

**REPORT ON SPRING 2019 EMERGING TOPICS
WORKING GROUP: “COHERENCE AND QUASI-CONVEX
SUBGROUPS”**

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Participants: Martin Bridson, Kasia Jankiewicz, Rob Kropholler, Lars Louder, Jason Manning, Kim Ruane, Emily Stark, Matthew Stover, Henry Wilton.

A group is *coherent* if every finitely generated subgroup is finitely presented. The understanding of whether a group is coherent often hinges on understanding the geometry of finitely generated subgroups. This was a dynamic and intense working group which investigated coherence and the geometry of subgroups in a variety of contexts.

The format for the working group was as follows. On the first morning we had a long problem session, where each participant spoke briefly about their personal take on the topic and gave suggestions for intermediate problems to tackle. Partial solutions to these questions were sometimes suggested immediately. In the afternoon and during subsequent days, we divided into smaller working groups and worked on various aspects of these questions. We intermittently met all together and discussed progress with the whole group, which was very helpful. Sometimes people moved groups or split off to discuss some particular aspect of a question. There were also three public talks. The first focused on an introduction to the topic, connections with (potential) 3-manifold groups, and exhibiting incoherence for various classes

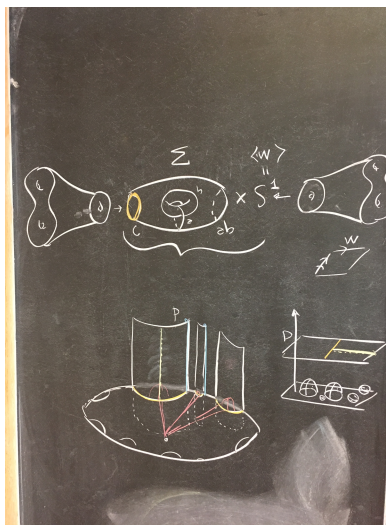


FIGURE 1. Gluing towards coherence

of groups via algebraic fibering and gluing constructions. The second talk described the problem and recent progress on understanding coherence of one-relator groups. The third talk focused on the coherence (and incoherence) of lattices in Lie groups, and described some techniques for showing certain of these lattices virtually algebraically fiber (i.e. map onto \mathbb{Z}^n with finitely generated kernel).

Throughout the workshop, using geometrically infinite subgroups to demonstrate incoherence was a theme. In particular, one way that subgroups arise with distorted geometry is when they are algebraic fibers (in the sense of being a kernel of a map as described above). The Virtual Fibering Conjecture for 3-manifolds, proved by Agol (and Wise in the non-compact finite-volume hyperbolic case) was one of the landmark results of the last decade. Recently, this has been put into an algebraic context by the work of Kielak, who substantially generalized Agol's results on the connection between fibering and the RFRS condition. This workshop drew various connections between algebraic fibering, coherence and a variety of topics, such as lattices in Lie groups, 1-relator groups, 3-manifold groups, hyperbolic groups, parafree groups and other classes.

The recent work of Louder and Wilton on building a structure theory for 1-relator groups played a central role in some of the working groups. In particular, questions about local convexity and coherence were front and center. In addition, connections between this work and some classical questions about "parafree" groups (those residually nilpotent groups which share the same lower central series as a free group) were made, suggesting a new approach to some long-standing open problems about these groups.

In a different direction, time was spent intensively studying $CAT(0)$ and hyperbolic groups whose boundary is a Sierpinski carpet. Conjecturally, all such groups have finite-index subgroups which are fundamental groups of 3-manifold groups (in the $CAT(0)$ setting) and more specifically Kleinian groups (in the hyperbolic setting). The geometry, algebraic structure, and coarse topology of these groups was studied, using the classical results about 3-manifolds as inspiration. Certain key technical obstacles were indentified, and the group left with renewed inspiration about these difficult topics.

Participants reported a very productive week, and particularly expressed the importance of seeing many different perspectives about the same questions. Many said that they left with new ideas about existing topics and difficult problems. It appears that at least one paper will result from the work in this group, on the subgroup structure of random 2-generator one-relator groups. Probabaly the most lasting impact of the workshop is the incredible number of problems, conjectures and questions raised in this area. Indeed, we are continuing to find more avenues that we would like to investigate, and the work continues.

We would like to express our appreciation to the IAS for their hospitality during the week and to Cynthia and Robert Hillas whose generous financial support made the working group possible.