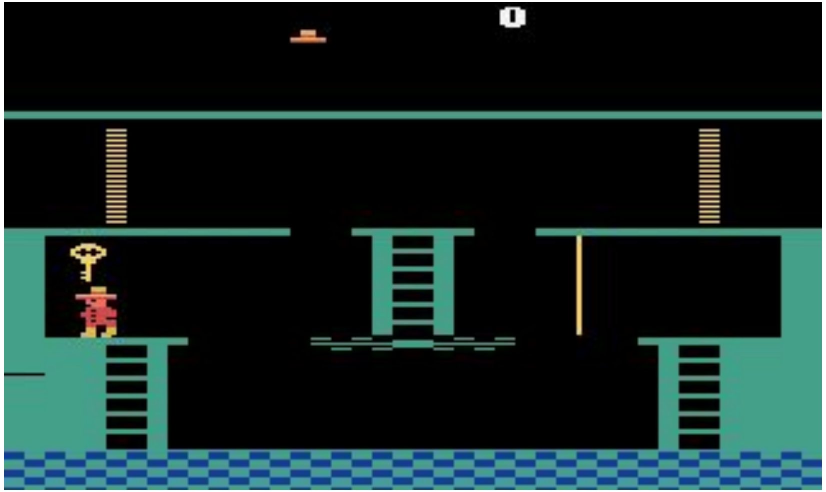
Emma Brunskill

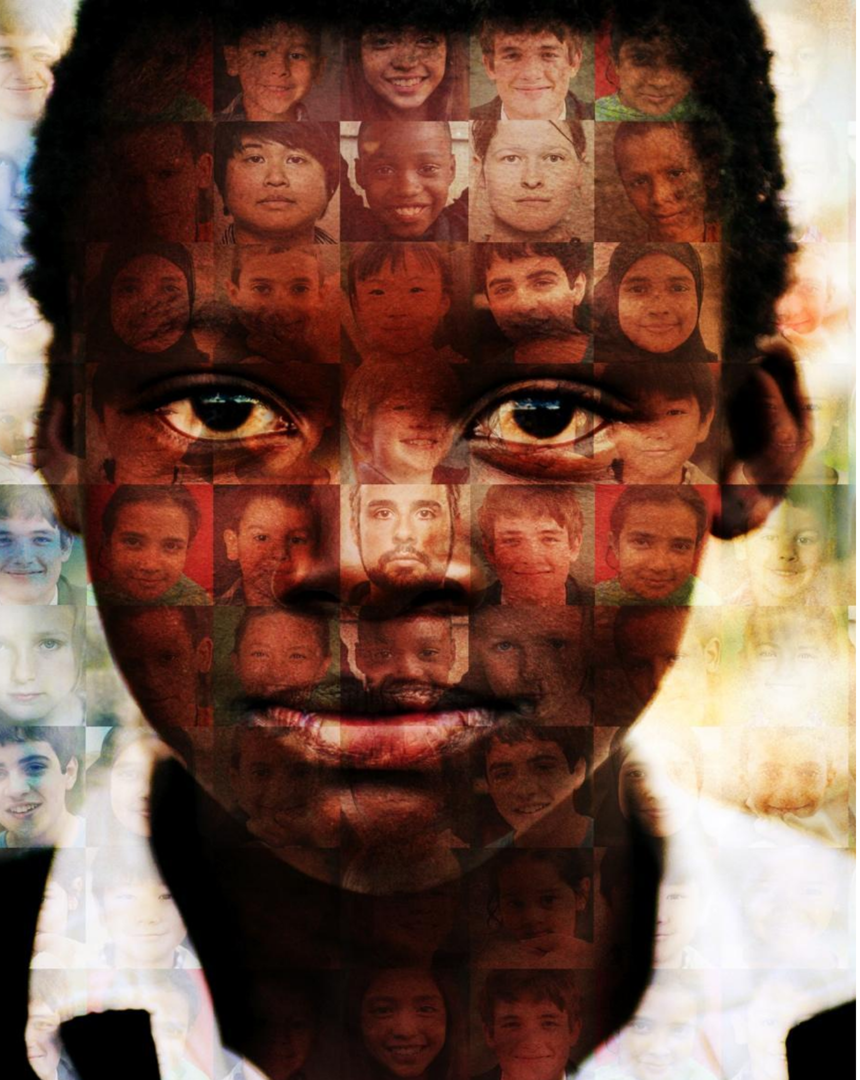
Assistant Professor, Computer Science, Stanford

IAS November 2019

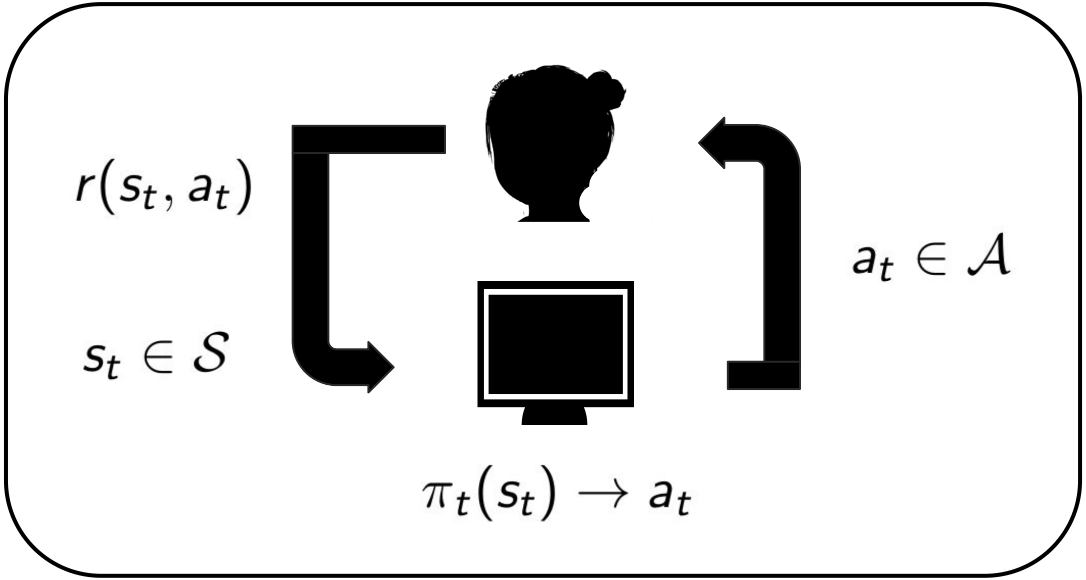
Work in collaboration with Ramtin Keramti, Christoph Dann & Alex Tamkin, preprint at: https://arxiv.org/abs/1911.01546

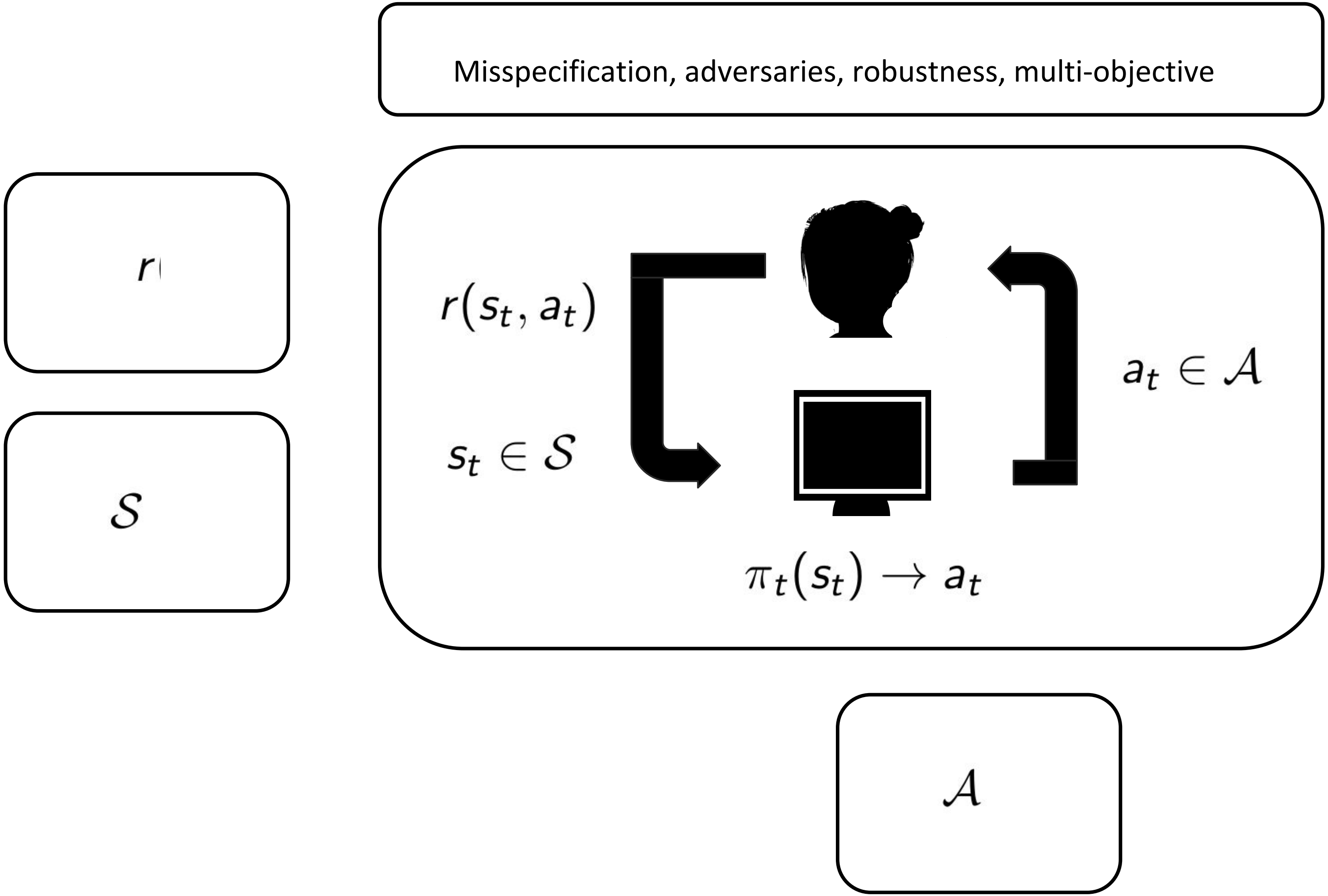
2010s: A New Era of RL





Reinforcement Learning to Improve People's Lives


$r(s_t, a_t)$
$s_t \in \mathcal{S}$
$a_t \in \mathcal{A}$
$\pi_t(s_t) \rightarrow a_t$


