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An Upper Bound on the Threshold Quantum Decoherence Rate

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Let η_0 be the supremum of those η for which every poly-size quantum circuit can be simulated by another poly-size quantum circuit with gates of fan-in ≤ 2 that tolerates random noise independently occurring on all wires at the constant rate η. Recent fundamental results showing the principal fact $\eta_0 > 0$ give estimates like $\eta_0 \geq 10^{-6}$–10^{-4}, whereas the only upper bound known before is $\eta_0 \leq 0.74$.

In this note we improve the latter bound to $\eta_0 \leq 1/2$, under the assumption $\mathbf{QP} \not\subseteq \mathbf{QNC^1}$. More generally, we show that if the decoherence rate η is greater than $1/2$, then we can not even store a single qubit for more than logarithmic time. Our bound also generalizes to the simulating circuits allowing gates of any (constant) fan-in k, in which case we have $\eta_0 \leq 1 - \frac{1}{k}$.

Keywords: Fault-tolerant quantum computation, noise rate

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1 Introduction

Whereas it is still too premature to rush for any definite conclusions about the prospects of practical quantum computing, some important issues presumably confronting any implementation scheme have been identified. And it seems to be of more or less universal agreement that the most serious of these issues is the problem of decoherence due to the interaction of the quantum system with its environment. For this reason it has become an extremely important task for theoreticians to identify the type and amount of noise against which arbitrary quantum computations can be protected with at most polynomial slow-down.

The noise model most often considered in the literature (and the one we are sticking to in the current note) is that of *local noise* (see e.g. [1]). In this model, errors occur on every wire in the quantum circuit independently with a certain probability η. They occur even on those wires which are currently not involved in the computation but simply transmit information from one level to another. As a compensation for this (otherwise devastating) assumption, the circuit is also allowed to have a parallel architecture and perform operations on disjoint qubits simultaneously. The basic question is how small should the noise rate η be so that

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every poly-time quantum computation can be transformed into another poly-time quantum computation resistant to this kind of noise.

The early (and relatively easy) lower bound on the threshold value η_0 below which the noise-resistant computation becomes possible was given in [6]. Namely, they showed $\eta_0 \geq \Omega(1/n)$, where n is the number of gates in the protected circuit. [14] improved this to $\eta_0 \geq (\log n)^{-O(1)}$. Finally, [3, 10] proved the fundamental result that η_0 is separated from 0 by some universal constant not depending on the circuit size at all, and various estimates of the quality of these error-correction schemes give lower bounds on η_0 in the range 10^{-6}–10^{-4} (we have listed above only some of the most central work in this direction; further references can be found e.g. in the monographs [12, 11] and in the expositions [1, 8, 13]).

Much less work has been done on *lower* bounds for fault-tolerant quantum computations (that is, *upper* bounds on the threshold η_0). [2] showed that $\eta_0 \leq 0.96$, as long as the simulating circuit is allowed to have gates of fan-in ≤ 2 only. Then it was established in the beautiful paper [4] that fault-tolerant computation is impossible for *any* constant rate of noise η without an uninterrupted supply of fresh qubits initialized in (known) pure states.

[7] considered a similar problem of estimating the quantum capacity of a noisy (depolarizing) quantum channel. As indicated by the referee of this paper, their methods also give $\eta_1 \leq 1/3$, where η_1 is the largest noise rate for which one can maintain, with high fidelity, a single qubit for an indefinitely long time. Whereas it is certainly true that all known schemes of fault-tolerant quantum computation use this feature in a very essential way, the author is not aware of any rigorous explanation of why it should necessarily be the case in general. In more mathematical terms, the inequality $\eta_0 \leq \eta_1$ (as well as the inverse one, of course) seems to be unknown and, therefore, the results of [7] are incomparable with ours.

Finally, using ideas quite different from those in [2, 4], [9] recently showed that $\eta_0 \leq 0.74$ (they were also able to prove $\eta_0 \leq 0.5$, but in a much more restricted model of the adversarial noise not necessarily independent on wires, and only for certain specific unitary quantum gates on two qubits).

As any normal (or, depending on the point of view, arrogant) complexity theorist, the author is rather wary about the activity of determining the exact value of various quantities previously established to be an absolute constant. Still, the range $\eta_0 \in [10^{-6}\text{–}10^{-4},\ 0.74]$ is rather embarrassing and humiliating. Also, it is conceivable that this is the order of magnitude of η_0 that will have a final say on whether quantum computers will some day become available. For these reasons, the author believes that this case should be granted an exception, and that the questions like $\eta_0 \stackrel{?}{\geq} 0.1$ or $\eta_0 \stackrel{?}{\geq} 0.01$ are of great *theoretical* value.

