# PROGRESS REPORT: IAS SUMMER COLLABORATORS 2018 ON LOCALITY AND UNIVERSALITY OF TOPOLOGICAL QUANTUM COMPUTING

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This summer, Shawn Cui, Kevin Tian, Jennifer Vasquez, Helen Wong, and I started collaborating on two projects: one on the search of leakage-free entangling Fibonacci braiding gates, and one on the identification of the exact images of the mapping class group representations from the Ising or the closely related $SU(2)_2$ TQFT. Both problems have theoretical and practical significance for topological quantum computing. We have made enough progress on the first project to warrant a paper, and set up a plan for continuing collaboration for the second one. Therefore, I will solely focus on the first project in this report.

Fibonacci anyons are universal for quantum computing by braidings alone. They are believed to exist in fractional quantum Hall liquids at $\nu = \frac{12}{5}$, and realistic engineering designs are proposed with superconductor-Abelian quantum Hall heterostructure and topological superconductors. Quantum algorithms such as Shor's integer factoring are written in the quantum circuit model, and are not convenient for implementation using Fibonacci anyons due to the explicit tensor products in qubits. Moreover, the universality proof of Fibonacci anyons only promises efficient approximations of two-qubit entangling gates. It has been a long interesting open problem whether or not there are leakage-free entangling Fibonacci braiding gates.

Our focus was on two fronts: either proving the non-existence of leakage-free Fibonacci entangling gates, or finding protocols to generate good approximations adequate for the experimental construction of a Fibonacci quantum computer. We've accomplished the following:

- We found a systematic construction of leakage-free braiding gates, which are then proved to be non-entangling.
- We set up a computer search with up-to-date computing technology and found no leakage-free entangling gates either.
- We discovered a much simpler protocol to generate approximately leakage-free entangling Fibonacci braiding gates than algorithms in the existing literature.

The first two results provide some evidence that such leakage-free Fibonacci braiding gates do not exist. The time complexity of our approximation algorithm for a leakage-free entangling gate is comparable to the standard Solovay-Kitaev algorithm; however, our algorithm performs worse for the length of words. The gain in simplicity and geometric intuition justifies such a sacrifice.