# Recursively Applying Constructive Dense Model Theorems and Weak Regularity

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## Abstract

Green and Tao [GT] used the existence of a dense subset indistinguishable from the primes under certain tests from a certain class to prove the existence of arbitrarily long prime arithmetic progressions. Tao and Ziegler [TZ] showed some general conditions under which such a model exists. Independently, Barak, Shaltiel and Wigderson [BSW] had also given such a characterization from a complexity-theoretic standpoint. In [RTTV], a quantitatively improved characterization was obtained using an argument based on Nisan's proof of the Impagliazzo hard-core set theorem [I95] from computational complexity. Gowers [Gow] independently obtained a similar improvement. Trevisan, Tulsiani and Vadhan [TTV] give a *decomposition* theorem that generalizes both the dense model theorem and the hard-core set theorem, and show connections between dense model theorems and the weak graph regularity results of Frieze and Kannan.

In this talk, we look at constructive and recursive versions of dense model theorems. A constructive version is one in which the model itself is definable in terms of the same class of distinguishing tests for which it is indistinguishable from the set. We show a decomposition theorem based on an iterative use of dense model theorems, and then show that it is equivalent to that of [TTV]. However, in addition, we can use this decomposition theorem recursively to get stronger decomposition theorems.

We give several applications, including generalizations of weak regularity lemmas [FK, Koh, COCF]. For example, we show that any graph $G$ with $\Delta n^2$ edges has a $\gamma$-cut-approximator of rank $2^{poly(1/\gamma, 1/\log(1/\Delta)}$, whereas direct application of [FK] gives rank $2^{O(1/\gamma^2\Delta^2)}$.

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