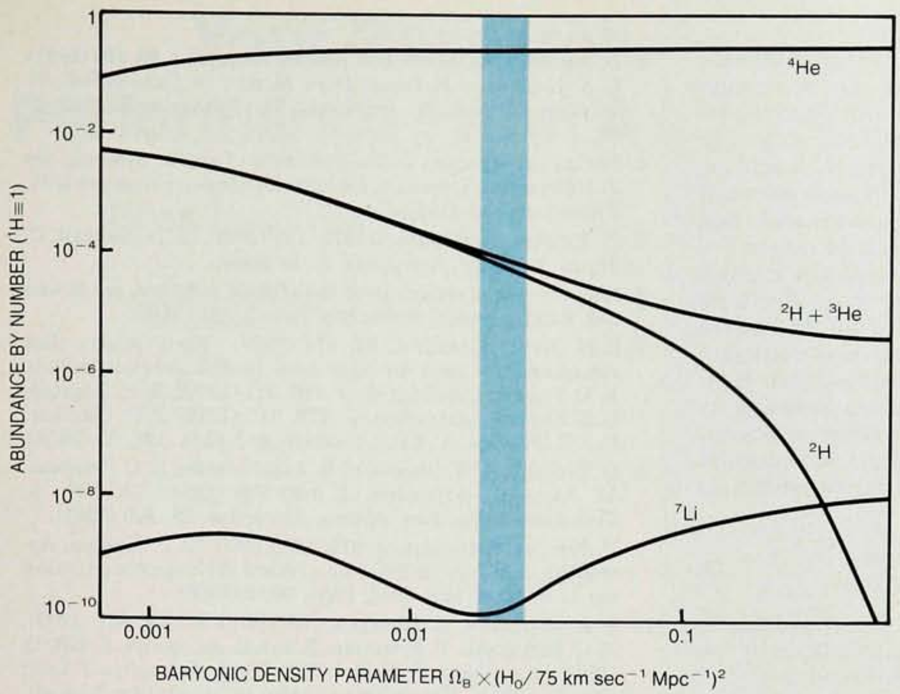
A strong constraint on the total density of baryons comes from the abundances of the light elements. In the standard FRW cosmology, ^{2}H, ^{3}He, ^{4}He and ^{7}Li are all formed in the first 10^3 seconds after the Big Bang, when the temperature exceeds 5×10^8 K. The abundance predictions of the standard model depend on a single parameter, which may be taken to be the present mean density of baryons, $\bar{\rho}_\mathrm{B}$. The observed abundances are consistent with the predictions of the standard model[21] if and only if the baryonic density parameter Ω_B, which is defined as $\bar{\rho}_\mathrm{B}/\rho_\mathrm{c}$, satisfies

$$0.02 \leqslant \Omega_\mathrm{B}(H_0/75\ \mathrm{km\ sec^{-1}\ Mpc^{-1}})^2 \leqslant 0.03 \qquad (7)$$

As figure 6 indicates, if Ω_B exceeds the upper limit, cosmological production of ^{2}H is too small. Nuclear reactions in stars do not provide a loophole, since they only destroy ^{2}H. Below the lower limit the abundance of ^{2}H plus ^{3}He is too large. (The combination is used because most of the ^{3}He is produced by burning ^{2}H.) Variations in either direction tend to produce too much ^{7}Li.

The principal uncertainty in Ω_B arises from the uncertain value of the Hubble constant H_0. Even if H_0 is as small as 50 km sec^{-1} Mpc^{-1}, the constraint in equation 7 implies that $\Omega_\mathrm{B} \lesssim 0.07$. This density is far too small to explain the Ω value inferred from the mass of the Coma cluster ($\Omega \simeq 0.25$) or from large-scale density fluctuations and peculiar velocities ($\Omega \simeq 0.75$–1.15 if the bias factor for IRAS galaxies is near unity) or to provide the density required by inflation ($\Omega = 1$). Thus most of the mass of the universe cannot be in baryons.

An appealing possibility is that the nonbaryonic dark matter consists of weakly interacting, massive, stable