Misspecification, adversaries, robustness, multi-objective
r(
$\mathcal{S}$
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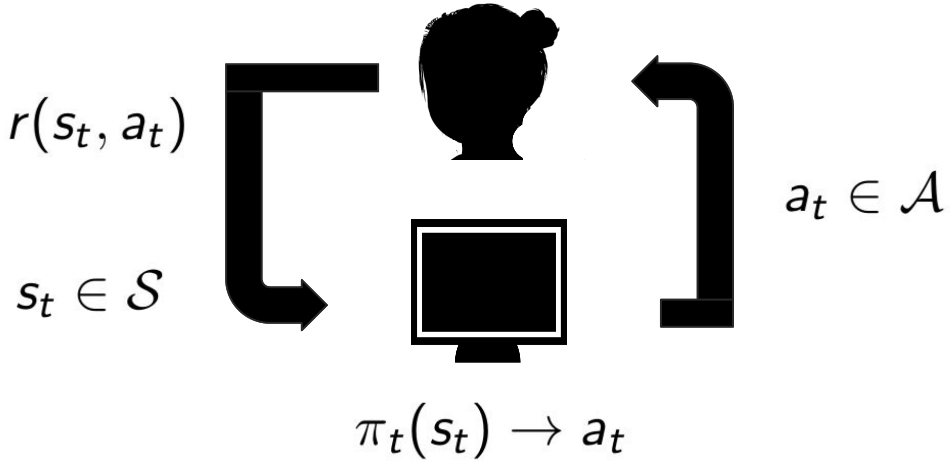
$\mathcal{A}$

Today: Risk Sensitive RL

$r($

$\mathcal{S}$



$\mathcal{A}$

Why is Risk Sensitive Control Important?

- Individuals experience single trajectory / 1 return

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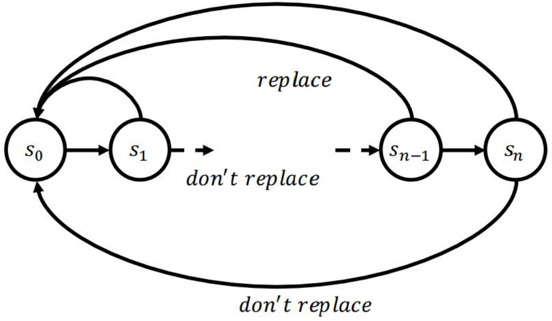
- Individuals experience single trajectory / 1 return

- Organizations often care about equity and fairness for everyone in distribution

Risk Sensitive Reinforcement Learning

Given data, Plan Safe Policy



Large body of literature in controls,
also work by Bagnell and many others

Risk Sensitive Reinforcement Learning

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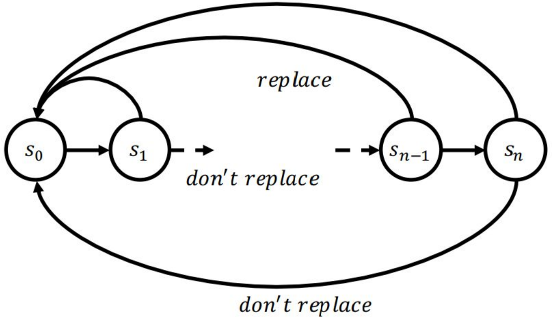

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Safely Learn a Safe Policy

Krause, Mannor, Tamar, Tomlin, Abbeel, Ghavamzadeh, Pavone, Schoellig...

Risk Sensitive Reinforcement Learning

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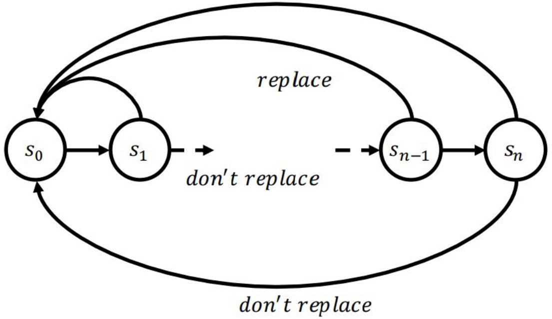

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Safely Learn a Safe Policy

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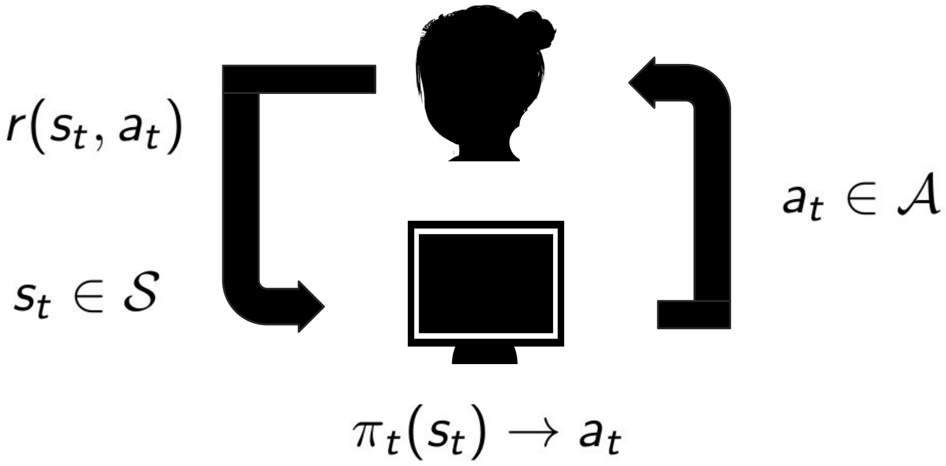
Quickly Learn a Safe

Image: createhealth.com/

Notation: Markov Decision Process Value Function



$$\underbrace{V^{\pi}(s)}_{\text{Value func.}} = \underbrace{r(s, \pi(s))}_{\text{Reward}} + \gamma \sum_{s'} \underbrace{p(s'|s, a)}_{\text{Dynamics}} V^{\pi}(s')$$

Notation: Reinforcement Learning

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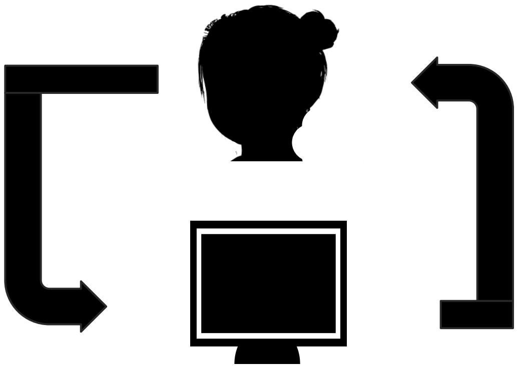
$s_t \in \mathcal{S}$



$a_t \in \mathcal{A}$

$\pi_t(s_t) \to a_t$

$$\underbrace{V^\pi(s)}_{\text{Value func.}} = \underbrace{r(s, \pi(s))}_{\text{Reward}} + \gamma \sum_{s'} \underbrace{p(s'|s, a)}_{\text{Dynamics}} V^\pi(s')$$

Only observed through samples (experience)

Background: Distributional RL for Policy Evaluation & Control

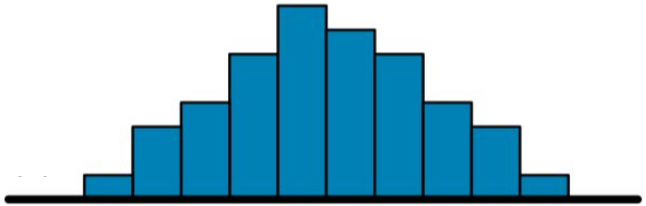
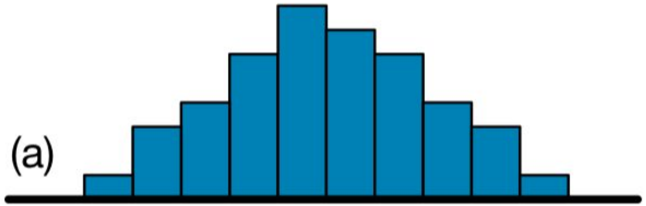


Figure from Bellemare, Dabney, Munos ICML 2017

Background: Distributional Bellman Policy Evaluation Operator for Value Based Distributional RL

$P^\pi Z$



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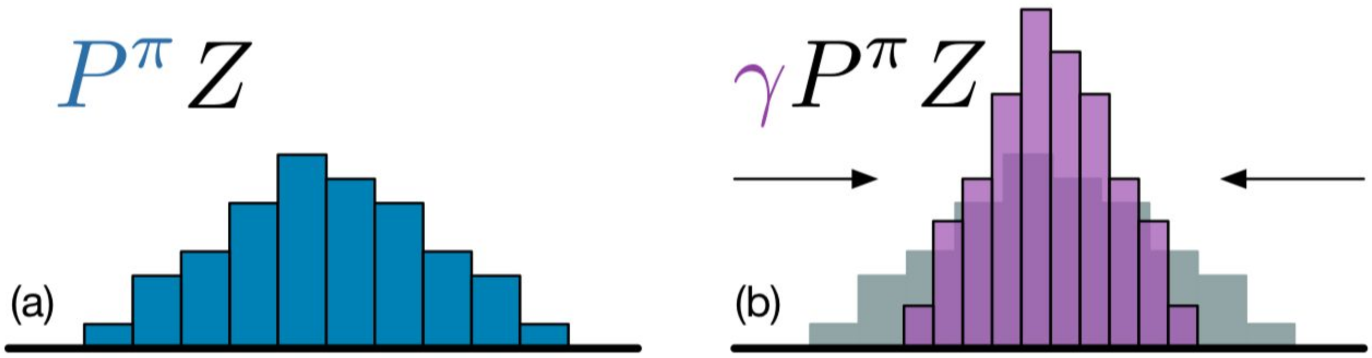

