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Title: On the local structure of the set of stationary flows to the 2D incompressible Euler equations

Abstract

Steady-states are of great interest in the study of the dynamics of 2D incompressible Euler flows. (They could, for instance, possibly shed some light on the long time behavior of typical non-stationary flows.) In this talk I will consider the local structure of the {\it set} of steady-states under certain non-degeneracy assumptions.

The space of vorticity fields is formally foliated into the coadjoint orbits of the group $\mathcal D_{\rm area}(\Omega)$ of area-preserving diffeomorphisms of the region Ω filled with the ideal fluid, and the Euler evolution is a Hamiltonian system on each orbit, with the kinetic energy as the Hamiltonian function H. In particular, critical points of H restricted to the orbits correspond to steady-states, and hence these should be, locally and under certain suitable assumptions, in one-to-one correspondence with the coadjoint orbits. In finite dimensions, that is, when one considers a ``genuine'' Lie group instead of $\mathcal D_{\rm area}(\Omega)$, this can be established by a routine application of the classical Implicit Function Theorem. In the case of $\mathcal D_{\rm area}(\Omega)$, it seems difficult to give the orbits a satisfactory structure as submanifolds in the space of vorticity fields. Nevertheless, the problem can be approached in an indirect way and an analogue of the finite-dimensional result can be established via the Nash-Moser Inverse Function Theorem.

This is joint work with V. Sverak.