TIME-SYMMETRIZED KUSTAANHEIMO-STIEFEL REGULARIZATION

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Abstract.
In this paper, we show a new algorithm to integrate the orbits of binaries. Our new algorithm has the good properties of both symmetrized timesteps and KS regularization: (1) no secular error in either energy or angular momentum; (2) a constant number of timesteps per orbit for a binary with arbitrary eccentricity (Funato et al., 1995).

1. Introduction

The long-term numerical integration of the binary orbit is important in the theoretical study of the evolution of dens stellar clusters. Binary evolution is also important from the observational point of view, since many interesting objects, such as X-ray sources, millisecond pulsars, high-velocity stars, and blue stragglers, are believed to be the result of binary interactions. In order to study the evolution of clusters, self-consistent N-body simulation is a most useful tool. However, its computational requirements would be prohibitive; in addition, both truncation and round-off error would be unacceptably large.

Here we present a new time-integration algorithm which has no secular error in either the binding energy or the eccentricity, while allowing variable stepsize. By contrast, the stabilization technique, which has been widely used in the field of stellar dynamics, conserves energy very well but does not conserve angular momentum.

2. Symmetrized KS Hermite Scheme with Variable Stepsize

We integrate the equations of motion of the KS binary and its specific energy (Aarseth, 1985) using the time-symmetrized Hermite scheme (Hut, Makino and McMillan, 1995). The procedure for a single step of the time integration is as follows.

[1] Predict the positions and velocities of all particles.
[2] Evaluate all accelerations and jerks using predicted positions and velocities.
[3] Calculate the fourth and fifth derivatives of the relative position of the binary components in KS coordinates at both the beginning and the end of the step.
[4] Evaluate the new stepsize ($\Delta\tau_{new}$) using the stepsize calculated at the end of the step ($\Delta\tau_e$) and the beginning of the step ($\Delta\tau_b$).
[5] Correct the integrated values using the new stepsize $\Delta\tau_{new}$.
[6] Repeat procedures [2]–[5] until both the stepsize and the integrated variables converge.

3. Numerical Tests

We have carried out integration of orbits of a binary with initial eccentricity $e = 0.9$. We compared the results of three schemes; symmetrized, stabilized and "plain". All calculations were done in double precision (16-digit accuracy). The result shows that the eccentricity of the binary is conserved by the symmetrized integrator, but not by either the plain or the stabilized schemes. The plain scheme conserves angular momentum but not energy, while the stabilized scheme conserves energy but not angular momentum. Thus neither scheme preserves the eccentricity. Only the symmetrized scheme conserves both energy and eccentricity up to round-off error.

We also experimented the case of a hierarchical triple in which the eccentricity of inner binary is 0.9 and that of outer is 0.0. Our result for the hierarchical triple case shows that our scheme can follow the evolution of weakly perturbed binary for a long time.

References

Aarseth, S. J., 1985, "Multiple Time Scales" Academic Press.
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Hut, P., Makino, J., and McMillan, S. : 1995, *ApJLetter*, **443**, L93–96.