EMERGING TOPICS WORKSHOP ON INTERACTIONS OF GEOMETRIC AND VARIATIONAL METHODS

1. Description of topic

Between April 9–13, 2018, the IAS hosted a workshop on Emerging Topics focused on geometric and variational methods in the Arnold diffusion problem, and related topics.

The Arnold diffusion problem is concerned with the phenomena of global instability in nearly integrable Hamiltonian system. It has originated with a conjecture by V.I. Arnold in 1964, that 'generic' integrable systems subjected to arbitrarily small, 'generic' perturbations, have 'diffusing orbits' that travel some distance that is independent of the size of the perturbation. A related problem, possed by Chirikov in 1979, is concerned with describing the stochastic process exhibited by diffusing orbits. A fundamental role in pursuing Arnold's conjecture, and in providing many ideas and inspiration, was played by John Mather.

The last several years have witnessed significant progress on the Arnold diffusion problem, and the emergence of new geometric approaches, as well as approaches at the confluence of geometric and variational methods.

This workshop was focused on a particular combination of such techniques. A first focal point was on the methods based on normally hyperbolic invariant manifolds, scattering maps, and shadowing lemmas, that were developed in works by Delshams, Gidea, de la Llave, Seara, in order to prove diffusion in the so called 'a priori unstable' case of the Arnold diffusion problem. A second focal point was on a method to obtain diffusing orbits by combining classical hyperbolic methods with Mather's theory, developed in works by Bernard, Kaloshin and Zhang, in the 'a priori stable' case, for two-and-a-half degrees of freedom Hamiltonian systems. A third focal point was on the construction of compact invariant cylinders with boundary, located near simple or double resonances, and the construction of diffusing orbits along these cylinders, by intertwining scattering dynamics and twist dynamics, developed by Marco and Gidea, in the 'a priori stable' case, for three-degrees of freedom Hamiltonian systems. All these advances were discussed at the workshop, which contributed to a better understanding of the different components of these works.

Another topic that was actively discussed is an approach of proving stochastic Arnold diffusion, developed by Guardia, Kaloshin, Zhang, for certain classes of a priori unstable systems. This approach not only provides existence of diffusing orbits, but also show that in a certain time scale small fluctuations accumulate to a stochastic diffusion process. This gives a partial justification for the name of the phenomenon: Arnold diffusion. Stochastic Arnold diffusion is being also investigated by Gidea and Capinski, in the context of the planar elliptic restricted three-body problem, through a computer assisted proof.

The methods mentioned above are of interest not only for the purpose of overcoming the Arnold diffusion problem, but also from the point of view of applications. Indeed, one informal way to describe this problem is that 'small perturbations can accumulate to large effects'. The range of applications include space mission design – when one would like to exploit instability to produce efficient trajectories –, and the dynamics in particle accelerators and plasma confinement – when one would like to prevent instability to disintegrate the system.

2. Participants

The workshop was organzied by Vadim Kaloshin (Univ. of Maryland) and Marian Gidea (Yeshiva Univ.). The participants were:

- Marie-Claude Arnaud (Univ. d'Avignon)
- Abed Bounemoura (CNRS & CEREMADE – Univ. Paris-Dauphine)
- Amadeu Delshams (Univ. Politècnica de Catalunya)
- Jaques Féjoz (Univ. Paris-Dauphine & Observatoire de Paris)
- Marcel Guardia (Univ. Politècnica de Catalunya)
- Rafael de la Llave (Georgia Tech)
- Jean-Pierre Marco (Univ. Pierre and Marie Currie)
- Tere Seara (Univ. Politècnica de Catalunya)
- Ke Zhang (Univ. of Toronto).

Local participants included:

- Thomas Spencer (IAS)
- Lai-Sang Young (Courant Institute)
- Javier Gómez Serrano (Princeton Univ.)
- Edward Belbruno (Princeton Univ. & Yeshiva Univ.)
- Pablo Roldan (Yeshiva Univ.)

3. Organization

There were three colloquium-style lectures, on the state of the art on the Arnold Diffusion problem, as well as on future directions, by J-P. Marco, R. de la Llave, and K. Zhang. Also, there were five seminar-style lectures focused on specific techniques, applications, and related topics, by A. Delshams, T. Seara, M. Gidea, M. Guardia, and M-C. Arnaud. All talks were open to the general public and are available as video lectures on the IAS website. Ample time was devoted to collaborative work, on pursuing ongoing projects as well as on launching new research ideas.

4. Outcomes

The workshop can be viewed as a first step towards making connections between several works, and building new bridges between various approaches.

The participants also started to look at several new problems, in which they anticipate to make advancements. Some specific research directions include:

- Existence of diffusion in the case when the unperturbed Hamiltonian is not convex;
- Describing the stochastic process underlying Arnold diffusion, in general systems as well as in concrete examples (e.g., from celestial mechanics);
- Robust transitivity in large classes of nearly integrable Hamiltonian systems;
- Existence of diffusion in system with weak dissipation.

It is anticipated that many of the research problems started at the IAS will be continued during the MSRI program on "Hamiltonian systems, from topology to applications through analysis", to be held in Fall 2018. This will be attended by many of the workshop participants.

Also, we plan to write a joint paper that surveys the state of the art of the Arnold diffusion problem through the lens of geometric-variational approaches, providing a clear picture on what has been done so far, and what needs further work.