post-core-collapse clusters, tend to be closer to the center of the Galaxy. The effect of giant molecular clouds, on the other hand, is completely negligible according to Chernoff and co-workers.

Various authors have postulated the existence of massive ($10^6$ $M_\odot$) black holes as the major constituents of the galactic dark halo responsible for the flat rotation curve of the Galaxy. Roland Wielen has studied the effect of such a population of massive components on the population of globular clusters. He finds that the effect is strongly dependent on the mean density of the clusters, and that for clusters of median mean density, the vast majority may have already been destroyed.

## *SUMMARY*

The effect of the galactic tidal field upon the system of globular clusters of our galaxy is quite important and has to be taken into account when we study or draw conclusions from it. It affects present day globular clusters and may have destroyed the vast majority of an initial population of clusters. In this sense, the present day characteristics of globular clusters may be largely dependent upon these external forces that shape them. The effect of the tidal field of the parent galaxy may also explain, at least in part, the observed differences in number of globular clusters pe unit luminosity between spiral and elliptical galaxies, because the latter do not have a disk that can disrupt their clusters.

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*See also* **Galactic Structure, Globular Clusters; Star Clusters, Globular, Formation and Evolution; Star Clusters, Globular, Mass Loss.**

# *Star Clusters, Globular, Gravothermal Instability*

*Piet Hut*

Globular clusters do not live forever. Just as a glass of water, left by itself, slowly loses its contents by evaporation, so do star clusters lose stars, in a process that resembles evaporation. In a glass of water, interactions between molecules occasionally give an unusually high velocity to an individual molecule, which can then escape from the water surface. In a star cluster, gravitational interactions between stars occasionally give a star a velocity that exceeds the escape velocity, causing the star to leave the cluster. This process of random exchange of energy between stars, as a cumulative effect of near and far hyperbolic encounters, is called two-body relaxation. As a consequence of the resulting escape of stars, the inner parts of a star cluster lose energy and therefore contract, a process that leads to a higher central density. In turn, this increase in density leads to a higher rate of two-body relaxation, which leads to a higher rate of escapes. Not all the stars that evaporate from the central region will actually leave the system; to sustain the central contraction, it is enough that they are removed from the core. The vicious circle of increased density leading to an increased rate of contraction leading to an even more increased density is a runaway process: Two-body relaxation predicts the occurrence of an infinite central density in a finite time.

Before discussing the solution to this paradox of predicting infinite increase in observational quantities, let us look at a few other self-gravitating systems that show similar behavior. The birth of a star, for example, is the end result of the contraction of a gas cloud: The cloud loses energy by radiation from its surface and therefore tends to grow colder, which lowers the internal pressure that was holding up the cloud against its own gravity. As a result, gravity takes the upper hand and the cloud contracts, paradoxically being heated (by an amount larger than the amount of heat lost by radiation at the surface) in the process. This is a consequence of the virial theorem, which states that the internal kinetic energy is proportional to the potential energy (half as large, in fact, and of opposite sign).

This phenomenon of *increasing* temperatures as a result of a *loss* of heat runs counter to our normal intuition, based on laboratory experiments, where the exact opposite holds true. It seems that the heat capacity of a self-gravitating object has a *negative* sign. In fact, this is a very basic aspect of gravity and is related to the fact that we cannot apply gravity to an infinite static and homogeneous medium: The Jeans catastrophe will break up such a medium into clumps with masses of the order of the Jeans mass, within a time scale corresponding to a crossing time through such a clump. In other words, we cannot apply the usual framework of thermodynamics to self-gravitating systems, partly because there is no meaningful thermodynamic limit. Another way of seeing this is by realizing that the energy of a self-gravitating system grows with the square of its mass, which implies that the energy grows faster than an extensive quantity. This poses a much more serious problem than some other laboratory phenomena, such as the surface tension of a soap bubble, where the associated energy grows at a speed in between that of extensive and intensive quantities.