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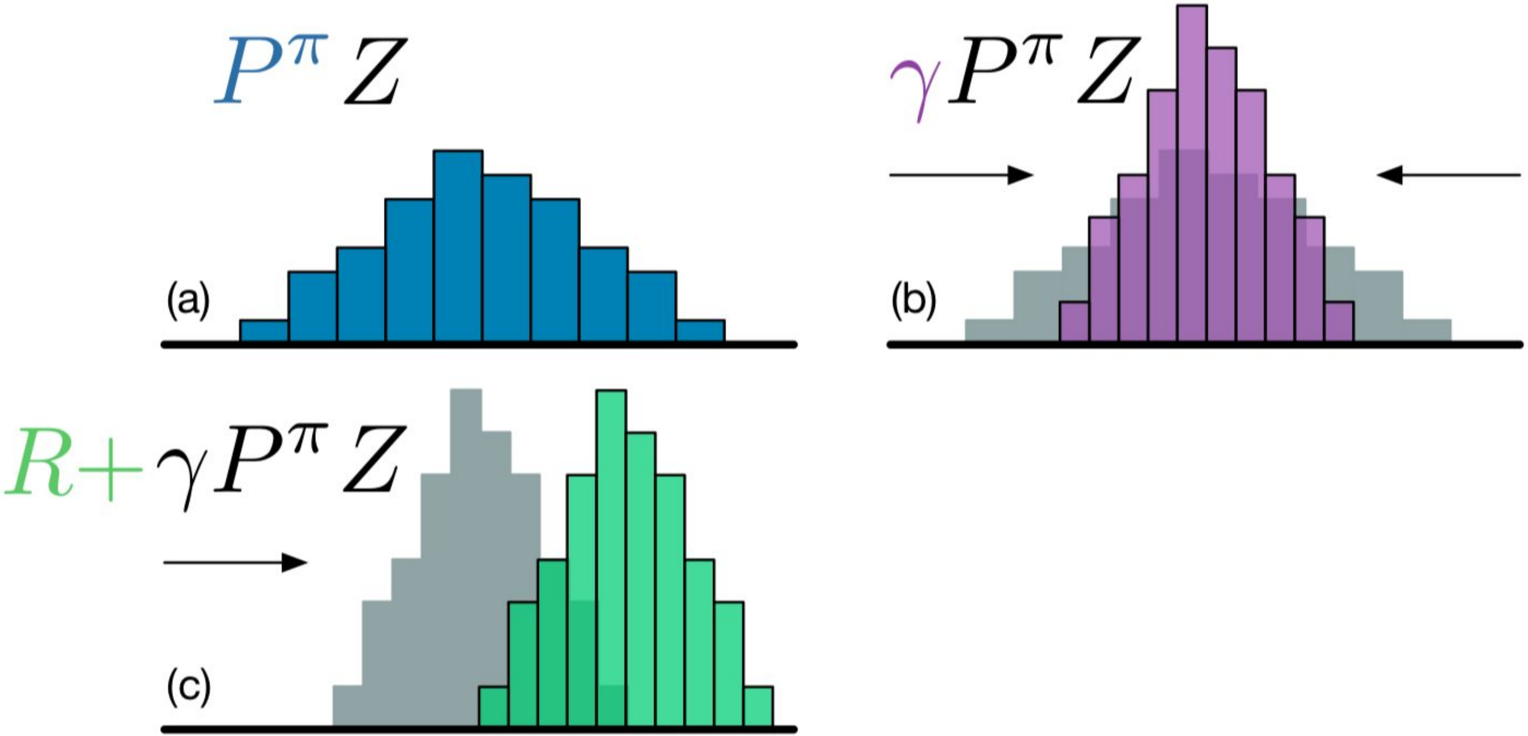

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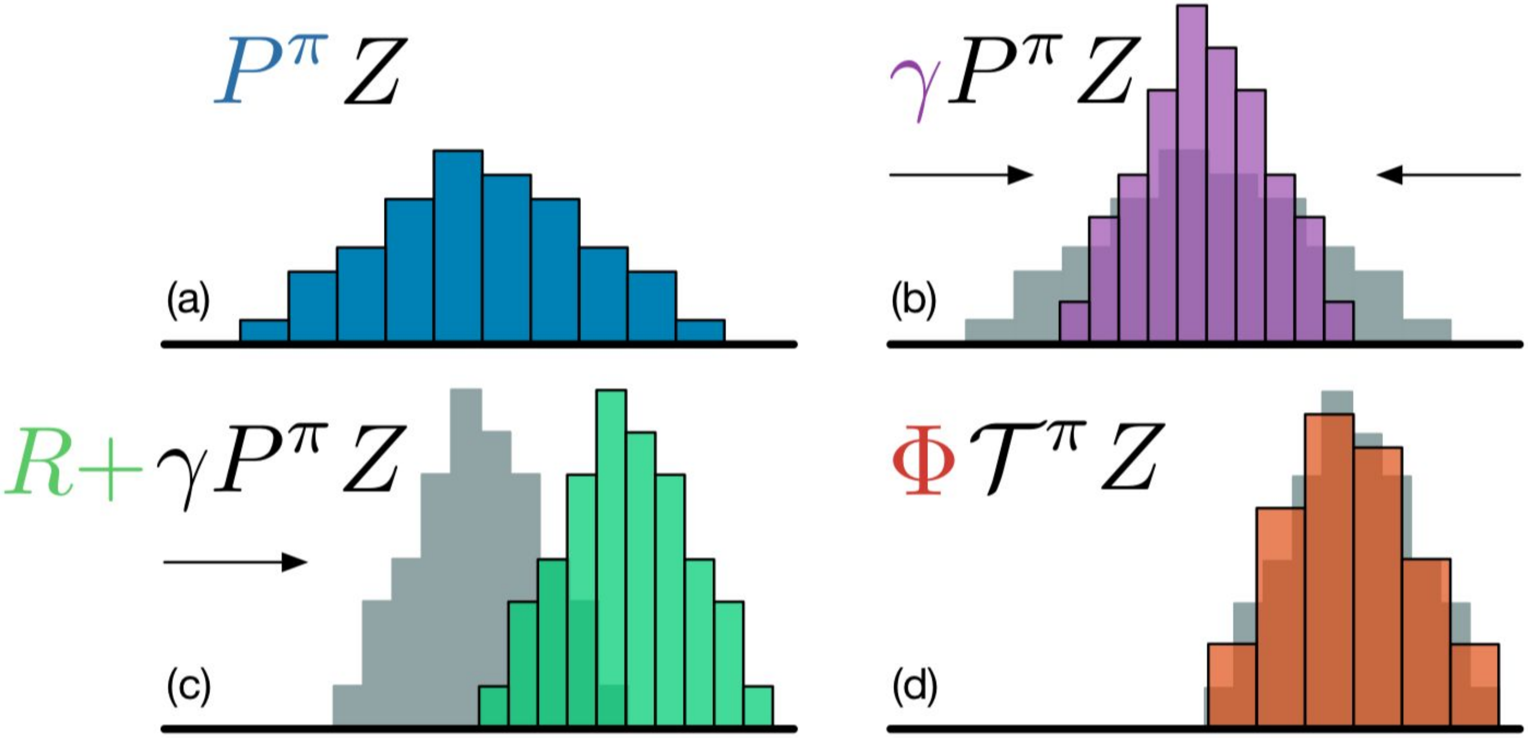

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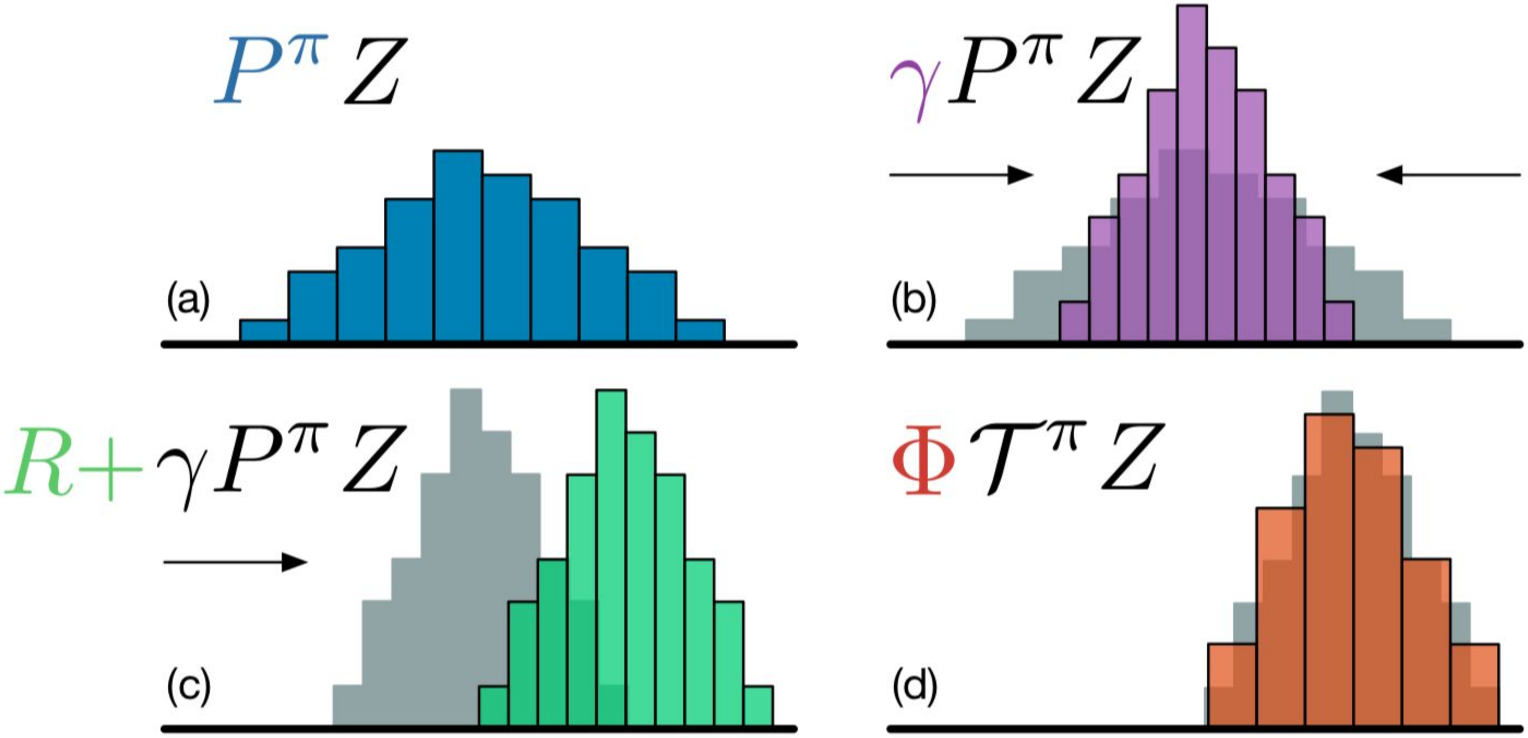
What About Control ?



