SUMMARY

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This is the first IAU symposium devoted specifically to elliptical galaxies. I would like to congratulate the organizers for initiating a symposium on such a well-defined topic with so many interesting puzzles. I have been impressed at this meeting by the great strides made in this subject over the last ten years, both in the quality and quantity of data and in the sophistication of the theoretical models. In addition, I'm sure that I express the sentiments of the great majority of the participants when I say what a pleasure it was to attend a meeting on galaxies in which there was not a single paper on spiral structure!

Rather than attempting to give a general summary, I have chosen to concentrate on four specific topics which reflect some of my personal impressions of the meeting.

NOTATION

Before discussing science I wanted to comment briefly on the vexing issue of the names of the fundamental equations which describe the evolution of stellar systems. If $f(\vec{x}, \vec{v}, t)$ is the density of stars in 6-dimensional phase space, we know that in a collisionless system

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left[v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} \right] = 0.$$

This is usually referred to as the "Vlasov equation", but Hénon (1982) has stressed that it is more properly called the "collisionless Boltzmann equation", since it is merely a simplified version of an equation which Boltzmann derived in 1872, long before Vlasov's work. A second equation involves the probability density f_N of an ensemble of N-body systems in $6N$-dimensional phase space. This satisfies

$$\frac{\partial f_N}{\partial t} + \sum_{i=1}^{3N} \left[v_i \frac{\partial f_N}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f_N}{\partial v_i} \right] = 0,$$

which is usually called the "Liouville equation". However, Liouville neither derived this equation nor even worked in statistical mechanics, and as far as I know it was first written down in a short abstract by Gibbs (1884), who states that "the object

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of this paper is to establish this proposition (which is not claimed as new, but which has hardly received the recognition which it deserves) and to show its applications to astronomy and thermodynamics". Although one wishes Gibbs had published more detail, it seems clear that he deserves priority (particularly in astronomical applications), and it might therefore be appropriate for us to replace "Liouville equation" by "Gibbs equation". Finally, the equations of stellar hydrodynamics or "Jeans equations"

$$\frac{\partial \overline{v}_i}{\partial t} + \sum_{j=1}^{3} \overline{v}_j \frac{\partial \overline{v}_i}{\partial x_j} = -\frac{\partial \Phi}{\partial x_i} - \frac{1}{\nu} \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j},$$

where n, $\overline{v}_i$, and σ_{ij} are the number density, mean velocity and dispersion tensor respectively, were first derived by Maxwell (1866). Unfortunately, he already has a set of eponymous equations, and since Jeans was the first to use the equations in stellar dynamics, the name "Jeans equations" is probably a reasonable compromise. My personal vote, then, is for "collisionless Boltzmann equation", "Gibbs equation" and "Jeans equations".

TRIAXIALITY

The modern era in the study of elliptical galaxies began with the paper by Bertola and Cappacioli (1975) showing that NGC 4697 had a rotation speed far smaller than would be expected if it were a rotating gaseous spheroid. This remarkable result was confirmed and considerably generalized by Illingworth's (1977) observations of thirteen ellipticals, most of which had rotation curves which fell far below the curves expected for rotating gaseous masses. At the same time, Binney (1976, 1978) stressed that slow rotation was a natural consequence of pancake theories of galaxy formation, which produce flattened or triaxial galaxies with little or no figure rotation. The hypothesis that elliptical galaxies may be triaxial has proved to be an extremely fertile one, and much of the theoretical and observational work over the last decade has been devoted to testing and elaborating on this idea.

On the observational side, the progress has been slow. It is discouraging that we still have no conclusive evidence and no clear consensus on whether most ellipticals are triaxial. It is true that observers can cite specific examples which are difficult to explain without triaxiality (see paper by Davies). However, I suspect that this issue, like many others in astronomy, will only be decided by statistical analysis of large samples of high-quality data, and one of the most encouraging features of this meeting was that such samples—CCD surface brightness distributions of dozens to hundreds of galaxies—are beginning to be reported (papers by Djorgovski, Jedrzejewski, and Kormendy). I echo Schechter's comment that these surveys deserve to be analyzed carefully to see whether some clues to the distribution of intrinsic shapes can be disentangled from projection effects.

