

**REPORT ON THE IAS SUMMER COLLABORATION:  
LOCAL COHOMOLOGY AND THICKENINGS OF PROJECTIVE VARIETIES**

BHARGAV BHATT, MANUEL BLICKLE, GENNADY LYUBEZNIK, ANURAG K. SINGH, AND WENLIANG ZHANG

During our week at the IAS, May 27–June 2, 2018, we made progress on the following three topics:

**Applications of the Riemann-Hilbert correspondence.** By systematically exploiting the Riemann-Hilbert correspondence, we are able to recover several results on the local cohomology of polynomial rings of characteristic zero: these include foundational results of Ogus [Og], the work on Eulerian-graded  $\mathcal{D}$ -modules [MZ], and an interpretation of Bass numbers of local cohomology modules in terms of singular cohomology [LSW]. What is more, we can adapt the Riemann-Hilbert framework to obtain corresponding results in positive characteristic.

**Cohomology of thickenings.** Our earlier paper [BBSZ] included a version of the Kodaira vanishing theorem for thickenings of local complete intersection (lci) subvarieties of projective space over a field of characteristic zero. While the Kodaira vanishing theorem fails in positive characteristic, we proved the following *asymptotic* version, while at IAS: Let  $X$  be a closed lci subvariety of  $\mathbb{P}^n$  over a field of arbitrary characteristic, and let  $X_t$  denote its  $t$ -th thickening. Then there exists an integer  $c$ , such that for all  $t \geq 1$ , one has

$$H^k(X_t, \mathcal{O}_{X_t}(j)) = 0$$

for each  $k < \dim X$  and  $j < -ct$ . This answers a question raised in the recent paper [DM].

**Frobenius on the cohomology of thickenings.** One of our long-term goals has been a search for uniform results on the injectivity of Frobenius maps on cohomology groups; this is related to universal bounds for the vanishing of the Hasse invariant on families of hypersurfaces. For example, we proved that for a smooth hypersurface  $X$  in  $\mathbb{P}^n$ , over a field of characteristic  $p > n$ , the Frobenius map

$$\tilde{F}_n: H^{n-1}(X, \mathcal{O}_X) \longrightarrow H^{n-1}(X_n, \mathcal{O}_{X_n}),$$

is injective; here  $X_n$  denotes the  $n$ -th thickening of  $X$  in  $\mathbb{P}^n$ . What is surprising is that the  $n$ -th thickening suffices for all large  $p$ . We made progress on corresponding characteristic-independent bounds for complete intersections.

REFERENCES

- [BBSZ] B. Bhatt, M. Blickle, G. Lyubeznik, A. K. Singh, and W. Zhang, *Stability of cohomology of thickenings*, Amer. J. Math., to appear.
- [DM] H. Dao and J. Montaño, *Length of local cohomology of powers of ideals*, Trans. Amer. Math. Soc., to appear.
- [LSW] G. Lyubeznik, A. K. Singh, and U. Walther, *Local cohomology modules supported at determinantal ideals*, J. Eur. Math. Soc. **18** (2016), 2545–2578.
- [MZ] L. Ma and W. Zhang, *Eulerian graded  $\mathcal{D}$ -modules*, Math. Res. Lett. **21** (2014), 149–167.
- [Og] A. Ogus, *Local cohomological dimension of algebraic varieties* Ann. of Math. **98** (1973), 327–365.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MICHIGAN, 530 CHURCH STREET, ANN ARBOR, MI 48109, USA  
E-mail address: bhargav.bhatt@gmail.com

INSTITUT FÜR MATHEMATIK, FACHBEREICH 08, JOHANNES GUTENBERG-UNIVERSITÄT MAINZ, 55099 MAINZ, GERMANY  
E-mail address: blicklem@uni-mainz.de

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MINNESOTA, 206 CHURCH ST., MINNEAPOLIS, MN 55455, USA  
E-mail address: gennady@math.umn.edu

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF UTAH, 155 S 1400 E, SALT LAKE CITY, UT 84112, USA  
E-mail address: singh@math.utah.edu

DEPARTMENT OF MATHEMATICS, STATISTICS, AND COMPUTER SCIENCE, UNIVERSITY OF ILLINOIS AT CHICAGO, 851 S. MORGAN ST., CHICAGO, IL 60607, USA  
E-mail address: wlzhang@uic.edu