IAS SUMMER COLLABORATORS REPORT

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This document provides a synopsis of what we accomplished during our two week collaboration meeting from June 17-30, 2018.

Coefficients of peak polynomials. Three days prior to our arrival at IAS, Ezgi Kantarci Oğuz posted the following paper on the arxiv: *Connecting descent and peak polynomials*. In this paper, Oğuz gave an expansion of descent polynomials in terms of peak polynomials and used the expansion to give a combinatorial interpretation of the coefficients of the peak polynomial in a binomial basis. Our previous work presented a proof of a conjecture regarding the positivity of the coefficients when expanding peak polynomials in a binomial basis and we gave a combinatorial interpretation for the meaning of the coefficients in the descent setting. Given the connections in these works, we spent a large portion of our time at IAS reading Oğuz's paper thoroughly and were able to determine counterexamples to the main result presented in the manuscript. During the two week period we corresponded with Oğuz and were able to provide her with feedback on her original manuscript to the point that she now has a correct result and proof of the interpretation of these coefficients. This required the writing of over a thousand lines of code to provide counterexamples and substantial computational evidence of the correctness of the new statements.

Pinnacles of permutations. In a second direction of research, we began a project on the study of pinnacles of permutations. This work was motivated by the following question posed by Robert Davis, Sarah A. Nelson, T. Kyle Petersen, Bridget E. Tenner in *The pinnacle set of a permutation*:

Question 4.2. For a given S, is there a class of operations (e.g., valley hopping) that one may apply to any $w \in S_n$ with $Pin(w) = S$ to obtain any other permutation $w' \in S_n$ with $Pin(w') = S$, and no other permutations?

In the spring of 2018, Isabella Huang (student of Pamela Harris) proved that the "valley hopping" operation, introduced by Foata and Strehl, preserved the pinnacles of a permutation, but that the orbits did not encompass all elements having the same set of pinnacles. During our meeting, we developed a new algorithm, which used valley hopping, that answers Question 4.2 in the affirmative and a manuscript is in the writing stages.

Peaks on graphs. During our meeting we edited one of our previous manuscripts. The manuscript extends the notion of peaks of permutations to peaks of labeled graphs. It contains an algorithm that enumerates the number of labelings of a graph with peaks at

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a predetermined set of vertices. In order to improve the exposition, we rewrote the paper *Counting peaks on graphs* and submitted it for publication.

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