

Newton's differential equations for the N-body problem have rotational and translational symmetry. Their flow is incomplete due to collisions. Symplectic reduction eliminates the symmetries. Levi-Civita regularization eliminates the binary collision singularities in the planar N-body problem. McGehee blow-up eliminates triple collision singularities. We show that when reduction, regularization, and blow-up are applied 'democratically' to the planar 4 body problem the resulting flow lives on the cotangent bundle of the cone over the K3 surface with a Kahler-twisted symplectic form. Key to our success is a modification of McGehee's real blow-up construction so that it becomes a complex blow-up. As a warm-up, we will begin with the planar 3-body problem, where a central feature of the reduced, regularized, blown-up dynamics becomes a 4:1 branched cover of the Riemann sphere, exhibiting octahedral symmetry. This work is joint with Rick Moeckel of the University of Minnesota.