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Distributional Bellman Backup Operator for Control for Maximizing Expected Reward

$$\mathcal{T}Q(x,a) = \mathbb{E}\,R(x,a) + \gamma\,\mathbb{E}_P \max_{a' \in \mathcal{A}} Q(x',a').$$

Figure from Bellemare, Dabney, Munos ICML 2017

Maximal Form of Wasserstein Metric on 2 Distributions

$$\mathcal{T}Q(x,a) = \mathbb{E}\,R(x,a) + \gamma\,\mathbb{E}_P \max_{a' \in \mathcal{A}} Q(x',a').$$

$$\bar{d}_p(Z_1, Z_2) := \sup_{x,a} d_p(Z_1(x,a), Z_2(x,a)).$$

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Distributional Bellman Backup Operator for Control for Maximizing Expected Reward **is Not a Contraction**

$$\mathcal{T}Q(x,a) = \mathbb{E}\,R(x,a) + \gamma\,\mathbb{E}_P \max_{a' \in \mathcal{A}} Q(x',a').$$

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⇒ Suggests convergence results may be hard

Figure from Bellemare, Dabney, Munos ICML 2017

Goal: Quickly and Efficiently use RL to Learn a Risk-Sensitive Policy using Conditional Value at Risk

$$\mathcal{T}Q(x,a) = \mathbb{E}\,R(x,a) + \gamma\,\mathbb{E}_P \max_{a' \in \mathcal{A}} Q(x',a').$$

$$d_p(Z_1, Z_2) := \sup_{x,a} d_p(Z_1(x,a), Z_2(x,a)).$$

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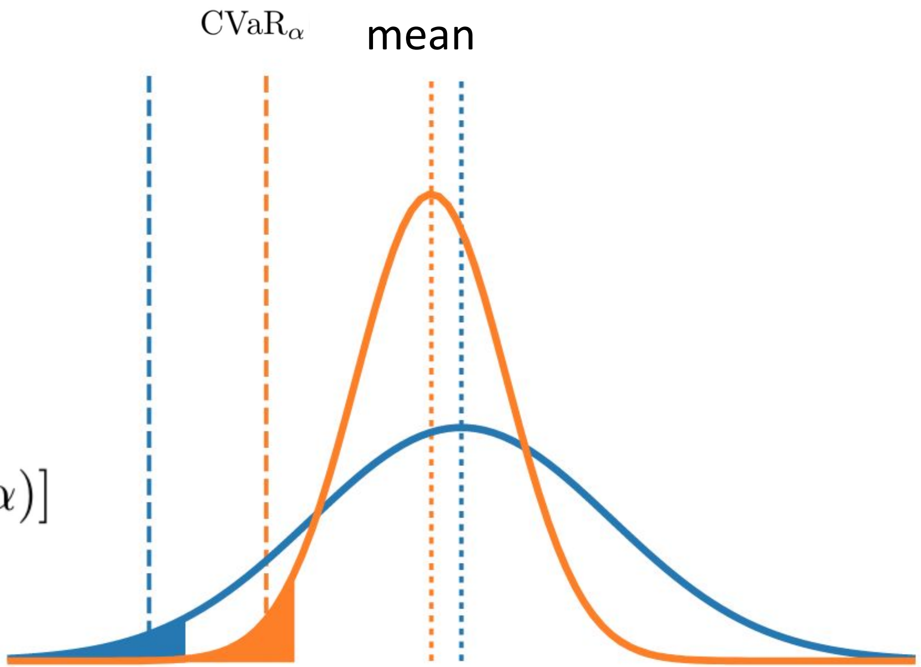
Figure from Bellemare, Dabney, Munos ICML 2017

Conditional Value at Risk for a Decision Policy

- Risk-level *alpha* in (0, 1]
- Expected sum of rewards of a policy in worst *alpha*-fraction of cases

$$F^{-1}(u) = \inf\{x : F(x) \geq u\}$$

$$\mathrm{CVaR}_\alpha(F) = \mathbb{E}_{X \sim F}[X | X \leq F^{-1}(\alpha)]$$



Goal: Sample Efficient RL to Optimize Conditional Value at Risk

- Risk-level *alpha* in (0, 1]
- Expected sum of rewards of a policy in worst *alpha*-fraction of cases

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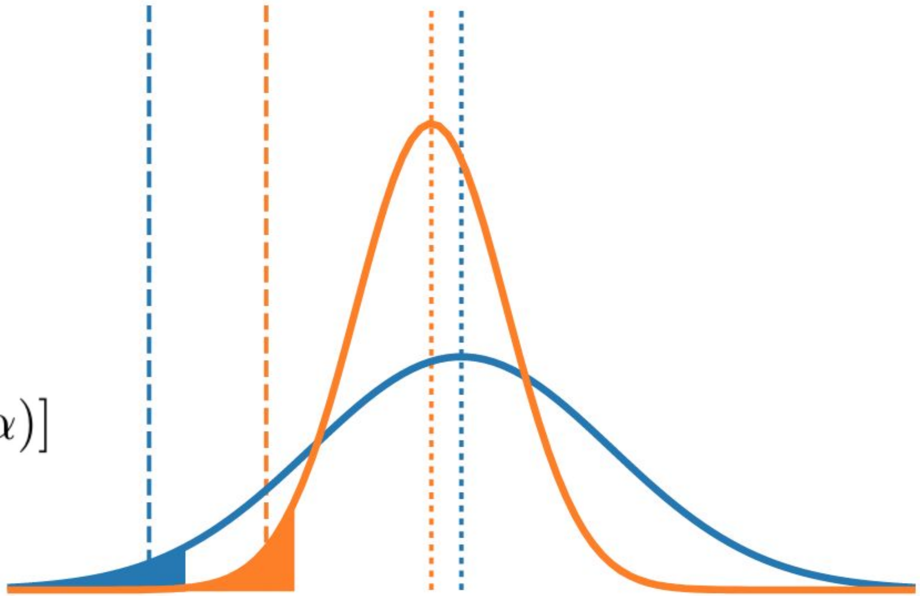

For Inspiration, Look to Sample Efficient Learning for Policies that Optimize Expected Reward

- Risk-level *alpha* in (0, 1]
- Expected sum of rewards of a policy in worst *alpha*-fraction of cases

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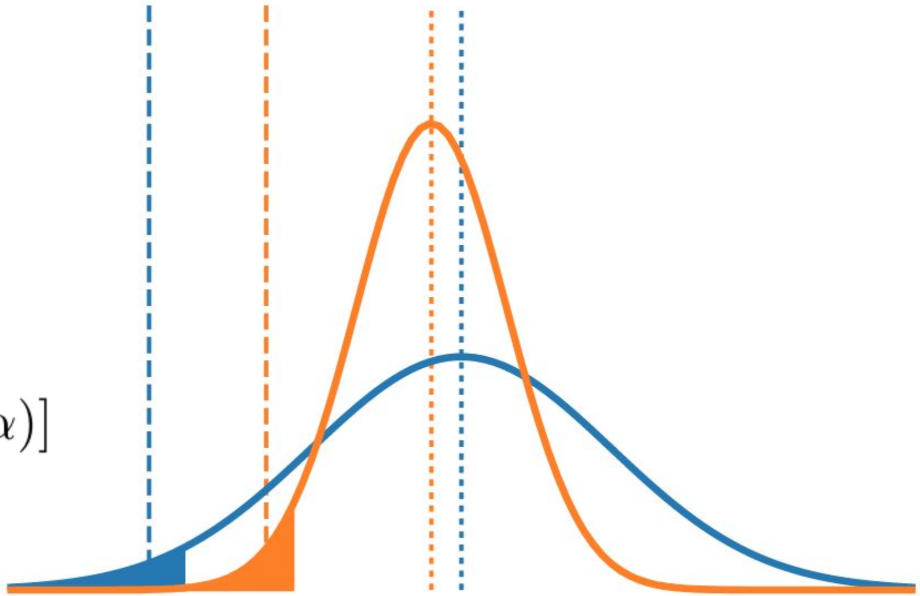

Problem Dependent Analysis

Lower Bound

$\tilde{O}\left(\left(\frac{SAH^2}{\epsilon^2}+\frac{S^2AH^3}{\epsilon}\right)\ln\frac{1}{\delta}\right)$.

(Dann, Wei, Li, B. 2019)

Efficient Exploration

(Dann & B 2015)

$\tilde{O}(\frac{|\mathcal{S}|^2|\mathcal{A}|H^2}{\epsilon^2}\ln\frac{1}{\delta})$

(Kakade 2003; Strehl & Littman 2005)

$\tilde{O}\left(\frac{S^2A}{\epsilon^3(1-\gamma)^6}\right)$

No Intelligent Exploration

$O(A\ S^H)$

PAC

Regret

$\tilde{O}(\sqrt{\mathbb{Q}^*SAT})$

(Zanette & B 2019)

$\tilde{O}(\sqrt{HSAT})$

(Azar et al. 2017)

$\tilde{O}(S\sqrt{HAT})$

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$\tilde{O}(H\sqrt{SAT})$

(Dann & B 2015)

$\tilde{O}(HS\sqrt{AT})$

(UCRL2, Jaksch et al. 2010)

$O(T)$

(greedy or epsilon-greedy)

$\mathbb{Q}^*$**: problem dependent constant that does not need to be known**

S: # states
A: # actions
T: # steps
H: time horizon

Problem Dependent Analysis

Lower Bound

Efficient Exploration

No Intelligent Exploration

$$\tilde{O}\left(\left(\frac{SAH^2}{\epsilon^2} + \frac{S^2AH^3}{\epsilon}\right)\ln\frac{1}{\delta}\right).$$

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$$\tilde{O}\left(\frac{S^2A}{\epsilon^3(1-\gamma)^6}\right)$$

$O(A\ S^H)$

PAC

Regret

We now have minimax bounds for regret and PAC learning (Dann, Wei, Li, B ICML 2019) in tabular episodic MDPs

Approaches use optimism under uncertainty

$\tilde{O}(\sqrt{\mathbb{Q}^* SAT})$

(Zanette & B 2019)

$\tilde{O}(\sqrt{HSAT})$

(Azar et al. 2017)

$O(S\sqrt{HAT})$

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Optimimism Under Uncertainty for Standard RL