In this note we take the next modest step toward narrowing the gap and show that $\eta_0 \leq 1/2$, unless $\mathbf{QP} = \mathbf{QNC^1}$. For doing that we prove, in the style of [4], that when the noise rate η is greater than $1/2$, any quantum computation completely collapses within logarithmic time. Our proof is much simpler than the proofs in [2, 9], and it also has a straightforward generalization to quantum circuits allowing gates of arbitrary fan-in k, with the corresponding bound $\eta_0 \leq 1 - \frac{1}{k}$.

2 Preliminaries and the main result

In this note we exclusively deal with mixed state quantum circuits [5, 10]. For this reason we completely skip the usual quantum formalism pertinent to the standard (unitary) model of quantum computing (for that see e.g. [12, 11]) and immediately proceed to density matrices.

For a finite dimensional Hilbert space $\mathcal{N}$, $\mathbf{L}(\mathcal{N})$ is the set of linear operators on $\mathcal{N}$. $\mathbf{D}(\mathcal{N}) \subseteq \mathbf{L}(\mathcal{N})$ is the set of *density matrices on* $\mathcal{N}$: $\rho \in \mathbf{D}(\mathcal{N})$ if and only if ρ is Hermitian, positive semi-definite and $\mathrm{tr}(\rho) = 1$. $\mathbf{D}(\mathcal{N})$ is a convex subset in $\mathbf{L}(\mathcal{N})$.

A linear mapping $T : \mathbf{L}(\mathcal{N}) \longrightarrow \mathbf{L}(\mathcal{M})$ is a *quantum operation* iff it is trace-preserving and completely positive (that is, sends positive semi-definite operators to positive semi-definite operators and retains this property even after taking a tensor product with the identity operator on an arbitrary Hilbert space). The set of all quantum operations $T : \mathbf{L}(\mathcal{N}) \longrightarrow \mathbf{L}(\mathcal{M})$ is denoted by $\mathbf{T}(\mathcal{N}, \mathcal{M})$. Quantum operations take density matrices to density matrices, and the tensor product of quantum operations is a quantum operation.

For $\eta \in [0, 1]$, *depolarization at the rate* η is the specific quantum operation $\mathcal{E}_\eta \in \mathbf{T}(\mathcal{N}, \mathcal{N})$ given by

$$\mathcal{E}_\eta(\rho) \stackrel{\mathrm{def}}{=} (1 - \eta)\rho + \frac{\eta \mathrm{tr}(\rho)}{\dim(\mathcal{N})} I_{\mathcal{N}}, \tag{1}$$

where by $I_{\mathcal{N}}$ we denote the identity operator on $\mathcal{N}$. Physically, it corresponds to the process in which the system described by the density matrix ρ gets depolarized (= replaced with the completely mixed state $I_{\mathcal{N}}/\dim(\mathcal{N})$) with probability η, and is left untouched otherwise.

Remark 1 Depolarization is exactly the noise model considered in [4, 9]. [2] used a slightly different model ("dephasing", or measurement in a given basis), but it is straightforward to see that their method actually applies to the depolarizing noise as well.

The *trace distance* $D(\rho, \sigma)$ between two density matrices ρ and σ on the same space $\mathcal{N}$ is defined as $D(\rho, \sigma) \stackrel{\mathrm{def}}{=} \frac{1}{2}||\rho - \sigma||_{\mathrm{tr}}$, where $|| \cdot ||_{\mathrm{tr}}$ is the trace norm. It is equal to the maximal difference in the results of measuring ρ and σ in the same basis one can achieve.

Proposition 1 *For any* $p_1, \ldots, p_n \geq 0$ *with* $p_1 + \cdots + p_n = 1$ *and* $\rho_1, \ldots, \rho_n, \sigma_1, \ldots, \sigma_n \in \mathbf{D}(\mathcal{N})$, $D(\sum_i p_i\rho_i, \sum_i p_i\sigma_i) \leq \sum_i p_i D(\rho_i, \sigma_i)$.

Proposition 2 ([12, Theorem 9.2]) *For any* $\rho, \sigma \in \mathbf{D}(\mathcal{N})$ *and* $T \in \mathbf{T}(\mathcal{N}, \mathcal{M})$, $D(T(\rho), T(\sigma)) \leq D(\rho, \sigma)$.

