1. Let $r < n/2$. What is the largest intersecting family contained in $X^{(\leq r)}$?

2. A set system $A \subset \mathcal{P}(X)$ is an *up-set* if whenever $x \in A$ and $x \subset y$ then also $y \in A$. Explain why every maximal intersecting family is an up-set of size 2^{n-1}. Conversely, if A is an up-set with $|A| = 2^{n-1}$, must A be intersecting?

3. Let $A \subset \mathcal{P}(X)$ with $|A| = 2^{n-1} + 1$, so that some pair $x, y \in A$ must be disjoint. What is the smallest number of such disjoint pairs that A can have? And what if $|A| = 2^{n-1} + 2$?

4. How large can a t-intersecting set system $A \subset \mathcal{P}(X)$ be, when $n + t$ is odd?

5. Give a direct proof of the Erdős-Ko-Rado theorem (without Kruskal-Katona or cyclic orders) for the case when r divides n.

6. A set system $A \subset \mathcal{P}(X)$ is an *antichain* if no member of A is contained in another member of A. Use Katona's method (averaging) to prove *Sperner's lemma*, that no antichain has size greater than $\binom{n}{\lfloor n/2 \rfloor}$. (Hint: take as auxiliary objects the permutations of X.)

7. Let $A_1, A_2, \ldots, A_d \subset \mathcal{P}(X)$ be intersecting families. Prove that $|A_1 \cup A_2 \cup \ldots \cup A_d| \leq 2^n - 2^{n-d}$.

8. Let $A \subset \mathcal{P}(\mathbb{N})$ be an intersecting family of finite sets. Must there exist a finite set $F \subset \mathbb{N}$ such that the family $\{x \cap F : x \in A\}$ is intersecting? And what if $A \subset \mathbb{N}^{(r)}$ for some fixed r?