Another, even simpler, example of the negative heat capacity associated with gravity can be found closer to home, in the form of a satellite orbiting the Earth: When such a satellite encounters some friction from the upper atmosphere, it tends to lose energy and, thereby, altitude. As a consequence, however, its orbital speed is actually *increased*, even though it has *lost* energy. Returning to our discussion of the dynamics of a globular cluster, we see that the accelerated collapse of the inner regions is a consequence of the inherent instability of gravity, which can be characterized by assigning a negative heat capacity to a self-gravitating system. This runaway process was first discovered, in the simpler but analogous case of a self-gravitating gas confined within a rigid sphere, by V. A. Antonov in 1962. It was later analyzed in more detail by Donald Lynden-Bell and Roger Wood who termed it a "gravothermal catastrophe."

## *GRAVOTHERMAL CATASTROPHE*

Antonov found that isothermal equilibrium configurations can be stable only when the density contrast (the ratio between the density near the confining sphere and the central density) is less than a critical value, which he determined to be $\approx 709$. He showed that for larger values, a small increase in central temperature leads to an accelerated contraction of the core. Conversely, a small decrease in the central temperature leads to a runaway expansion of the core. Direct confirmation of the validity of such gas models for stellar dynamical applications was provided by Shogo Inagaki and Haldan Cohn in 1980. Inagaki performed a linear stability analysis of a stellar dynamical system enclosed in a sphere, using the Fokker–Planck approximation to describe the interactions between the particles, and recovered Antonov's instability criterion for stellar dynamics. Cohn solved the Fokker–Planck equations for diffusion in energy space numerically, and closely reproduced the results of self-similar collapse calculations by Lynden-Bell and Peter P. Eggleton, based on gas dynamics. For more details we refer the reader to the monograph on globular cluster dynamics by Lyman Spitzer, Jr.

With the existence of the gravothermal catastrophe being firmly established, we have to investigate the fate of a globular cluster after core collapse. For guidance, let us again look at the corresponding situation of a gas cloud. The contraction of a protostar is halted when the core of the star starts burning nuclear fuel at a rate that balances the energy loss at the surface. Similarly, gravothermal collapse of the core of a globular cluster will be halted when a central source generates more energy than is effectively conducted out of the core by two-body relaxation. This notion of a central energy source was first discussed by Michel Hénon, who suggested that a small subsystem of the core could shrink enough to generate the energy required. For example, even the formation of a single close binary, with an orbital velocity about an order of magnitude larger than the velocity dispersion in the cluster, would release an amount of energy comparable to that of a hundred single stars.

Hénon's ideas were confirmed several years later by $N$-body calculations. For all values of $N$ explored ($10^1-10^3$) a contraction of the central region produced at least one tight binary star with a binding energy comparable to the total binding energy of the system. Formation and hardening of such a binary then fueled the expansion and evaporation of the $N$-body system. (A review of this topic was given by Sverre J. Aarseth and Myron Lecar in 1975.)

However, a significant difference between the $N$-body calculations performed to date and globular clusters is that for clusters $N \sim 10^5-10^6$, whereas for the calculations $N \leq 10^{3.5}$. The large number of stars in a globular cluster implies that a single hard binary can absorb at most $\sim 1\%$ of the binding energy of the cluster: Attempts to form harder binaries lead either to a merging of the stars or to an escape of the binary from the cluster by the recoil momentum gained in an encounter with a third star. Thus, although in simulations each binary plays a dominant role, in real globular clusters only the cumulative effect of many binaries can change the local energy budget of the cluster significantly.

The first attempt to study the effects of binaries in systems with large $N$ was made by Hénon in 1975 using a Monte Carlo Fokker–Planck code. He introduced an artificial energy source in the innermost part of his cluster model, which was tuned to give off just the amount of energy necessary to avoid collapse locally at the inner shell. He found that the cluster reached a maximum central density, after which the collapse was reversed into an overall expansion.

A decade later, several authors confirmed Hénon's results by following the evolution of much more detailed models. Around that time, observations by Stanislav G. Djorgovski and Ivan R. King of central brightness excess of the cores of several globular clusters suggested that a significant fraction of all globular clusters may already have undergone core collapse. A review of both the theoretical and observational situation was given by Rebecca Elson and co-workers in 1987.

