Abstract: Introduced by R. Schwartz about 20 years ago, the pentagram map acts on plane n-gons, considered up to projective equivalence, by drawing the diagonals that connect second-nearest vertices and taking the new n-gon formed by their intersections. The pentagram map is a discrete completely integrable system whose continuous limit is the Boussinesq equation, a completely integrable PDE of soliton type. In this talk, I shall describe a new family of completely integrable discrete dynamical systems, including the pentagram map, and explain their connection to the theory of cluster algebras, a new and rapidly developing area with numerous connections to diverse fields of mathematics.