

# REPORT ON THE IAS SUMMER COLLABORATION: LOCAL COHOMOLOGY AND THICKENINGS OF PROJECTIVE VARIETIES

BHARGAV BHATT, MANUEL BLICKLE, GENNADY LYUBEZNIK, ANURAG K. SINGH, AND WENLIANG ZHANG

During our week at the IAS, May 27–June 2, 2018, we made progress on the following three topics:

**Applications of the Riemann-Hilbert correspondence.** By systematically exploiting the Riemann-Hilbert correspondence, we are able to recover several results on the local cohomology of polynomial rings of characteristic zero: these include foundational results of Ogus [Og], the work on Eulerian-graded  $\mathcal{D}$ -modules [MZ], and an interpretation of Bass numbers of local cohomology modules in terms of singular cohomology [LSW]. What is more, we can adapt the Riemann-Hilbert framework to obtain corresponding results in positive characteristic.

**Cohomology of thickenings.** Our earlier paper [BBSZ] included a version of the Kodaira vanishing theorem for thickenings of local complete intersection (lci) subvarieties of projective space over a field of characteristic zero. While the Kodaira vanishing theorem fails in positive characteristic, we proved the following *asymptotic* version, while at IAS: Let  $X$  be a closed lci subvariety of  $\mathbb{P}^n$  over a field of arbitrary characteristic, and let  $X_t$  denote its  $t$ -th thickening. Then there exists an integer  $c$ , such that for all  $t \geq 1$ , one has

$$H^k(X_t, \mathcal{O}_{X_t}(j)) = 0$$

for each  $k < \dim X$  and  $j < -ct$ . This answers a question raised in the recent paper [DM].

**Frobenius on the cohomology of thickenings.** One of our long-term goals has been a search for uniform results on the injectivity of Frobenius maps on cohomology groups; this is related to universal bounds for the vanishing of the Hasse invariant on families of hypersurfaces. For example, we proved that for a smooth hypersurface  $X$  in  $\mathbb{P}^n$ , over a field of characteristic  $p > n$ , the Frobenius map

$$\tilde{F}_n: H^{n-1}(X, \mathcal{O}_X) \longrightarrow H^{n-1}(X_n, \mathcal{O}_{X_n}),$$

is injective; here  $X_n$  denotes the  $n$ -th thickening of  $X$  in  $\mathbb{P}^n$ . What is surprising is that the  $n$ -th thickening suffices for all large  $p$ . We made progress on corresponding characteristic-independent bounds for complete intersections.

## REFERENCES

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MICHIGAN, 530 CHURCH STREET, ANN ARBOR, MI 48109, USA  
E-mail address: bhargav.bhatt@gmail.com

INSTITUT FÜR MATHEMATIK, FACHBEREICH 08, JOHANNES GUTENBERG-UNIVERSITÄT MAINZ, 55099 MAINZ, GERMANY  
E-mail address: blicklem@uni-mainz.de

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MINNESOTA, 206 CHURCH ST., MINNEAPOLIS, MN 55455, USA  
E-mail address: gennady@math.umn.edu

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF UTAH, 155 S 1400 E, SALT LAKE CITY, UT 84112, USA  
E-mail address: singh@math.utah.edu

DEPARTMENT OF MATHEMATICS, STATISTICS, AND COMPUTER SCIENCE, UNIVERSITY OF ILLINOIS AT CHICAGO, 851 S. MORGAN ST., CHICAGO, IL 60607, USA  
E-mail address: wlzhang@uic.edu