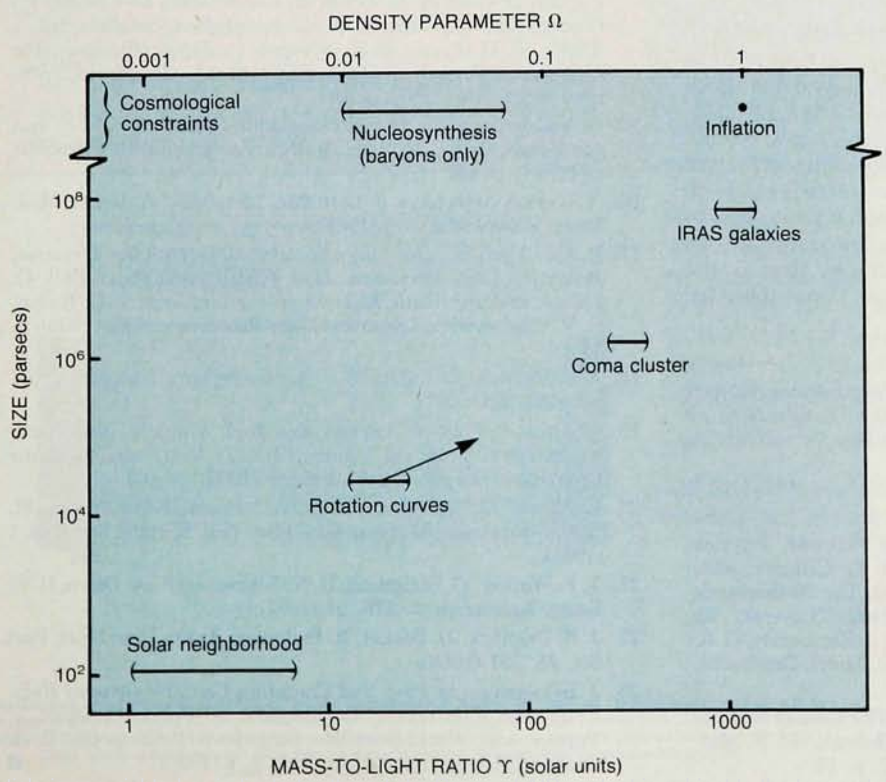


Predicted abundances from cosmological nucleosynthesis as a function of Ω_B (the ratio of the baryon density to the critical density) and the Hubble constant H_0. The shaded band marks the range consistent with observations. (Adapted from ref. 21.) **Figure 6**

elementary particles—"WIMPs"—formed in the hot, dense early universe. There are several plausible but hypothetical candidates in nonstandard models of particle physics, including massive neutrinos, neutralinos and axions, some of which may be detectable in laboratory experiments.[22] (See Leo Stodolsky's article in PHYSICS TODAY, August 1991, page 24.)

The constraint in equation 7 also implies that $\Omega_B > 0.01$, even if H_0 is as large as 100 km sec^{-1} Mpc^{-1}. The mean mass-to-light ratio of baryonic material thus exceeds $12\Upsilon_\odot$, from equation 5. This is substantially larger than the mass-to-light ratio of the stars and gas in the solar neighborhood, which is typical for disk galaxies. Thus there must be a substantial component of baryonic dark matter as well, most likely in the form of brown dwarfs or compact stellar remnants—white dwarfs, neutron stars or black holes.

Finally, it is possible that the apparent evidence for dark matter arises from inadequacies in the conventional laws of gravity or dynamics. There is little evidence that Newtonian gravity is accurate on scales much larger than 0.1 pc, the size of the solar system comet cloud. Thus, for ex-



Constraints on the mass-to-light ratio Υ and density parameter Ω as a function of scale, with determinations based on the standard FRW cosmological model plotted at the top, beyond the size scale. The plot assumes that the Hubble constant H_0 is 75 km sec^{-1} Mpc^{-1}, except that the error bar for the nucleosynthesis constraint includes the uncertainty in the Hubble constant. The constraint from galaxies detected by IRAS, the Infrared Astronomical Satellite, assumes that the bias factor b is 1. The arrow on the rotation curve determination is a reminder that we can measure only a lower limit on the halo mass and size from rotation curves. The relation between Υ and Ω is given by equation 5. **Figure 7**

ample, we might consider modifying the gravitational acceleration from a point mass M from the Newtonian expression $a = GM/R^2$ to $a = (GM/R^2) + (GM/R_0R)$, where R_0 is some new fundamental length. Then the circular speed around a mass M at distances $R \gg R_0$ would be $v_c = (GM/R_0)^{1/2}$, consistent with the flat rotation curves of disk galaxies. One difficulty (among several) with this proposal is that if the mass-to-light ratio is constant—which it should be, if there is no dark mass—the circular speed should scale as $v_c \propto L^{1/2}$, where L is the total luminosity of the galaxy. This contradicts the observation that for disk galaxies $v_c \propto L^{0.25}$ over more than two orders of magnitude in luminosity (the infrared Tully–Fisher law).

A much more interesting modification, proposed by Mordehai Milgrom of the Weizmann Institute of Science in Rehovot, Israel, is to introduce a new fundamental acceleration a_0, so that the acceleration from a point mass M is

$$a = \begin{cases} GM/R^2, & \text{for } a \gg a_0 \\ (GMa_0)^{1/2}/R, & \text{for } a \ll a_0 \end{cases} \qquad (8)$$

In this case the circular speed at large distances is $v_c = (GMa_0)^{1/4}$, and for constant mass-to-light ratio we have $v_c \propto L^{1/4}$, consistent with the Tully–Fisher law.

The modified acceleration specified by equation 8 is surprisingly successful at explaining most of the dynamical evidence for dark matter, with the constant $a_0 \simeq 1 \times 10^{-8}$ cm/sec^2. The modified acceleration can be derived from a nonrelativistic Lagrangian, but so far there is no fully satisfactory replacement for general relativity that yields equation 8 in its weak-field limit.[23]

Figure 7 summarizes the evidence for dark matter described in this article. The following general trends seen in the figure are confirmed by many other dynamical arguments, of varying accuracy and rigor, that are not described here:

▷ The dark mass exceeds the luminous mass in virtually all systems of galaxy size or larger.

▷ The ratio of dark to luminous mass generally increases with scale.

▷ On scales greater than about 10 Mpc the ratio of dark to luminous mass is independent of scale and of order 10^3 in solar units, large enough that the total density could equal or exceed the critical value needed to close the universe.

▷ At least 90% of the mass in the universe is not baryonic.

At present we must admit with some embarrassment that we do not know what most of the universe is made of. A more positive view is that the light that has been shed on dark matter over the past two decades is the first stage of a revolution against "barycentric" cosmology that is the direct descendant of the revolution that Copernicus led against geocentric cosmology.

* * *

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