Quantum circuits with mixed states [5, 10] are just ordinary parallel circuits allowing gates from a prescribed set $\mathcal{G}$ of quantum operations. More formally, let $\mathcal{B}$ be a two-dimensional Hilbert space ("qubit"), and let $\mathcal{B}^{\otimes n} \stackrel{\mathrm{def}}{=} \mathcal{B} \otimes \cdots \otimes \mathcal{B}$ (n times). For any partition $[n] = I_1 \dot{\cup} \ldots \dot{\cup} I_w$ of the ground set $[n] \stackrel{\mathrm{def}}{=} \{1, \ldots, n\}$, $\mathcal{B}^{\otimes n}$ can be naturally decomposed as $\mathcal{B}^{\otimes |I_1|} \otimes \cdots \otimes \mathcal{B}^{\otimes |I_w|}$, and we will freely use this notation. A *quantum gate* is a quantum operation $T \in \mathbf{T}(\mathcal{B}^{\otimes k}, \mathcal{B}^{\otimes \ell})$, k being called *fan-in* of the gate T. Let in particular $\mathcal{G}_k$ be the set of all gates with fan-in $\leq k$.

For a family $\mathcal{G}$ of quantum gates, a *quantum circuit* Q *over* $\mathcal{G}$ is any sequence of quantum operations

$$\mathcal{B}^{\otimes n_0} \xrightarrow{T_0} \mathcal{B}^{\otimes n_1} \xrightarrow{T_1} \ldots \xrightarrow{T_{t-1}} \mathcal{B}^{\otimes n_t}, \tag{2}$$

in which every T_i allows a decomposition of the form

$$\left.\begin{array}{rcl} [n_i] & = & K_{i1} \dot\cup K_{i2} \dot\cup \ldots \dot\cup K_{iw_i}; \\ [n_{i+1}] & = & L_{i1} \dot\cup L_{i2} \dot\cup \ldots \dot\cup L_{iw_i}; \\ T_i & = & \bigotimes_{\nu=1}^{w_i} T_{i\nu},\ T_{i\nu} \in \mathbf{T}(\mathcal{B}^{\otimes|K_{i\nu}|}, \mathcal{B}^{\otimes|L_{i\nu}|}) \cap \mathcal{G}. \end{array}\right\} \qquad (3)$$

The *depth* $Depth(Q)$ of the circuit (2) is t, and its *width* $Width(Q)$ is $\max\{n_0, n_1, \ldots, n_t\}$. For an input $\rho \in \mathbf{D}(\mathcal{B}^{\otimes n_0})$, the *final state of running Q on ρ* is simply

$$Q(\rho) \stackrel{\rm def}{=} T_{t-1}T_{t-2}\ldots T_1 T_0(\rho).$$

For $\eta \in [0,1]$, the *perturbed circuit* Q_η is obtained by interlacing the computational steps $T_0, T_1, \ldots, T_{t-1}$ with the noise operators $\mathcal{E}_\eta$ independently depolarizing all qubits with probability η. That is,

$$Q_\eta(\rho) \stackrel{\rm def}{=} T_{t-1}\mathcal{E}_\eta^{\otimes n_{t-1}} T_{t-2}\mathcal{E}_\eta^{\otimes n_{t-2}} \ldots \mathcal{E}_\eta^{\otimes n_2} T_1 \mathcal{E}_\eta^{\otimes n_1} T_0(\rho). \qquad (4)$$

Remark 2 Introducing fresh qubits into the system (unavoidable due to the result of [4]) can be thought of as a special quantum operation from $\mathbf{T}(\mathcal{C}, \mathcal{B})$ of fan-in 0 (cf. [10]), and does not require any special treatment in our framework.

[4] called the quantum circuit Q of the form (2) *worthless* if for every $\rho \in \mathbf{D}(\mathcal{B}^{\otimes n_0})$, $D(Q(\rho),\ 2^{-n_t} \cdot I_{\mathcal{B}^{\otimes n_t}}) \leq 1/100$. Following their suit, we introduce the following (slightly more relaxed) definition:

Definition 1 A quantum circuit Q is *practically worthless* if for every $\rho, \sigma \in \mathbf{D}(\mathcal{B}^{\otimes n_0})$, $D(Q(\rho), Q(\sigma)) \leq 1/100$.

Thus, whereas worthless circuits do not produce any output at all, practically worthless circuits can compute only constant Boolean functions 0 and 1.

