IAS SUMMER COLLABORATORS FINAL REPORT

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We met at the Institute for Advanced Study for two weeks of collaboration from July 17–31, 2017. These dedicated weeks for focused discussion were instrumental to forging progress on our research project, which aims to develop the Goresky-Kottwitz-MacPherson (GKM) approach to the equivariant cohomology of the affine Grassmannian. We are immensely grateful to the IAS for the opportunity to carry out several intensive weeks of work in such a stimulating and yet idyllic environment for mathematical research.

The primary goal of our project is influenced by a deep result of Peterson, later published by Lam and Shimozono, which gives an isomorphism between the equivariant homology of the affine Grassmannian and the equivariant quantum cohomology of the complete flag variety, suitably localized. Over the past several decades, researchers have approached this result from many different perspectives, using algebra (affine nilHecke algebra), combinatorics (k-Schur functions), physics (Toda lattice), and symplectic methods (nilpotent cones). Using the Peterson isomorphism to leverage one (co)homology theory against the other often requires explicit formulas for representatives of the (co)homology classes, but these computational tools have only been developed in special cases. Our current project centers on developing polynomial representatives for the equivariant cohomology classes of the affine Grassmannian, which then provides information about comultiplication in the equivariant quantum cohomology of the complete flag variety through the Peterson isomorphism.

The aim of our collaborative visit at the Institute for Advanced Study was to find an algebra basis for the equivariant cohomology of the affine Grassmannian which can be more naturally described using the underlying geometry and representation theory than the typical Schubert basis. Employing GKM theory as generalized by Harada, Henriques, and Holm then naturally leads to explicit polynomial representatives for the dual k-double Schur functions representing the Schubert classes, in terms of this geometric basis. During a two-week collaborative visit to the Max Planck Institute for Mathematics in May 2016, we carried out this program for the degree one classes. At the IAS in July 2017, we began by computing examples, employing a root-theoretic version of Billey's formula to identify the image of an affine Schubert class under the localization map. By the end of our stay, we successfully extended this method to these higher degree examples, resulting in a conjectural basis for the equivariant cohomology of the affine Grassmannian with the desired geometric properties.

Two dedicated weeks to work in the Summer Collaborators program has afforded us the opportunity to put the final pieces together for this long-term collaborative project. As such, we expect that our time at the Institute for Advanced Study will result in a forthcoming publication. Our stay at the IAS and the resulting opportunity for extended daily discussions has contributed to decisive mathematical progress on our project, and we are grateful to the Institute for its support.