

1. Write down the elements of  $Q_5$  in simplicial order.
2. Let  $A \subset [9]^{(3)}$  with  $|A| = 28$ . How small can the lower shadow of  $A$  be? And the upper shadow?
3. Let  $A \subset Q_6$  with  $|A| = 26$ . How small can the vertex-boundary of  $A$  be?
4. The *diameter* of a set  $A \subset Q_n$  is  $\max\{d(x, y) : x, y \in A\}$ . Deduce from Harper's theorem that if  $A \subset Q_n$  has diameter  $d$ , where  $d < n$  and  $d$  is even, then  $|A| \leq |X^{(\leq d/2)}|$ .
5. Find a set system  $A \subset X^{(r)}$  (for some  $n$  and  $r$ ) for which equality holds in the Kruskal-Katona theorem but which is not isomorphic to an initial segment of colex.
- <sup>+</sup>6. Let  $A \subset X^{(r)}$  and  $B \subset X^{(r+1)}$  be initial segments of colex with  $|A| = |B|$ . Do we always have  $|\partial A| \leq |\partial B|$ ?