On the theoretical side, the last decade has produced giant leaps in our understanding of the structure of triaxial galaxies—so much so, that it would be a real shame if the observers found that they were not triaxial after all! Here the turning point was Schwarzschild's (1979) construction of a realistic triaxial galaxy using linear programming. My belief is that the most lasting and important accomplishment of this paper was not the introduction of linear programming, which has by now been superseded in many cases by other approaches, such as Lucy's algorithm and maximum entropy methods (see paper by Richstone). Rather, it was

Schwarzschild's recognition that the traditional construction of models by what might be called "integral-based methods"—using Jeans' theorem and the integrals of motion in a given potential—could be replaced by "orbit-based methods", in which the model builder simply integrates a selection of orbits, computes the fractional time each orbit spends in each spatial grid cell, and combines the orbits so as to make a self-consistent model. For the first time, orbit-based methods enable us to construct galaxy models with realistic triaxial potentials, for direct comparison with both N-body experiments and real galaxies.

MERGERS

One striking aspect of this meeting has been the impressive display of evidence that mergers are a common and ongoing process in ellipticals. Although the importance of this process was clearly recognized by Toomre and Toomre (1972), it is only in the last few years that the "smoking gun"—evidence for recently completed mergers in single galaxies—has been found. Some of the most convincing candidates for recent mergers are the radio galaxies Fornax A (Schweizer 1980) and Centaurus A (Malin *et al.* 1983), and the "double core" elliptical NGC 5813 (Kormendy 1984). The features suggestive of recent mergers in many other candidates include (i) the presence of warped or misaligned HI disks in ellipticals (see papers by Knapp and Schweizer), in particular the presence of over $10^{10}M_\odot$ of HI in the giant elliptical NGC 807 (Dressel); (ii) the presence of dust and ionized gas (Bertola and others); (iii) ripples and shells (Quinn and others); (iv) polar rings; and (v) the IRAS starburst galaxies. A nice recent review of this subject is given by Schweizer (1986).

VIOLENT RELAXATION

N-body experimenters now have codes which can reliably follow the collapse and violent relaxation of galaxies from cold initial conditions (see papers by van Albada and White). It appears that whenever the relaxation is violent enough, the final state is largely independent of the initial conditions and has an $R^{1/4}$ surface density profile. It would be nice to have a simple analytic distribution function which both reflects the physics of violent relaxation and fits the results of these numerical simulations. I have been inspired by two of the papers presented here to offer a heuristic derivation of such a function.

Violent relaxation redistributes the energies and angular momenta of stars through the strong potential fluctuations in the central core of the collapsing galaxy. Any relaxation process of this form tends to produce a Maxwell-Boltzmann distribution with temperature inversely proportional to energy, $f(\vec{x},\vec{v}) \propto \exp(-\beta E)$, where E is the energy per unit mass. However, violent relaxation is not complete, since the potential fluctuations only last for a limited time t_{eff}, which is of order one or two crossing times. Thus highly eccentric orbits, whose radial orbit period t_r exceeds t_{eff}, will be underpopulated, by a factor of order t_{eff}/t_r. We may crudely account for this effect by replacing the Maxwell-Boltzmann distribution by $f \propto \min(1, t_{\text{eff}}/t_r)\exp(-\beta E)$. Since the minimum orbit period is of order t_{eff}, we can simplify this result to $f \propto t_r^{-1}\exp(-\beta E)$. Indeed, since the correction is most important for highly elongated orbits, and since these orbits are nearly Keplerian with $t_r \propto |E|^{-3/2}$, we can simplify again, to obtain the distribution function $f \propto |E|^{3/2}\exp(-\beta E)$. Furthermore, the potential fluctuations are effective only in a limited region, and hence cannot place stars on orbits whose angular momenta are so large that their pericenters lie outside this region. We must therefore expect that the distribution function exhibits a cutoff at large angular momenta, which we may incorporate by a multiplicative factor $\exp(-\alpha J^2)$, to arrive at our final guess at the distribution function arising from violent relaxation:

$$f(\vec{x},\vec{v}) = K|E|^{3/2}\exp(-\beta E - \alpha J^2).$$

This turns out to be precisely the distribution function which Bertin and Stiavelli (1984, and this conference) have advocated for violently relaxed systems, although on the basis of quite different arguments. They have already shown that it can provide a good match to the $R^{1/4}$ profile.

