1 Automorphic cohomology I
(General theory)

2 Automorphic cohomology II
(Carayol's work and an application)

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These two talks will be about automorphic cohomology in the non-classical case. By definition, automorphic cohomology are the groups $H^q(\Gamma\backslash D, L)$ where D is a homogeneous complex manifold $G_\mathbb{R}/H$, $G_\mathbb{R}$ is a real, semi-simple (or reductive) Lie group, H is a compact subgroup containing a compact maximal torus T, Γ is a discrete subgroup (usually an arithmetic one) of $G_\mathbb{R}$, and $L \to D$ is a homogeneous vector bundle. The non-classical case is when D does not fibre holomorphically over an Hermitian symmetric domain. If Γ, assumed for simplicity to operate freely on D, is co-compact or is an arithmetic group where we use L^2-cohomology $H^q_{(2)}(\Gamma\backslash D, L)$, one has

$$H^0_{(2)}(\Gamma\backslash D, L) = 0$$

for all L (there are no automorphic forms in the classical sense), whereas for Γ co-compact and "very regular" (to be defined in the talks), for one integer $d \neq 0$, dim D

$$H^d(\Gamma\backslash D, L) \qquad \text{is "big"}$$

while all of the other groups are zero. The central questions of these talks will be:

- (1) What is the geometric interpretation of automorphic cohomology?

- (2) In case Γ is an arithmetic group, what is the arithmetic interpretation of the automorphic cohomology? In particular, can one define and give properties of arithmetic automophic cohomology classes?

Automorphic cohomology first arose from Hodge theory in the 1960's, and it has been the subject of periodic study since then (c.f. Wells-Wolf [WW], Fels-Huckleberry-Wolf [FHW], and Williams [Wi]). But the progress on some of the deeper issues awaited the work of Eastwood-Gindikin-Wong [EGW] and, especially, Carayol [C1], [C2], [C3].

The first talk will trace the historical development of the subject, including its differential-geometric origins, its connection to representation theory (realizations of the discrete series [S]), **n**-cohomology expressions for automorphic cohomology, the cycle spaces which give an initial geometric interpretation of automorphic cohomology, the Penrose transform methods in [EGW] which in a very special case give a more subtle such realization, and finally an introduction to [C1], [C2]. It will be of a general character.

The second talk will be on [C3], building on Carayol's talk at the IAS in the fall giving more details - especially those concerning the Hodge theoretic aspects. It will also give an application of [C2] dealing with the second question above as it relates to CM points in Mumford-Tate domains.

References

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