# SUMMER COLLABORATORS FINAL REPORT

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The collaboration between myself and Petter Brándën went quite well. The main goal was to come up with a multivariate extension for what we call the "polynomial convolutions" and how one might go about extending the quantitative bounds that are known for the univariate case. We made substantial progress — in particular, I believe we (eventually) found the correct formulation using the intersection of hyperbolicity cones. We also made progress in the development of the tools needed to prove such bounds.

The known univariate bounds were proved using a technique called "pinching." This technique had a number of issues that prevented easy generalization. Firstly, each univariate convolution used a different pinch, which were developed ad hoc. Secondly, the way in which these pinches were constructed used the fact that the zeros of a polynomial form a discrete set (something not true in the multivariate case). Our first step was to realize that the polar derivative could be used to make "universal" pinches.

**Lemma 1.** *Let $T$ be a linear operator preserving real rooted polynomials, $p$ a real rooted polynomial with degree $d$, and $x$ a point larger than the largest root of $p$. Then there exists a $\xi$ such that the polar derivative at $\xi$ of $p$:*

$$D_\xi[p] = p(x) - \frac{(x - \xi)}{d} p'(x)$$

*such that $D_\xi[p]$ forms a valid pinch of $p$ with respect to $T$ at the point $x$ (there are a number of technical properties needed to be a valid pinch, so I won't list them all here).*

Using this, we were able to give quantitative root bounds for arbitrary linear operators preserving real rootedness (a vast generalization of the bounds known for the three convolutions).

The use of polar derivatives also gave us a way to extend the pinching technique to multivariate polynomials. These multivariate pinches are "universal" in the respect that they can work for any point above the roots of a real stable polynomial and for any directional derivative in the direction of a vector in the positive orthant. However, they can only handle one directional derivative at a time — it is unclear whether we can find pinches that preserve the needed quantities on multiple directions simultaneously (this is slated for future research).

The collaboration will certainly result in at least one paper (the extension of the quantitative bounds to arbitrary linear operators). There are still some details to work out in the multivariate case, but this was a great start and really could not have been accomplished without the framework of the summer collaborators program.