

# SUMMER COLLABORATORS 2017 REPORT

ANA-MARIA BRECAN AND LAURE FLAPAN

Our collaboration during the first half of July 2017 at the Institute of Advanced Study as part of the 2017 Summer Collaborators program was very productive and we had a very pleasant stay. It allowed us for an extended period of inquiry and lively discussion that led us to much greater understanding and would not have been possible without such a program. We are very grateful to the IAS for providing us with this opportunity and wonderful work environment.

The goal for our stay at the Institute was to jump-start a new project together exploring the relationship between arithmetic properties of curves and corresponding arithmetic properties of their holonomy representations. In particular, we were interested in exploring connections with Simpson's conjecture, which predicts when such representations should occur as a summand of a variation of Hodge structures.

A strong motivating question for us was the following:

**Question.** When does a representation of the fundamental group of an algebraic curve occur as a summand of a variation of Hodge structures?

As both of our primary mathematical backgrounds lie in Hodge theory, the bulk of our visit was spent reading and discussing the necessary arithmetic and geometric background needed to attack the problem. In particular, we quickly realized that we would need some understanding of Teichmüller space and its relationship to  $SL(2, \mathbb{R})$ -representation varieties, both of which were topics with which we had limited familiarity. It was thus crucial that we devote time to learning and computing many simple examples in these areas.

After gaining greater familiarity with the necessary background, we then focused our attention on understanding examples where we could understand the relationships we wished to explore. For instance, we observed that if a curve  $C$  is a Shimura curve, then  $C$  is defined over a number field and parametrizes a variation of  $\mathbb{Q}$ -Hodge structures whose corresponding monodromy representation is defined over  $\overline{\mathbb{Q}}$ . We also developed a similar statement for curves uniformized by triangle groups.

Our goal for the near future is to be able to describe more examples where similar behavior can be observed, in the hopes of gaining greater intuition about the general principles underlying this behavior.

Our stay at the Institute allowed us to make great strides forward in what promises to be a very interesting and stimulating project. We intend to keep developing this research program in the months ahead. We have created a schedule to plan our future meetings related to this project with our next meeting scheduled for August 2017 in Germany.

As the two of us live and work on separate continents, this progress could not have been achieved without the framework and generous support of the Institute and the Summer Collaborators program in particular. We, once again, would like to offer heartfelt thanks for this wonderful opportunity.