1. Draw the graph $K(5,2)$.

2. Give a direct proof that $\chi(K(n,2)) = n-2$.

3. What is the girth (the length of a shortest cycle) of $K(n,r)$? What is its odd girth (the length of a shortest odd cycle)?

4. The *n-ball* B^n is $\{x \in \mathbb{R}^n : \|x\| \leq 1\}$. The *Brouwer fixed-point theorem* states that if $f : B^n \to B^n$ is a continuous map then f has a fixed point. Deduce Brouwer from Borsuk-Ulam, by showing that if f has no fixed point then we may obtain a continuous antipodal map from S^n to S^{n-1}.

5. Two boys wish to share a ham sandwich, consisting of bread, ham and mustard. By applying Borsuk-Ulam for S^3, show that it is possible to achieve this with one straight cut, in other words to find a plane that bisects each of the ingredients.

$^+$6. An *involution* is a function that squares to the identity. Prove that every continuous involution $f : \mathbb{R}^n \to \mathbb{R}^n$ has a fixed point.