IAS Emerging Topics on

Scalar Curvature and Convergence

Organized by Misha Gromov and Christina Sormani

October 15-19, 2018

Description of the Topic:

When studying sequences of Riemannian manifolds with nonnegative sectional curvature, one obtains limits under Gromov-Hausdorff convergence to Alexandrov spaces with Alexandrov curvature bounded below (as studied in the work of Gromov and Burago-Gromov-Perelman). For sequences of manifolds with nonnegative Ricci curvature one applies metric measure notions of convergence and obtains metric measure spaces satisfying a variety of notions of generalized nonnegative Ricci curvature (as in the work of Cheeger-Colding, Lott-Villani, Sturm, and Ambrosio-Gigli-Savare). It is not yet clear what notion of convergence is best suited to Riemannian manifolds with nonnegative scalar curvature, although the notion of intrinsic flat convergence introduced by Sormani-Wenger in [SW-JDG] using the work of Ambrosio-Kirchheim seems promising. Even less well understood is an appropriate definition for a generalized notion of nonnegative scalar curvature on a limit space.

Gromov has proposed a variety of conjectures in this direction in [Gromov-12] [Gromov-Dirac] [Gromov-Plateau]. Sormani has refined the statements of some of these conjectures and stated others, including key examples in [Sormani-Scalar]. With the new Schoen-Yau proof of the Positive Mass Theorem in all dimensions, there is a new approach to understanding nonnegative scalar curvature in settings with lower regularity [Schoen-Yau-PMT]. The Positive Mass Theorem states that a complete asymptotically flat manifold with nonnegative scalar curvature has nonnegative ADM mass and the ADM mass is zero iff the manifold is isometric to Euclidean space. Lee-Sormani conjectured that if the ADM mass is small the manifold is close in the intrinsic flat sense to Euclidean space [Lee-Sormani-1]. This has been proven in various special cases by Lee-Sormani, Huang-Lee-Sormani, and Sormani-Stavrov using techniques by Lakzian-Sormani. There are teams currently working on Gromov's conjectured Stability of the Scalar Torus Rigidity Theorem working in the same special cases. However, all progress in this direction has required additional hypotheses. See the Intrinsic Flat Convergence Website for links to all these papers. While these recent results provide insight, they do not address the fundamental question of defining a notion of generalized nonnegative scalar curvature on a limit space.

Organizers: Misha Gromov and Chistinas Sormani

IAS funded Participants: Bernhard Hanke, Lan-Hsuan Huang, Sajjad Lakzian, Yashar Memarian, Pengzi Miao, Jacobus Portegies, Raquel Perales, Rick Schoen, Guofang Wei (Michael Eichmair was invited but unable to attend last minute and contributed by email).

Additional Participants: Brian Allen, Luca Ambrozio, Mauricio Bustamante, Alessandro Carlotto, Otis Chodosh, Fernando Coda Marques, Brian Frieden, Benedikt Hunger, Jeff Jauregui, Nicos Kapouleas, Demetre Kazaras, Anusha Krishnan, Dan Lee, Chao Li, Yevgeny Liokumovich, Siyuan Lu, Elena Maeder-Baumdicker, Andrea Malchiodi, Robin Neumeyer, Andre Neves, Daniel Rade, Shengwen Wang, Ruobing Zhang, Xin Zhou (some at IAS already, and some from nearby institutions and some were young mathematicians brought in with the research funding of the lead participants)

Organization:

There were 12 open lectures, 12 closed discussions, and 1 open discussion reporting on the week's ideas. The first 2 open lectures focussed on the existing conjectures in the area. The rest focused on specific progress providing more precise restatements of conjectures in light of these results. At the discussions teams of participants worked together to recombine the ideas from the various talks and formulate new approaches. The complete schedule of all topics and links to papers that were discussed appears here:

https://sites.google.com/site/professorsormani/home/conferences/ias-working-group-2018

We have continued to communicate by email, and are pulling together a more precise list of specific problems that younger mathematicians may work on as they move into this area of research. Many of those young people were among the additional participants who attended the open lectures and final discussion.

Outcomes:

The workshop was a great success with intense participation and overflowing classrooms. Others were watching the lectures online and sending emails with their thoughts. As the week progressed, more and more local people asked to participate and join the research teams. We have already arranged to have a reunion meeting at NYU on December 6-7, 2018, which will enable people to begin their projects. Later in the Spring there will be another meeting at NYU and finally a meeting of many of the participants in a workshop at SCGP on Low Regularity in General Relativity.

As originally proposed, we examined a number of rigidity theorems for manifolds with lower bounds on their scalar curvature. We spent significant time discussing how to state these theorems on generalized spaces with generalized notions of scalar curvature. There was much discussion of the distinction between theorems proven using minimal surface methods vs dirac operator techniques. Should different notions of generalized scalar curvature be used to capture different properties? Or should we combine these methods and try to prove new results even about Riemannian geometry by combining them? Indeed it seems there are some theorems and examples one may be able to prove about Riemannian manifolds that have not been proven before. These ideas will be written up in forthcoming papers by various teams.

We will also be writing a joint survey article or two clearly stating the conjectures developed during this meeting and subsequent ones. One of the original conjectures before we met stated that a sequence of three dimensional manifolds with nonnegative scalar curvature and uniform upper bound on volume and diameter and a uniform lower bound on the minimum area of a closed minimal surface, must subconverge in the intrinsic flat sense to a limit space which has generalized scalar curvature. We now clarify this to say that it should subconverge in the volume preserving intrinsic flat sense to a limit space which has Euclidean tangent planes almost everywhere and satisfies the prism inequality. We've also itemized which rigidity theorems should hold on the volume preserving intrinsic flat limits of manifolds with lower bounds on their scalar curvature and will list these in one of our upcoming survey articles.