Although this distribution function ought to provide a good fitting formula, its exact functional form is only a guess—the arguments above only show that the distribution function should have the general form

$$f \propto \begin{pmatrix} \text{function which is} \\ \text{proportional to} \\ t_r^{-1} \text{ when } t_r \text{ is large} \end{pmatrix} \times \begin{pmatrix} \text{cutoff in peri-} \\ \text{center or } J^2 \end{pmatrix} \times \begin{pmatrix} \text{decreasing function} \\ \text{of } |E| \text{ which is} \\ \text{non-zero at } E = 0 \end{pmatrix}.$$

The condition that the third function is non-zero at $E = 0$ arises because the potential fluctuations have no way to recognize the special role of the escape energy, and hence must populate energy space smoothly around $E = 0$. Jaffe (this meeting) has used similar arguments to show that the asymptotic density distribution in a violently relaxed galaxy must be $\rho \propto r^{-4}$, a condition which the Bertin-Stiavelli distribution function satisfies.

The Bertin-Stiavelli distribution function has one free parameter once the total mass and energy are fixed. This parameter reflects the size of the region in which the potential fluctuations occur. It would be most interesting to compute a family of these models and to compare them in detail with a sequence of violent relaxation simulations.

This completes my detailed comments. Of course, the selection of topics I have made leaves out many of the most interesting aspects of the meeting. A more balanced treatment would spend much of its effort on the topic of X-rays from ellipticals and cooling flows, which was the subject of some of the most lively and controversial discussions of the meeting. Unfortunately I do not feel that I can add anything useful to the reviews already given here by Fabian, O'Connell and Sarazin.

Let me close with one final remark. The Hubble classification for spirals is useful because many properties of spirals (gas content, color, spiral arm morphology, bulge prominence, etc.) all correlate with Hubble time. By contrast, almost nothing correlates with the elliptical Hubble sequence E1 to E7. In view of the rapid increase in quality and detail of our data on ellipticals, is it not time for someone to come up with a new classification scheme for ellipticals to replace Hubble's?

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DISCUSSION

Ostriker: You were looking for a classification parameter for ellipticals to replace Hubble's shape parameter. Perhaps the simplest such parameter would be the luminosity which several authors have shown correlates with metallicity, color, rotation, core radius etc. This parameter has the added virtue that, when applied to bulge luminosity in spirals, it provides a classification parallel to the Hubble sequence.

Whitmore: Since we seem to be talking about objective classification systems, I'd like to comment that although you state that there are many correlations for spirals and only a few correlations for ellipticals, a principal component analysis shows that there are two basic parameters for spirals 1) luminosity and/or radius 2) bulge/disk ratio and/or color (Whitmore 1984, *Astrophys. J.*, **278**, 61). I am therefore not so sure that we would have to treat ellipticals and spirals completely differently.

White: I think your suggested prescription for obtaining the distribution function of a violently relaxed system is incomplete because the parameters are not specified. They must depend on the initial conditions of the relaxation phase. It seems to me that a function with only one shape parameter will not be able to accommodate the results both of cold collapses and of mergers between fully formed stellar systems. Although the density profiles can look similar in the two cases, their anisotropy radii are very different.

Tremaine: The simple models I proposed have three parameters—energy, mass and an anisotropy radius. Of course, at some level they are incomplete, because they are spherical, while some initial conditions lead to non-spherical final states. However, they do seem to be much more appropriate than King–Michie models, isotropic de Vaucouleurs models, or the other models which have been traditionally compared to numerical simulations of violent relaxation.

Jaffe: Another way to formulate Simon White's comment is to ask for a quantitative explanation for where or how the transition from core to halo occurs.

Tremaine: The core–halo transition presumably occurs at the boundary of the region where the violent relaxation takes place.