1. Compute an optimistic estimate of Q(s,a)

2. Select the action which maximizes optimistic Q

Optimimism Under Uncertainty for Standard RL: Use Concentration Inequalities

1. Compute an optimistic estimate of Q(s,a):

$$\underbrace{|Q^\star(s,a) - \widehat{Q}^\star(s,a)|}_{\text{Gap between optimal and estimated}} \lessapprox \frac{H}{\sqrt{n}}$$

(Hoeffding Inequality)

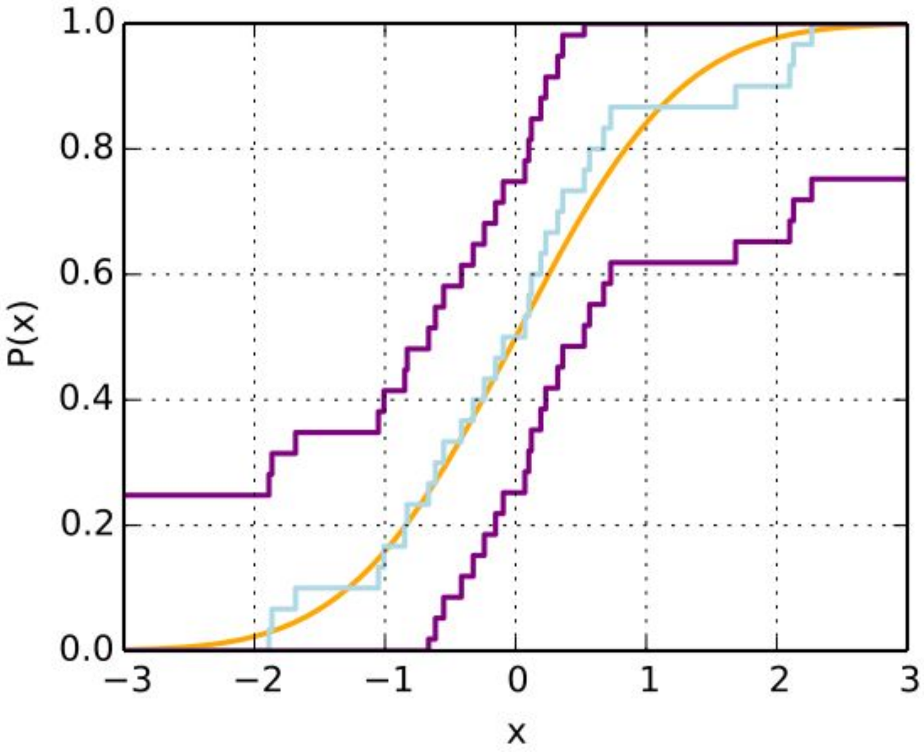
2. Select the action which maximizes optimistic Q

Suggests a Path for Sample Efficient Risk Sensitive RL

1. Compute an optimistic estimate of **distribution** of Q(s,a)

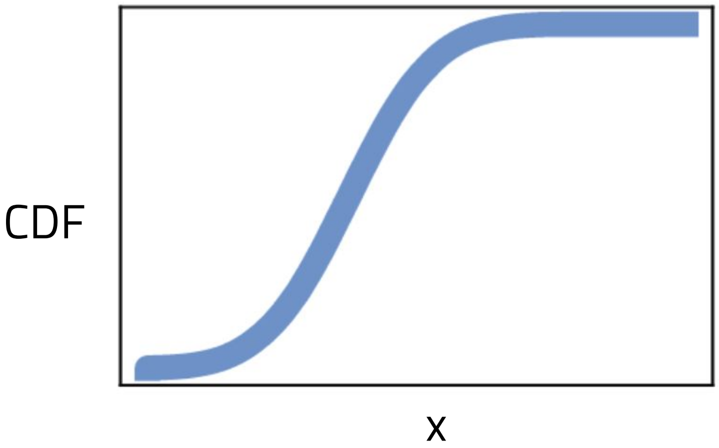
2. Select the action which maximizes **cVar** (Q(s,a)

Use DKW Concentration Inequality to Quantify Uncertainty over Distribution



Source: Wikipedia

Creating an Optimistic Estimate of Distribution of Returns



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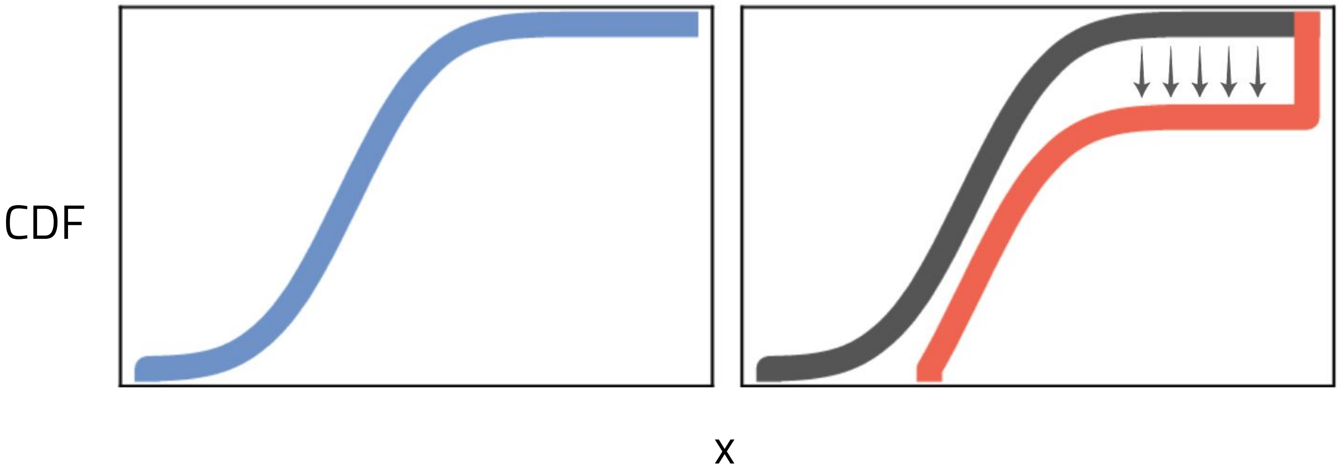

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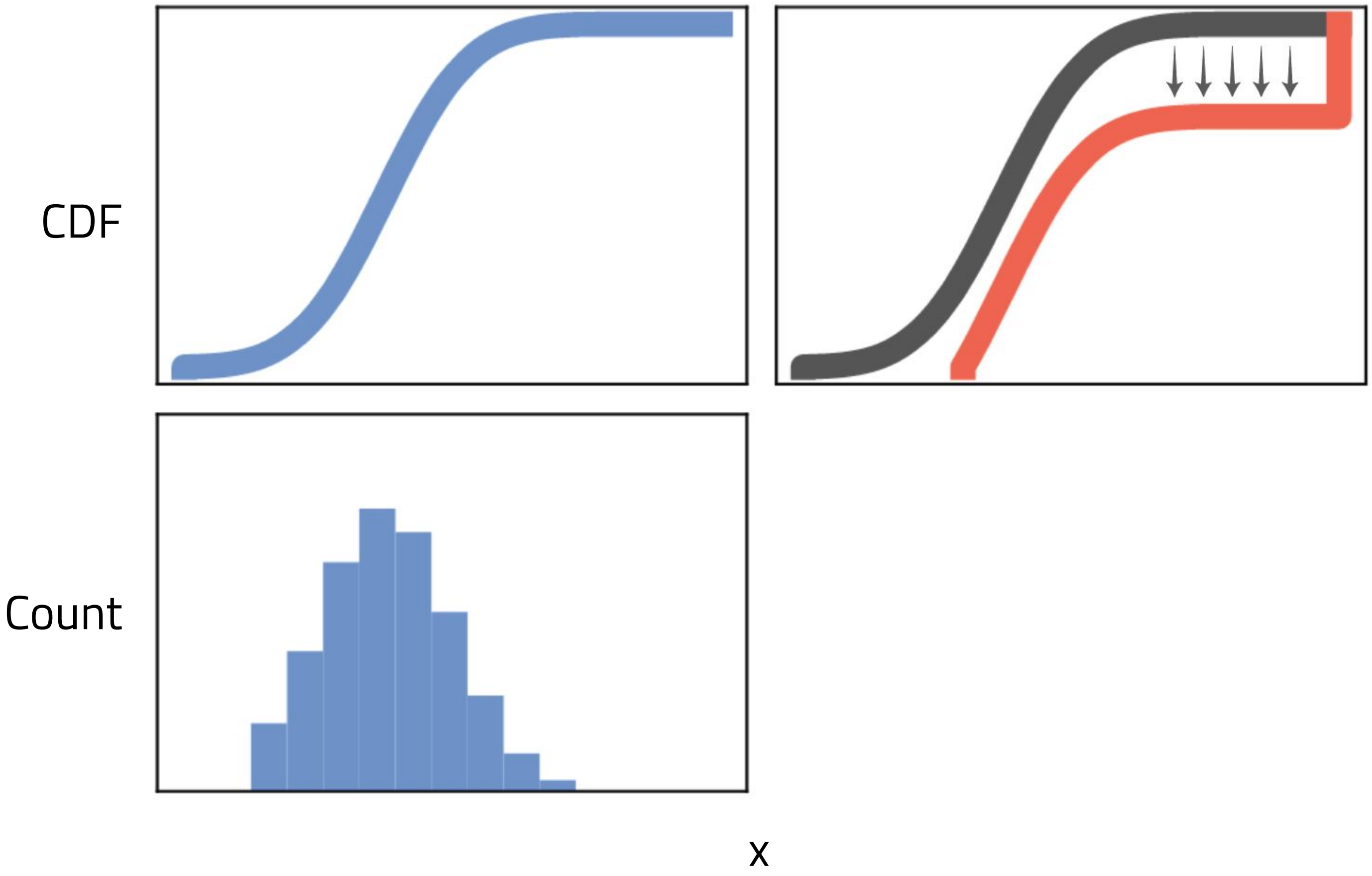