## *POSTCOLLAPSE EVOLUTION*

The evolution of a globular cluster after core collapse has only recently been studied intensively, and many aspects of our understanding of it remain uncertain and may change in the years following this review. The *mean* behavior of the cluster after core collapse, however, is firmly established: The half-mass radius of an isolated cluster expands according to $r_h(t) \propto t^{2/3}$, where $t$ is the time since core bounce, whereas the velocity dispersion drops according to $v \propto t^{-1/3}$. This relation may be derived from general principles, without any knowledge of the mechanism of energy generation in the core, as was done by Hénon in 1965 and 1975, in a manner analogous to Arthur S. Eddington's prediction in 1926 of the mass–luminosity relation for stars, which requires no precise knowledge of the nature of their internal energy generation. The derivation goes as follows: (1) The half-mass relaxation time $t_{hr}$ in a self-similar solution scales as $t_{hr} \propto t$, the time since core bounce; (2) $t_{hr} \propto N t_{hc}$, where $N$ is the number of stars in the cluster, $t_{hc}$ is the crossing time at the half-mass radius, and we have neglected a factor $\log N$; (3) if we neglect the slow change in mass and particle number due to escape, the virial theorem gives $t_{hc} \propto r_h^{3/2}$; (4) combining these gives $t \propto r_h^{3/2}$, which leads to the results quoted previously. In contrast, the rate of expansion of the *core* does depend on the details of the central engine. This was illustrated by detailed calculations, performed by Cohn, by Ostriker, and by Statler and coworkers.

There are, however, strong indications that the evolution of globular clusters after core collapse is quite a bit more complicated than the simple picture described here. Several years ago, Erich Bettwieser and Daiichiro Sugimoto followed the evolution of gas sphere models, and found large oscillations in the size of the core radius, which they interpreted as a new physical phenomenon: gravothermal oscillations. They interpreted these gravothermal oscillations as yet another consequence of the negative heat capacity of gravity. In the previous section we saw how the gravothermal instability can lead to core collapse: If the central temperature is slightly too high, the core will lose more heat than it gains, and this will lead to a contraction and therefore a density increase, which in turn will produce a higher central temperature. After core collapse is reversed into core reexpansion, the opposite may occur: The expansion may lower the central temperature, leading to an energy flow into the core and, in turn, to a lowering of both core density and temperature. The result is a runaway expansion that proceeds much faster than dictated by the boundary conditions at the surface of the cluster. The expanding region in the cluster center will grow radially until it reaches a region of radially decreasing temperature. At this point the expansion halts and the central region starts to collapse again.

The gravothermal character of core oscillations in gas sphere models was confirmed explicitly by Jeremy Goodman, in a linear stability analysis of a new regular self-similar model for postcollapse evolution, which he constructed in the same paper. Also for a more realistic stellar dynamics model, based on Fokker–Planck approximations for the two-body relaxation, Cohn, the author, and Michael Wise confirmed the existence and gravothermal nature of core oscillations. However, it is not yet clear to what extent these models apply to real globular clusters, as opposed to gas-sphere and equal-mass point-particle models. The fact that the core of a cluster around the time of core collapse typically contains fewer than a hundred stars makes the use of statistical models questionable. Ultimately, full $N$-body calculations are needed to provide the answer to the question of the nature of gravothermal oscillations, as was shown in a detailed analysis by Hut and coworkers in 1988.

## *SUMMARY AND OUTLOOK*

In many respects, models of globular cluster evolution have reached a point comparable to the state of stellar evolution theory in the 1950s, when nuclear reaction rates describing the physics of energy generation became available, and the computers necessary for the construction of detailed models were just being developed. The analogous two- and three-body gravitational reactions between stars were studied in detail by the author in 1985. These reactions turn on after the initial core collapse and heat the central regions of the cluster both directly, through the effects of energetic reaction products, and indirectly, via mass loss. They power the "main-sequence" phase of a globular cluster, and drive a slow but steady loss of stars by evaporation. Computationally, the ongoing development of new supercomputers and parallel computers promises, within a few years, speeds orders of magnitude greater than those available today.

