

Collaboration Summary:  
**The Role of Foraging in the Formation of Locust Hopper Bands**

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We would like to thank the Institute for Advanced Study for this invaluable time working in direct collaboration. The setting and facilities were extremely conducive to productive research, and we especially would like to thank Michelle Huguenin and the rest of the staff!

At the IAS, we continued our investigation into the shape of aggregate groups of foraging locusts. Specifically, we are studying the role of food on the formation of traveling waves in an agent-based model (ABM) and a continuum model. Before coming to the IAS, we had preliminary results and an in-progress preprint. During our two weeks of collaboration, our goal was to complete the preprint and begin applying its results to a two-dimensional model. We have finalized analytical, numerical, and statistical results, and our preprint is essentially complete with minor edits remaining. We will acknowledge the IAS in our submission and notify the IAS upon publication.

We divided our time at the IAS among five main tasks. We identified several metrics by which to analyze solutions in the traveling wave frame. We also finished an exhaustive search of the existing literature for biologically relevant parameter values. Using these values, we quantified the sensitivity of model parameters to measurable outcomes using Sobol indices [2, 3, 4]. We implemented our ABM in the language R, which is commonly used in the fields of ecology and animal behavior. This programming language change will make our work accessible to a wider audience. Lastly, we spent time writing collaboratively and completed the rough manuscript [1]. One of the main results from our work is that resource (food) dependence drives the formation of a locust hopper band (traveling wave). Specifically, the probability function to determine how likely a locust marches (or stops) must depend upon the density of nearby available resources - without this dependence on resources, the locusts spread through a diffusive process (see Figure 1).

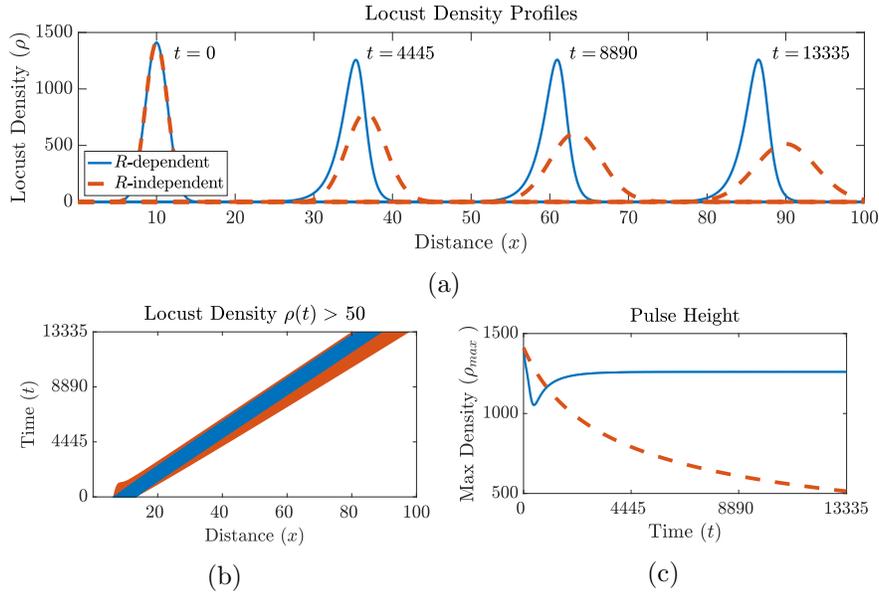


Figure 1: Locust density profiles with resource ( $R$ )-dependent (solid blue line) and  $R$ -independent (dashed red line) switching rates. Each profiles evolves from the same initial condition. Figure 1a shows snapshots of the density profiles over distance and time for both types of switching rates. Figure 1b illustrates the width of the bands where color represents a locust density greater than 50. Figure 1c displays the peak density of each pulse over time.

In Figure 1a, the dashed red lines illustrate the decay of a locust density profile with resource-independent switching rates, while the solid blue line shows the asymmetrical traveling wave created by resource-dependent switching rates. Figures 1b and 1c compare the width and peak of locust density profiles for switching rates with (blue) and without (red) resource dependence. In Figure 1b, colored regions correspond to a locust density greater than 50 with red and blue corresponding to  $R$ -independent and  $R$ -dependent switching rates, respectively. As the locust band without resource dependence progresses, the width of the red region increases in time, showing diffusive spreading. On the other hand, the width of the locust band with resource-dependence remains constant over time. Note that the locust band with resource dependence also reaches a constant height. Alternatively, the peak locust height with  $R$ -independent switching rates decreases over time as the locusts diffuse.

Formation of hopper bands corresponds to existence of a traveling wave solution in our PDE model for density of resources  $R(x, t)$ , stationary locusts  $S(x, t)$ , and marching locusts  $M(x, t)$ :

$$\begin{aligned} R_t &= -\lambda SR \\ S_t &= -k_{sm}(R)S + k_{ms}(R)M \\ M_t &= k_{sm}(R)S - k_{ms}(R)M - vM_x \end{aligned} \quad x \in \mathbb{R}, \quad t \in [0, \infty). \quad (1)$$

where  $k_{sm}$  and  $k_{ms}$  are defined as

$$k_{sm}(R) = \eta - (\eta - \alpha)e^{-\gamma R}, \quad k_{ms}(R) = \theta - (\theta - \beta)e^{-\delta R}. \quad (2)$$

Note that we select parameters  $\gamma, \delta > 0$ ,  $0 \leq \beta \leq \theta$ , and  $0 < \eta \leq \alpha$  such that  $k_{sm}(R)$  is a non-increasing function and  $k_{ms}(R)$  is a non-decreasing function of  $R$ .

We prove an existence theorem for traveling waves that relates the speed of the hopper band directly to the number of locusts. This theorem demonstrates the sufficiency of the resource dependence condition. We show necessity through a computation of the first and second moments in a simplified model where  $k_{sm}, k_{ms}$  are constants.

## References

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