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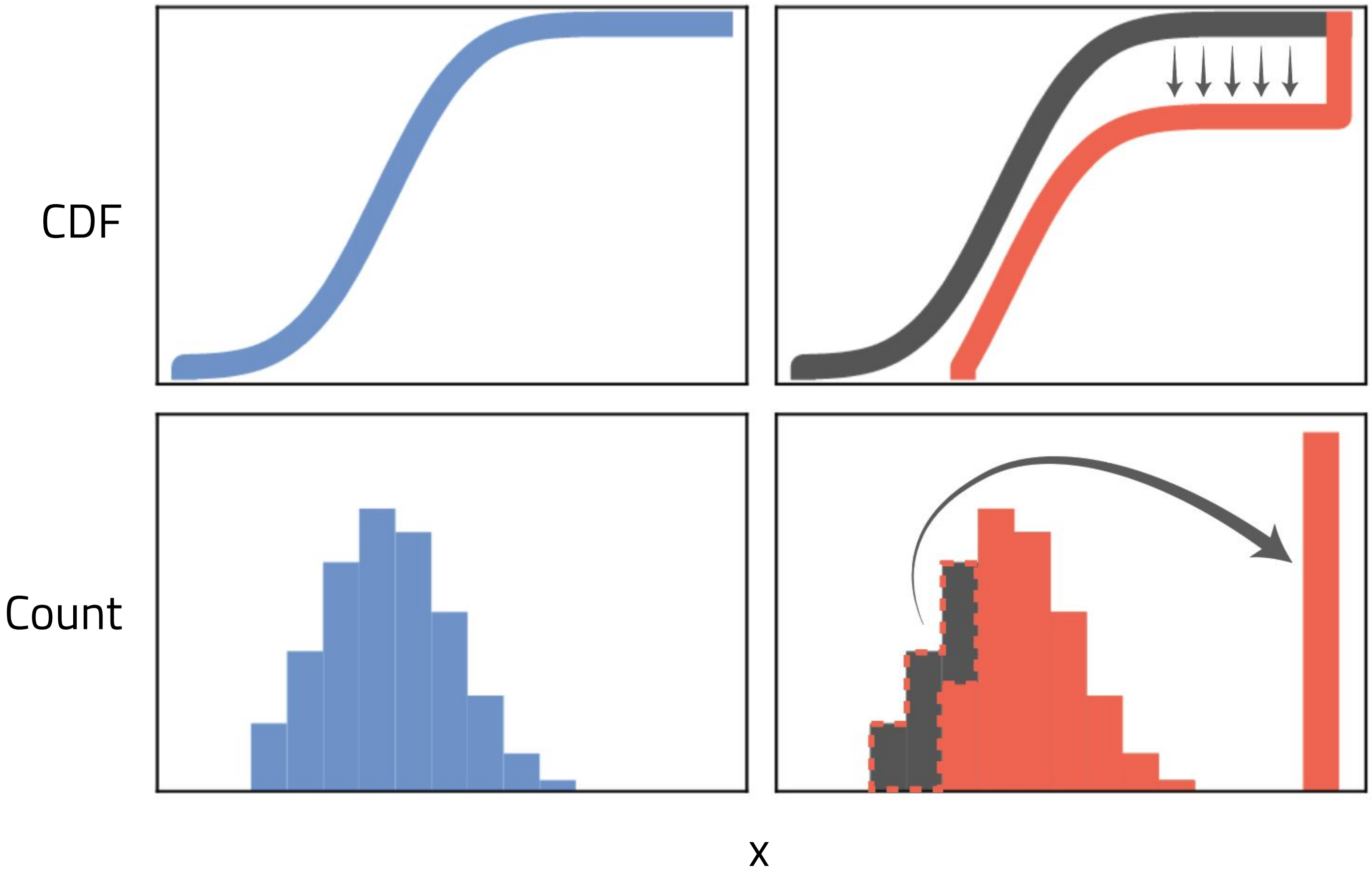
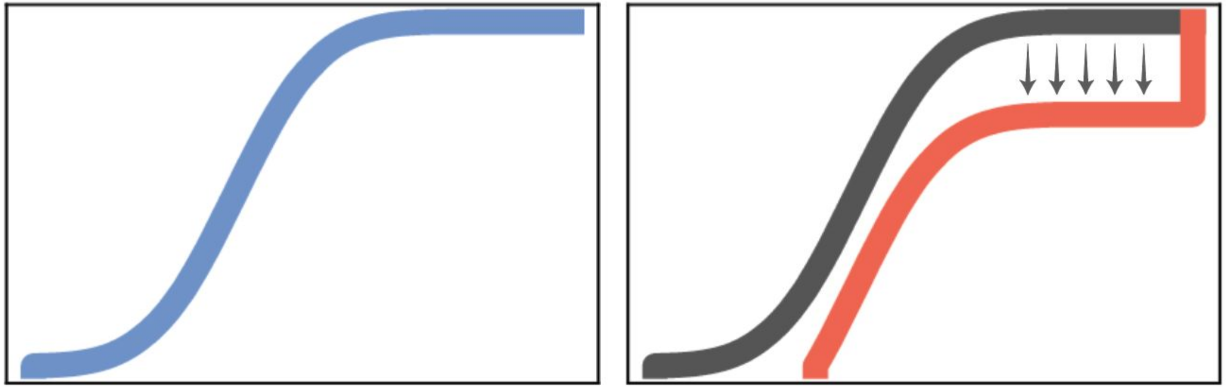


Optimism Operator Over CDF of Returns

CDF



$$F_{O_c Z(s,a)}(x) = \left(F_{Z(s,a)}(x) - c \frac{\mathbf{1}\{x \in [V_{\min}, V_{\max})\}}{\sqrt{n(s,a)}} \right)^{+}$$

Optimistic Risk Sensitive RL

1. Compute an optimistic estimate of **distribution** of Q(s,a)

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2. Select the action which maximizes **cVar** (Q(s,a)

Optimistic Operator for Policy Evaluation Yields Optimistic Estimate

Theorem 2. *Let the shift parameter in the optimistic operator be sufficiently large which is* $c = O\left(\ln(|\mathcal{S}||\mathcal{A}|/\delta)\right)$. *Then with probability at least* $1 - \delta$, *the iterates* $\mathrm{CVaR}_\alpha((O_c\hat{\mathcal{T}}^\pi)^m Z_0)$ *converges for any risk level* α *and initial* $Z_0 \in \mathcal{Z}$ *to an optimistic estimate of the policy's conditional value at risk. That is, with probability at least* $1-\delta$,

$$\forall s, a : \mathrm{CVaR}_\alpha((O_c\hat{\mathcal{T}}^\pi)^\infty Z_0(s,a)) \geq \mathrm{CVaR}_\alpha(Z_\pi(s,a)).$$

Concerns about Optimistic Risk Sensitive RL

1. Resulting actions may not be safe. Yes!
   - No guarantees on return for each episode
   - Not suitable for extremely high stakes scenarios

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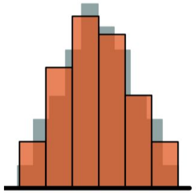
2. How do we compute optimistic distributions with infinite state spaces?

Optimistic Exploration for Risk Sensitive RL in Continuous Spaces

1. Maintain discretized representation of optimistic distrib of returns (similar C51, Bellemare, Dabney, & Munos 17)



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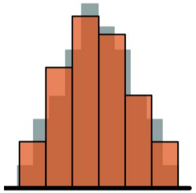

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2. For current state, for each action *a*
   - Compute CDF of current distributional Q^o for *a*
   - Apply optimism operator

Recall Optimistic Operator for Distribution of Returns for Discrete State Spaces, Uses Counts

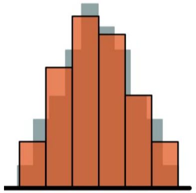
$$F_{O_c Z(s,a)}(x) = \left(F_{Z(s,a)}(x) - c \frac{\mathbf{1}\{x \in [V_{\min}, V_{\max})\}}{\sqrt{n(s,a)}} \right)^{+}$$

Optimistic Operator for Distribution of Returns for **Continuous** State Spaces, Uses **Pseudo**-Counts

$$F_{O_c Z(s,a)}(x) = \left(F_{Z(s,a)}(x) - c \frac{\mathbf{1}\{x \in [V_{\min}, V_{\max})\}}{\sqrt{n(s,a)}} \right)^{+}$$

$$\hat{n} = \frac{1}{\exp(\kappa t^{-1/2} \alpha (\nabla \log \rho_\theta (s_{t+1}, a'))^2) - 1}$$

Optimistic Exploration for Risk Sensitive RL in Continuous Spaces



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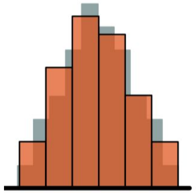

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2. For current state, for each action *a*
   - Compute CDF of current distributional Q^o for *a*
   - Apply optimism operator
3. Choose action that maximizes $Cvar_{alpha}(Q^o(s,a)$
4. Update optimistic distribution of returns

Simulation Experiments

Baseline Algorithms

- Epsilon-greedy CVaR

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- IQN epsilon-greedy CVaR: implicit quantile network (IQN) that also uses -greedy method for exploration (Dabney et al. 2018). Used dopamine implementation of IQN (Castro et al. 2018)

Baseline Algorithms

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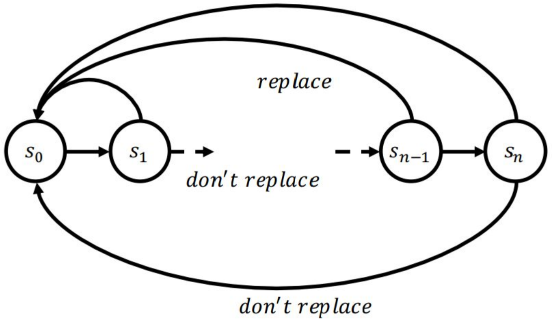
- IQN epsilon-greedy CVaR: implicit quantile network (IQN) that also uses -greedy method for exploration (Dabney et al. 2018). Used dopamine implementation of IQN (Castro et al. 2018)

- CVaR-AC: An actor-critic method that maximizes the expected return while satisfying an inequality constraint on the CVaR (Chow and Ghavamzadeh 2014). Relies on stochastic policy for exploration.