Remark 3 We would like to stress that the sole reason for our usage of the term "practically worthless" is to make the terminology reasonably consistent with [4] whose methods we extent. Whereas the ability to finally output a non-constant function is obviously a *necessary* condition for a quantum circuit to be "useful", we do not intend to imply by this choice that it is in any sense sufficient. In fact, another quite legitimate requirement comes already from the paper [2]: any useful quantum circuit should not allow a *local* (= step-by-step) simulation by efficient classical circuits. This comparison, however, also suggests that a comprehensive definition of an "useful" quantum circuit might be just very difficult to come by. Indeed, [4] and the current paper collapse (under certain conditions) fault-tolerant $\mathbf{QP}$ to $\mathbf{QNC^1}$, whereas [2] collapses it to $\mathbf{P}$. Thus, any comprehensive definition of an "useful" quantum circuit should presumably correspond to a complexity class $\mathcal{C}$ such that $\mathbf{QNC^1} \cup \mathbf{P} \subseteq \mathcal{C} \subseteq \mathbf{QP}$ and the last containment $\mathcal{C} \subseteq \mathbf{QP}$ is "believed to be proper". The author is not aware of any "natural" candidate $\mathcal{C}$ to satisfy these properties.

Our main result can be stated as follows:

Theorem 3 *For every constants $k > 0$ and $\eta > 1 - \frac{1}{k}$ there exists $C > 0$ such that the following holds. For every quantum circuit Q over $\mathcal{G}_k$ with $Depth(Q) \geq C \log Width(Q)$, the perturbed circuit Q_η is practically worthless.*

This theorem says that as long as the noise rate exceeds $1 - \frac{1}{k}$, every two initial states become totally indistinguishable within $O(\log n)$ steps, where n is the number of qubits allowed in the system. In particular, no Boolean function $f \in \mathbf{QP} \setminus \mathbf{QNC^1}$ can be computed fault-tolerantly. Also, it will become clear from the proof that the same conclusion holds if n is understood as the number of qubits *participating in the final measurement*. If, for example, the circuit Q_η is required to output the result by measuring only one qubit in the final state (as is often the case in the standard unitary model), then it becomes practically worthless already within a *constant* number of steps.

3 The proof

Before we start, let us remark that the proof can be re-casted in the completely combinatorial style of [2]. Namely, the same recursion as the one used below can also show that if $\eta > 1 - \frac{1}{k}$ then with high probability the computational graph of the circuit becomes disconnected after deleting from it faulty wires. We, however, prefer more analytical version of the proof (that was actually found first, and is inspired in part by [4]) as it appears to us more suggestive of potential improvements and generalizations.

Let Q be a quantum circuit of the form (2), and $\eta \in [0, 1]$. For $\rho \in \mathbf{D}(\mathcal{B}^{\otimes n_0})$, denote by ρ_i the density matrix obtained at the ith level of the faulty computation (4) before applying the noise operator $\mathcal{E}_\eta^{\otimes n_i}$. That is,

$$\rho_i \stackrel{\text{def}}{=} T_{i-1}\mathcal{E}_\eta^{\otimes n_{i-1}} T_{i-2}\mathcal{E}_\eta^{\otimes n_{i-2}} \dots \mathcal{E}_\eta^{\otimes n_2} T_1 \mathcal{E}_\eta^{\otimes n_1} T_0(\rho).$$

Note for the record that $\rho_0 = \rho$ and $\rho_t = Q_\eta(\rho)$.

Next, for $A \subseteq [n_i]$, let $\rho_i|_A$ be the result of tracing out the density matrix ρ_i with respect to all qubits in $[n_i] \setminus A$ (cf. [4]). Then the effect of applying the noise operator $\mathcal{E}_\eta^{\otimes n_i}$ on ρ_i, and, more generally, on all its reduced submatrices $\rho_i|_B$ can be expressed as follows (see [4]):

$$\mathcal{E}_\eta^{\otimes n_i}(\rho_i|_B) = \sum_{A \subseteq B} \eta^{|B|-|A|}(1-\eta)^{|A|}(\rho_i|_A \otimes (I_{\mathcal{B}}/2)^{\otimes(|B|-|A|)}), \tag{5}$$

where the notation $\rho_i|_A \otimes (I_{\mathcal{B}}/2)^{\otimes(|B|-|A|)}$ corresponds to the partition of B into A and $B \setminus A$.

