Abstract: Let $I = (f_t)_{t\in[0,1]}$ be an identity isotopy on an oriented surface M and denote by $Cont(I)$ the set of points $z \in M$ whose trajectory $I(z)$ is a contractible loop of M. A recent result of Olivier Jaulent asserts that there exists an open set U whose complement is included in $Cont(I)$ and an isotopy $I = (f'_t)_{t\in[0,1]}$ on U such that every trajectory $I'(z)$, $z \in U$, is homotopic to $I(z)$ in M and such that the set $Cont(I') \subset U$ is empty. This implies that there exists an oriented foliation F on U such that every trajectory $I'(z)$ is homotopic in U to a path that is transverse to F. This foliation F seen as singular foliation on M with singularities in $M\ Cont(I')$ is gradient like if M is a closed surface and I is a Hamiltonian isotopy.

We will state two applications of this result to dynamical systems on surfaces. The first one, a joint work with Francois Beguin, Sebastiao Firmo and Tomasz Miernowski, asserts that for every C^1 diffeomorphism f of $\mathbb{R}^2$ that extends to a C^1 diffeomorphism of S^2 and that admits an invariant finite measure not included in its fixed point set, there exists a subset $X \subset Fix(f)$ which is invariant by the centralizer of f in $Homeo(\mathbb{R}^2)$. The second one, a joint work with Andres Koropecki and Meysam Nassiri, asserts that if U is an open subset of a closed surface M that is invariant by an area preserving homeomorphism of M and if p is a fixed regular end of U with an irrational prime end rotation number, then the boundary $Z(p)$ is an annular set with no periodic point or a cellular set with a unique periodic (and so fixed) point.