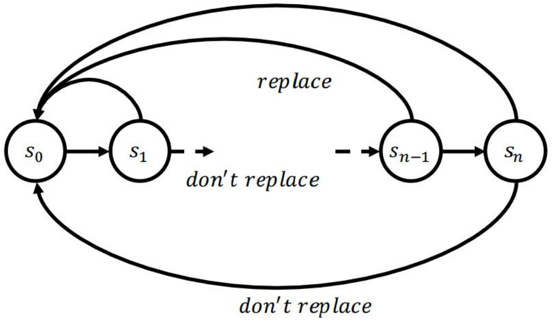
Simulation Domains

Machine Repair



Simulation Domains

Machine Repair



Structured Treatment simulator for HIV [Ernst et al CDC 2006]

- Simulator state: Infected CD4+ T-lymphocytes, number of infected macrophages, the number of free virus particles, …
- Action: start / stop treatment
- Reward is a function of cytotoxic T-lymphocytes

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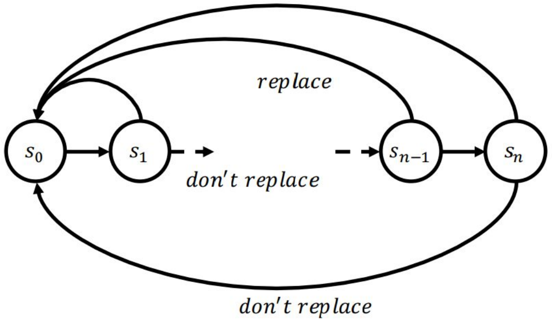

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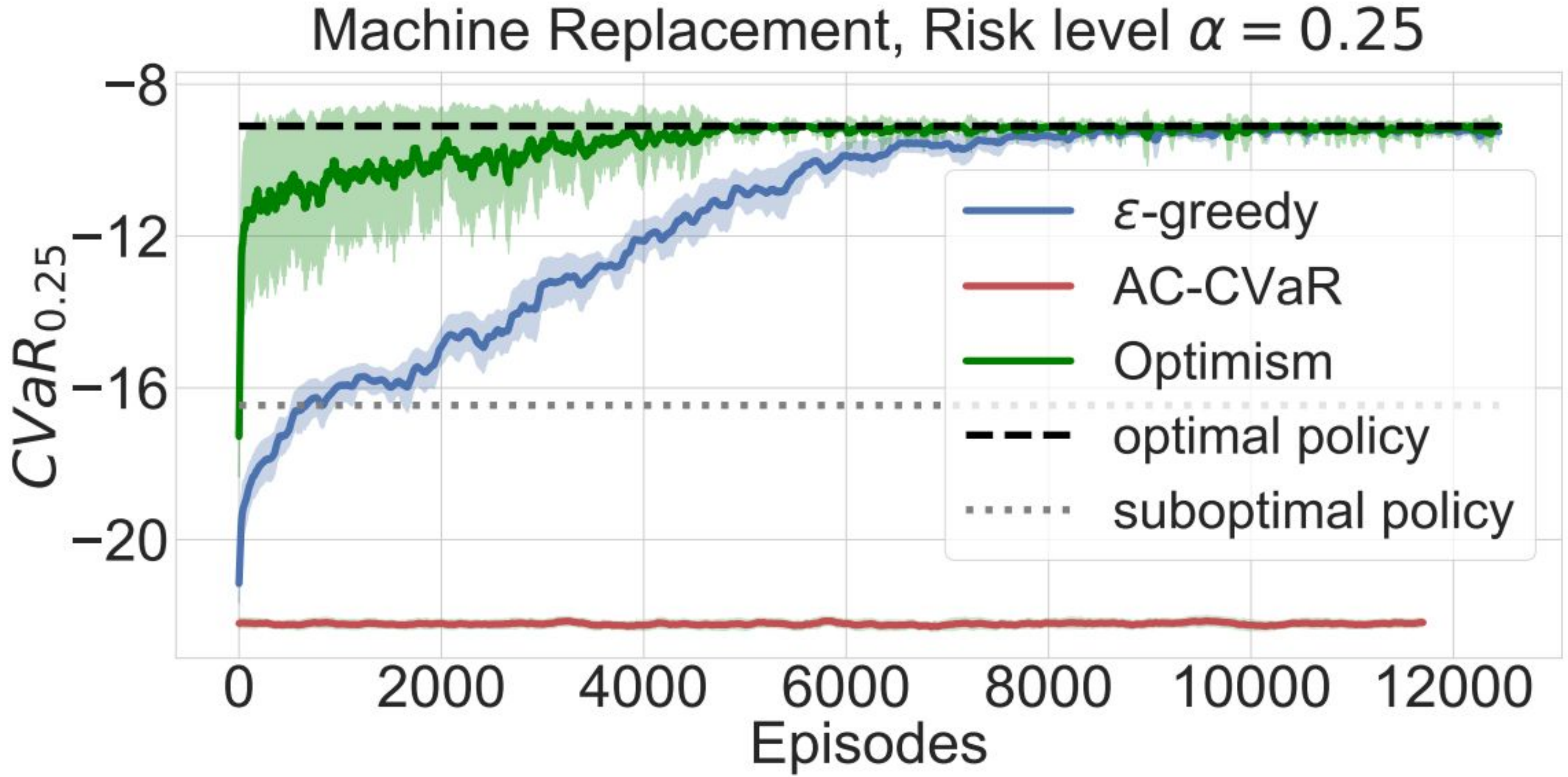
Diabetes Blood Glucose Control Simulator [Man et al]

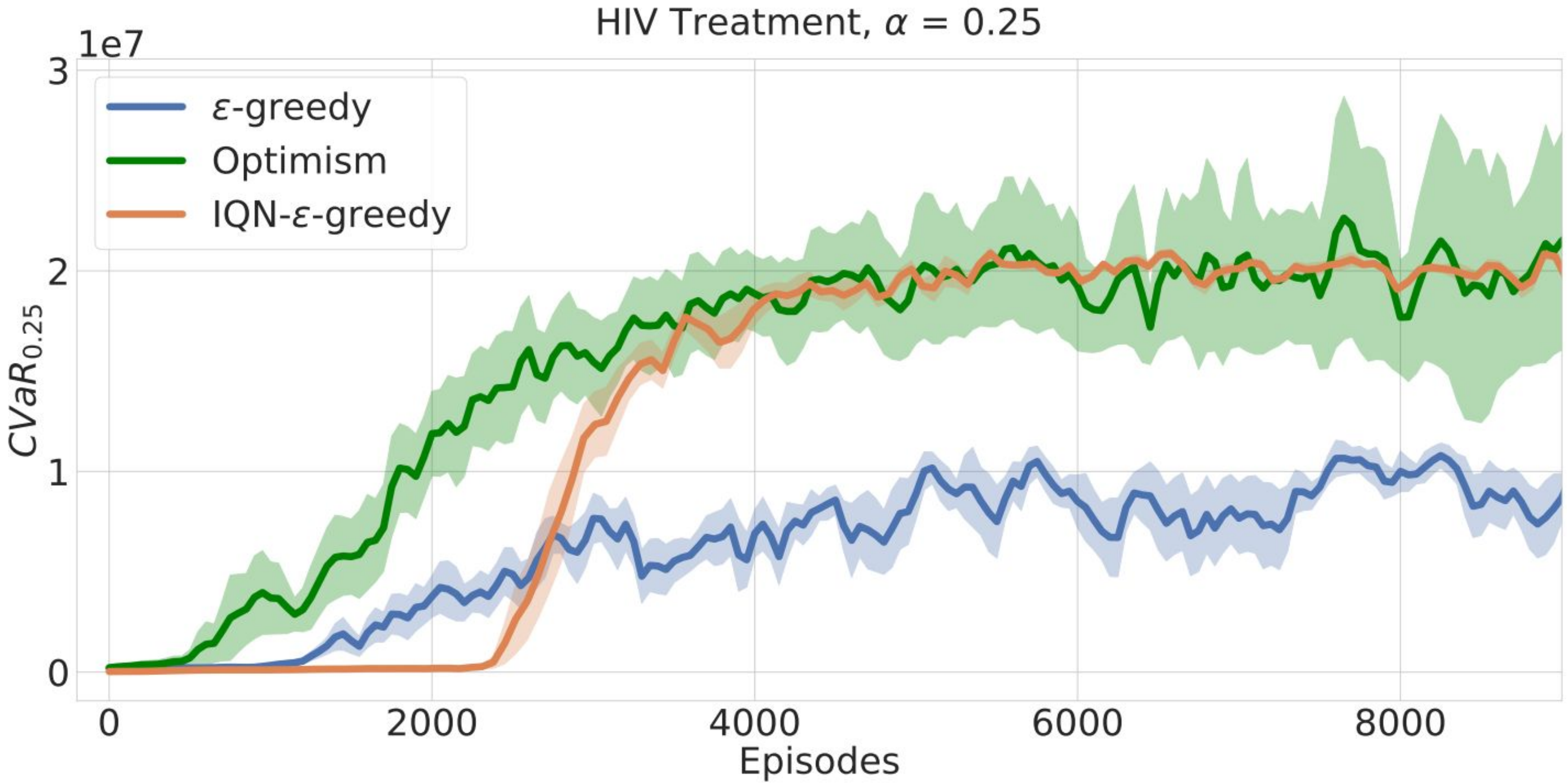
- Simulator state: Blood glucose (bg) + carb intake
- Action: 6 bolus insulin dosage injection levels
- Reward

$$r(bg) = \begin{cases} -\frac{(bg'-6)^2}{5} & if\ bg' < 6 \\ -\frac{(bg'-6)^2}{10} & if\ bg' \geq 6 \end{cases}$$

