Report of the 2008-2009 IAS special year program in Geometric Partial Differential Equation

Looking back ten years from now, the program in 2008-2009 has made great impact in the recent developement of geometric analysis. Major development includes:

(1) One of the theme is to study the connection between fully non-linear PDE and the upcoming new development in Optimal transport method. A number of leaders in this field have been invited for long term visits and gave series of talks –Luis Caffarelli, Neil Trudinger, Cedric Villani among them. Cedric Villani later won the field medal in 2014, partially based on his work done at IAS on the special year, his joint work with A. Figalli (field medal in 2018) was also partially done at IAS. This field has developed exceptionally well, with connections also to important problems in differential geometry.

(2) The other main theme is the connection between conformal geometry and non-linear PDE. A group of young mathematicians have been invited for long term visits–Andrea Malchiodi, Fernando Marquez, Matthew Gursky among them. F. Marquez (jointly with S. Brendle and R Schoen) solved an important problem on the compactness of the Yamabe metrics, he also developed the min-max method and together with A. Neves solved the famous Willmore problem; he was invited to be the distinguished visiting professor at IAS in charge of the special year program in Calculus of Variation and Geometric analysis in year 2018-2019. Matthew Gursky and Anrea Malchioid developed a long term co-workers relation, together they have made important contribution to the study of Q-curvatures, and the study of the log-determinant functionals in conformal geometry.

(3) One of the central problem in conformal geometry in conjunction to conformal field theory is the study the existence ”conformal compact Einstein fill in” with a given conformal infinity. There have been progress on the existence, non-existence, compactness issues related to the problem by Gursky-Han-Stolz, Chang-J. Qing-P. Yang, Chang-Y. Ge and J.Qing partially based on discussions started in the special year.