The progress in our understanding of globular cluster dynamics during the last three decades has been impressive indeed. We now have a consistent standard picture of precollapse evolution, initiated during the 1960s and developed in detail during the 1970s. Some of the main ingredients of postcollapse evolution have emerged during the 1980s, filling in parts of what was still a blank spot on the map only a few years ago. However, major questions about the postcollapse phase remain unanswered. Our insight into the further stages of evolution, during and after core bounce, is much less complete. Fundamental questions, such as whether gravothermal oscillations will occur in realistic cluster models, are largely unsettled. We are still far from the point where we can construct models that can be compared directly with observations.

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*See also* **N-Body Problem; Star Clusters, Globular, Formation and Evolution; Star Clusters, Globular, Mass Loss; Star Clusters, Globular, Mass Segregation.**

## *Star Clusters, Globular, Mass Loss*

*Donald J. Faulkner*

Current ideas about the late stages of the evolution of low mass stars, such as those presently ending their lives in the globular clusters of our galaxy, suggest that they lose an appreciable fraction of their mass before collapsing to become white dwarfs. In spite of this, observational searches for interstellar material within clusters have largely proved negative and have set upper mass limits for the cluster gas and dust content that are well below the amounts that would be present, in many clusters, were they to retain all the gas lost from stars. Many attempts at resolving this discrepancy have been made during the last 20 years, but no proposed explanation has, as yet, been conclusively verified.

### *STELLAR MASS LOSS IN CLUSTERS*

Evidence that mass loss occurs from stars within globular clusters is as follows:

1. Studies of field white dwarf stars indicate that their masses are typically 0.5–0.6 times that of the Sun. However, a star that began life with such a mass would take considerably longer than the age of the Universe to evolve. In a 0.6 $M_\odot$ star, the hydrogen-burning phase takes well over 25 Gyr for any reasonable stellar composition. Thus observed white dwarfs must have lost mass at some earlier stage of their evolution. In globular clusters, the stars that are presently finishing the long-lived, hydrogen-burning, main sequence phase of their lives are $\sim 0.8\ M_\odot$, so that we can expect them to lose ~25% of this mass before becoming white dwarfs.

2. After a low mass star has exhausted its central hydrogen and has swollen up to become a red giant for the first time, there follows a phase in which it burns both helium at its center and hydrogen in a surrounding shell. In the case of globular clusters, such stars occupy a distinctive horizontal branch in the cluster color-magnitude diagram. An acceptable match between this observational feature and the theoretical models for the stars that correspond to it can be obtained only if the mass assumed for them is $\sim 0.2\ M_\odot$ less than that of stars presently leaving the main sequence. Thus an appreciable fraction of the anticipated pre-white-dwarf mass loss seems to occur as early as during the first excursion into the red giant domain.

3. A number of spectroscopic studies of the brighter giant stars in globular clusters show that many of them display asymmetric H$\alpha$ emission signatures indicating possible mass loss. Loss rates reported are $10^{-9}$–$10^{-7}\ M_\odot\ \text{yr}^{-1}$, which could account for a total loss of $\sim 0.2\ M_\odot$ over the lifetime on the brighter part of the giant branch, fitting in well with the horizontal branch mass requirements. Furthermore, infrared observations of long period variables (LPVs) in globulars have shown 3.5-, 10-, 12-, and 25-$\mu$m excesses and strong $H_2O$ absorption, indicating that circumstellar dust shells and extended atmospheres are present. Loss rates have been estimated at $\sim 5\times10^{-6}\ M_\odot\ \text{yr}^{-1}$. The LPVs are *asymptotic* giant branch stars, so mass loss seems to occur during the second excursion into the giant domain as well.

4. It is known that low mass stars in the galactic field can experience still further mass loss immediately prior to the white dwarf collapse, through the ejection of a shell of surface material in the form of a planetary nebula. The shell expands and disperses on a time scale of about 30,000 yr. If the stars evolving in globular clusters also participate in this form of mass loss, then, on statistical grounds, one would expect one or two planetary nebulae to be observable in the globulars of the Galaxy. One has indeed been observed, K 648 in M 15, confirming the existence of planetary nebula gas ejection in globulars.

Thus there is strong evidence that all the usual processes of mass loss known to occur in the late stages of the evolution of low mass stars are active in the stars of globular clusters, and we need to consider the fate of the ejected material in the cluster systems.