Stephen Preston

Title: The inextensible string as a toy model of fluids

Abstract: The Euler equation for an ideal fluid can be viewed as a geodesic equation on the space of volume-preserving diffeomorphisms, in the L^2 Riemannian metric. As a simple geometric model, we consider the motion of a string in the plane as a geodesic equation on the space of curves parametrized with unit speed (in either an L^2 or H^1 metric).

The L^2 metric yields the geometric wave equation $ x_{tt} = \partial_s (\sigma x_s)$ with inextensibility constraint $|x_s| = 1$,
where the tension σ is determined implicitly much like the pressure in an ideal fluid. The H^1 metric is a regularized model. We prove that the L^2 equation is locally well-posed in certain Sobolev spaces but does not give a smooth Riemannian exponential map. However the regularized H^1 equation is actually a smooth ODE on the manifold of C^1 curves.

We also discuss global well-posedness of both equations and the connection to ideal fluids. This is joint work with Ralph Saxton.