Mamon: I would like to give a word of caution to the builders of self–consistent models of spherical galaxies. Many of you have been building your models on the model that James Binney and I had developed for M87. That model is clearly wrong, for at least three reasons. First, our model was constructed with no central mass in M87, whereas I don't see how the large jet could be present without a very large black hole on the center. Second, our models were constrained by the velocity dispersion in the very center of the galaxy, as measured by Dressler. However, his measurement is very difficult to interpret because of finite slit size and atmospheric seeing. The 'true' line–of–sight velocity dispersion in the core of M87 may rise much faster than what we assumed. Finally, Kormendy's surface brightness profile of M87 obtained with the excellent seeing in Hawaii shows a much smaller core, and a naive application of the Binney–Mamon algorithm would yield a radial anisotropy peak occuring 3 times deeper inside the galaxy. So I think people should base their self–consistent studies on a different galaxy model. Unfortunately, I don't know of any galaxy for which a decent model exists.

Goodman: The observational results that most surprised me were the X–ray results, particularly those for cooling flows in cluster ellipticals. If, as Dr. Fabian tells us, 100 to 1000 solar masses per year of hot gas is forming very low mass stars in M87, then even if these stars can't burn hydrogen, they should release gravitational binding energy comparable to the X–ray luminosity of the hot gas. (At least, this is what Binney and I estimated during the coffee break). Can't we detect these low–mass stars at infrared or millimeter wavelengths?

Gilmore: It is possible to measure the mass function in low mass stars directly by spectroscopy of the gravity sensitive 2-3mm CO absorption band (Arnaud & Gilmore, 1986 *Mon. Not. R. astr. Soc.*, **220**, 759; this volume, p. 445). We have done this for a sample of ellipticals with cooling flows and a large number of ellipticals with no cooling flows. These observations exclude the possibilty that the mass in the flow is forming stars with mass greater than ~ 0.2 solar mass. If the mass is locked up in very low mass objects, they will radiate their binding energy during formation at a temperature of $\sim 1500 - 2000K$, and they will be detectable only as a low surface brightness excess in the near infrared. Such data await suitable array detectors. The available spectroscopic data show no differences between X–ray and non X–ray galaxies. They are also consistent with the stellar mass function in elliptical galaxies being very similar to that of the solar neighbourhood.

Schweizer: In their relation to other morphological types in the Hubble sequence, ellipticals seem to be the closest to what one might call dead galaxies. They barely form stars anymore, whereas the later Hubble types do. I visualize the E's as being in the central graveyard of a little city, whose surrounding inhabitants are the more lively spiral and irregular galaxies. In the past, we have concentrated our research on the central part of that graveyard, figuring out some properties of the oldest skeletons. During recent years and especially at this meeting, we have begun exploring the outer regions of the graveyard, where the fresh graves are. There has been a lot of excitement about ellipticals with shells, ripples, and inclined gas disks, presumably the aftermaths of some recent collisions involving disk galaxies. It seems to me that we stand to gain even more by turning our attention to where the open graves are, watching for the arrival of fresh victims of head–on collisions. We need to study the most luminous IRAS galaxies, if we wish to learn about elliptical formation. Having simulated this formation in a rather abstract manner from mergers in various cosmological models with given sets of rules, we should now try to model in much more detail the collisions occurring in IRAS galaxies like Arp 220 and NGC 6240, collisions that produce abundant molecular gas and vigorous star formation and that seem likely to lead to elliptical remnants. A first promising attempt in that direction has been made by Negroponte and White (1983, *Mon. Not. R. astr. Soc.*, **205**, 1009).

Toomre: Could we ask Martin Schwarzschild for the last comment?

Schwarzschild: A good portion of what we have discussed in this symposium was triggered by the decisive early radial velocity observations in ellipticals by Bertola and by Illingworth. The challenge of their surprising data was picked up by Binney and other theoreticians—I think with much success. But now—as this symposium has surely shown—we will again be in trouble if new key observational data can not be obtained. Brilliant progress is being made in superprecise photometry as well as in velocity dispersion measurements. The greatest difficulties are encountered, it seems, in the observations of mean streaming patterns in the inner portions of big ellipticals. But such mean stellar velocities are a most powerful diagnostic—if they can be obtained with an accuracy of about ±5 km/s, which is a severe task in view of the typical velocity dispersions of 300 km/s. Some recent observational mean velocity data still appear beset by unidentified systematic errors of about ±15 km/s which spoil their full diagnostic power. Here then lies a decisive challenge for new observational inventiveness.