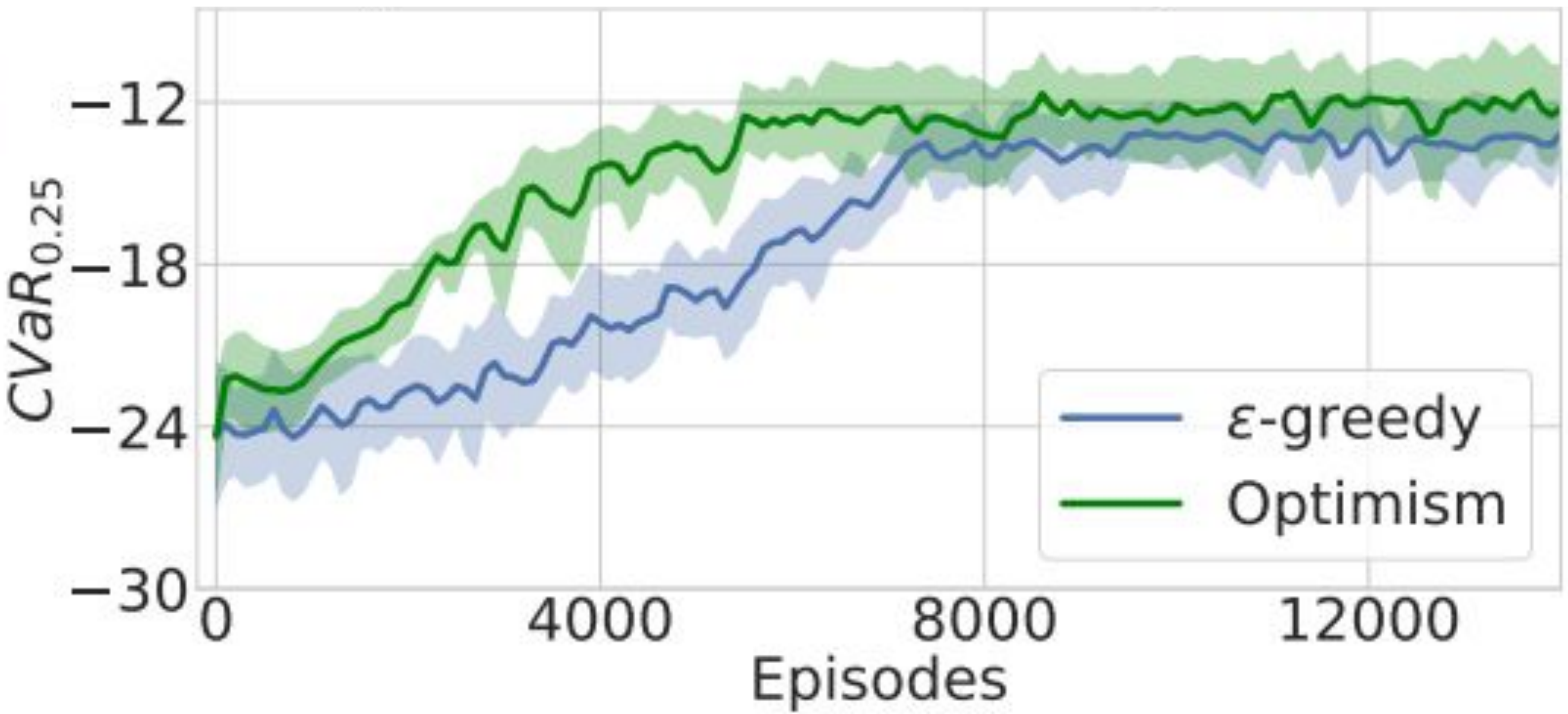


Machine Replacement, Risk level $\alpha = 0.25$
$CVaR_{0.25}$
−8
−12
−16
−20
0
2000
4000
6000
8000
10000
12000
Episodes
ϵ-greedy
AC-CVaR
Optimism
optimal policy
suboptimal policy

HIV Treatment

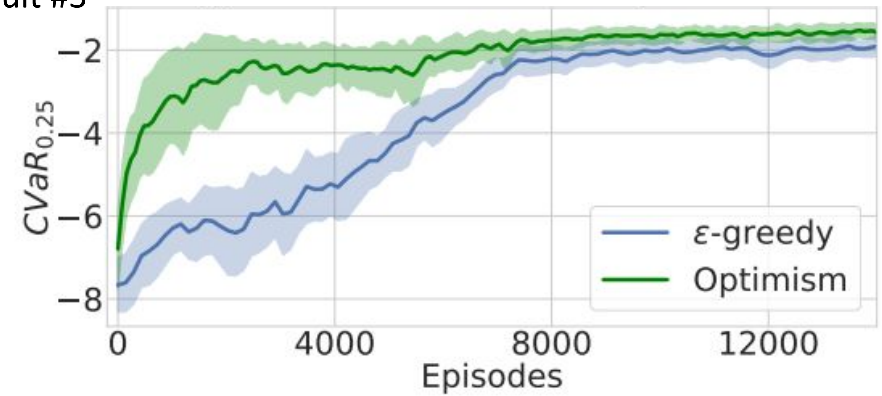


Blood Glucose Simulator, Adult #5

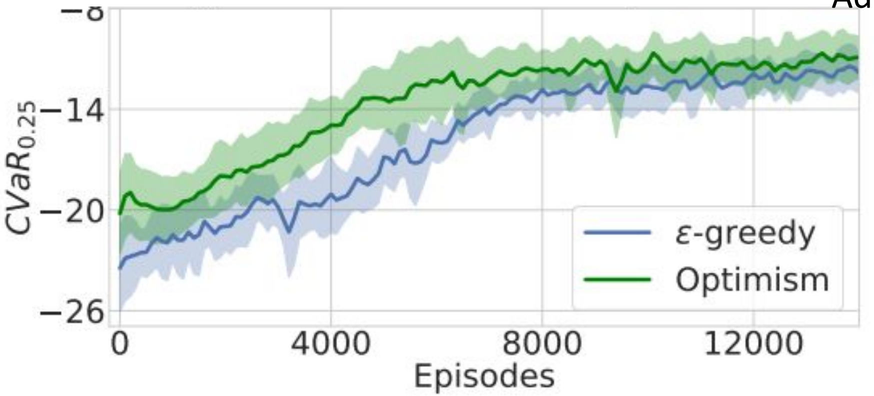


Blood Glucose Simulator, 3 Patients

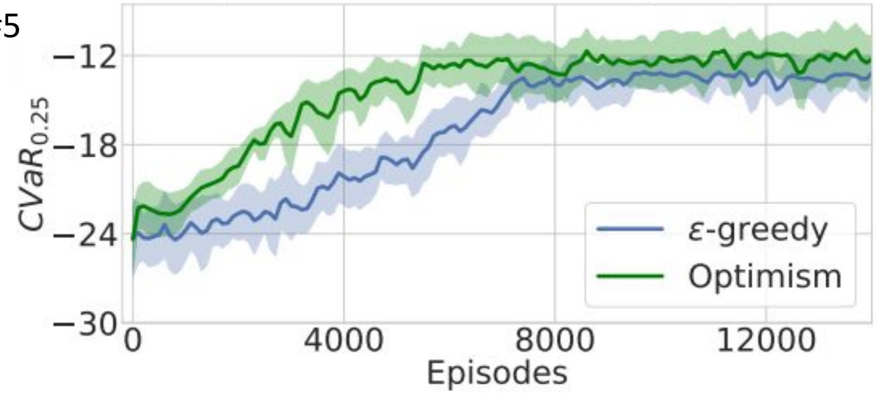
Adult #3



Adult #4



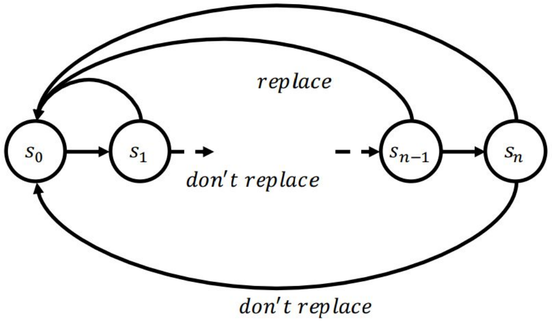
Adult #5



Hyperparameters optimized from held out patient for each algorithm, then fixed

In All 3 Domains, Optimism Significantly Speed Learning Optimal CVaR Policy

Machine Repair



Structured Treatment simulator for HIV [Ernst et al CDC 2006]

- Simulator state: Infected CD4+ T-lymphocytes, number of infected macrophages, the number of free virus particles, …
- Action: start / stop treatment
- Reward is a function of cytotoxic T-lymphocytes

Diabetes Blood Glucose Control Simulator [Man et al]

- Simulator state: Blood glucose (bg) + carb intake
- Action: 6 bolus insulin dosage injection levels
- Reward

$$r(bg) = \begin{cases} -\frac{(bg'-6)^2}{5} & if\ bg' < 6 \\ -\frac{(bg'-6)^2}{10} & if\ bg' \geq 6 \end{cases}$$

A Sidenote on Safer Exploration: Faster Learning also Reduces # of Bad Events During Learning

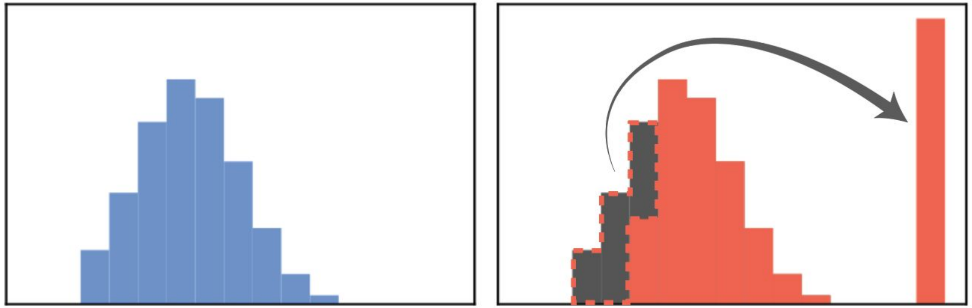
	ϵ-greedy	CVaR-MDP
Adult#003	11.2% ± 3.6%	**4.2% ± 2.3%**
Adult#004	2.3% ± 0.3%	**1.4% ± 0.6%**
Adult#005	3.3% ± 0.3%	**1.7% ± 0.6%**

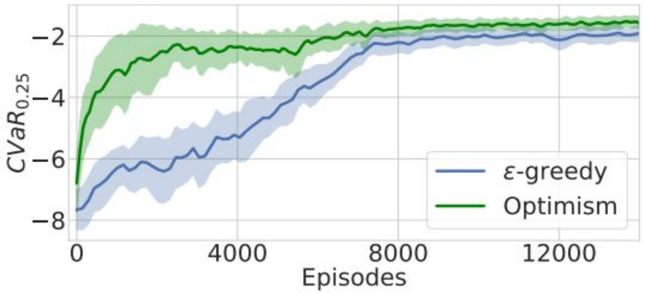
Figure 6: Type 1 Diabetes simulator, percent of episodes where patients experienced a severe medical condition (hypoglycemia or hyperglycemia), averaged across 10 runs

Many Interesting Open Directions

- Optimism operator is over the returns, could be used when policy is to maximize other features of the return (worst case, other statistics)
- Sample complexity bounds
  - Requires progress on distributional Bellman backup operator
- Combining safe exploration and fast learning
- Other forms of constrained learning
- Robustness to misspecification and adversarial inputs
- Learning robust policies to handle nonstationarity and covariate shift

Optimisim for Conservatism: Fast RL for Learning Conditional Value at Risk Policies





- Compute optimistic estimate of distribution of returns
- Easy to incorporate into existing distributional deep RL algorithms
- Enables substantially faster learning of CVaR policies in our simulations