Let now $\sigma \in \mathbf{D}(\mathcal{B}^{\otimes n_0})$ be another initial density matrix. For $i = 0, \dots, t$ and $n \geq 0$ let

$$d_{in}(\rho, \sigma) \stackrel{\text{def}}{=} \max_{\substack{A \subseteq [n_i] \\ |A| \leq n}} D(\rho_i|_A, \sigma_i|_A).$$

Then, clearly,

$$d_{00}(\rho, \sigma) = 0, \;\; d_{0n}(\rho, \sigma) \leq 1 \;\; (n \geq 1). \tag{6}$$

In order to get a recursive bound on d_{in}, we additionally assume that the fan-in of all gates in the circuit Q is bounded by k. Fix $A \subseteq [n_{i+1}]$ with $|A| \leq n$. Recalling the decomposition (3), let $\Gamma \stackrel{\text{def}}{=} \{\nu \in [w_i] \,|\, L_{i\nu} \cap A \neq \emptyset\}$ be the set of all quantum gates involved in the computation of qubits from A, and $B \stackrel{\text{def}}{=} \bigcup_{\nu\in\Gamma} K_{i\nu}$ be the set of all their inputs. Then $|B| \leq k \cdot |\Gamma| \leq kn$, and by Proposition 2 (applied to the quantum operation $\bigotimes_{\nu\in\Gamma} T_{i\nu}$ and density matrices $\mathcal{E}_\eta^{\otimes n_i}(\rho_i|_B), \mathcal{E}_\eta^{\otimes n_i}(\sigma_i|_B)$),

$$\begin{aligned} D(\rho_{i+1}|_A, \sigma_{i+1}|_A) &\leq D(\rho_{i+1}|_{\bigcup_{\nu\in\Gamma} L_{i\nu}}, \sigma_{i+1}|_{\bigcup_{\nu\in\Gamma} L_{i\nu}}) \\ &\leq D(\mathcal{E}_\eta^{\otimes n_i}(\rho_i|_B), \mathcal{E}_\eta^{\otimes n_i}(\sigma_i|_B)). \end{aligned}$$

Furthermore, applying Proposition 1 to the representation (5),

$$\begin{aligned} D(\mathcal{E}_\eta^{\otimes n_i}(\rho_i|_B), \mathcal{E}_\eta^{\otimes n_i}(\sigma_i|_B) &\leq \sum_{A\subseteq B} \eta^{|B|-|A|}(1-\eta)^{|A|} D(\rho_i|_A, \sigma_i|_A) \\ &\leq \sum_{m=0}^{|B|} \binom{|B|}{m} \eta^{|B|-m}(1-\eta)^m d_{im}(\rho,\sigma) \\ &\leq \sum_{m=0}^{kn} \binom{kn}{m} \eta^{kn-m}(1-\eta)^m d_{im}(\rho,\sigma), \end{aligned}$$

where for the last inequality we additionally used the fact that, by definition, $d_{im}(\rho,\sigma)$ is monotone in m. Combining the last two inequalities, and recalling that $A \subseteq [n_{i+1}]$ with the property $|A| \leq n$ was chosen arbitrarily, we get

$$d_{i+1,n}(\rho,\sigma) \leq \sum_{m=0}^{kn} \binom{kn}{m} \eta^{kn-m}(1-\eta)^m d_{im}(\rho,\sigma). \tag{7}$$

It is now straightforward to see that the recursion (6), (7) has the exact solution

$$d_{in}(\rho,\sigma) \leq 1 - f_i^n,$$

where the coefficients $f_i \in [0,1]$ are given by

$$f_0 \stackrel{\text{def}}{=} 0, \quad f_{i+1} \stackrel{\text{def}}{=} (\eta + (1-\eta)f_i)^k$$

(and $0^0 \stackrel{\text{def}}{=} 1$). Applying the inequality $(1-x)^k \geq 1-kx$ $(0 \leq x \leq 1)$ with $x := (1-\eta)(1-f_i)$, we get from here $f_{i+1} \geq 1 - \theta(1-f_i)$, where $\theta \stackrel{\text{def}}{=} k(1-\eta) < 1$ is a constant. This gives us $f_i \geq 1 - \exp(-\Omega(i))$. In particular, if $t \geq C \log n_t$ for a sufficiently large constant $C > 0$ then $f_t \geq 1 - \frac{1}{n_t^2}$ and $d_{t,n_t}(\rho,\sigma) \leq O(1/n_t)$.

We have shown that $D(Q_\eta(\rho), Q_\eta(\sigma)) \leq O(1/n_t)$ for every pair $\rho,\sigma \in \mathbf{D}(\mathcal{B}^{\otimes n_0})$, which completes the proof of Theorem 3.

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