Abstract: The Euler equations for the dynamics of a fluid domain with a free surface can be formulated as a Hamiltonian system in an infinite dimensional phase space. The Hamiltonian has been derived by Zakharov, and the Hamiltonian vector field evokes the Hadamard variational formula on the derivative of the Green's function with respect to the domain. The questions of existence of solutions, of normal forms transformations, and of the construction of particular classes of solutions all involve problems in harmonic analysis, some of which have been addressed.