100 Years of Noetherian Rings

Institute for Advanced Study

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Talk titles and abstracts (alphabetically by speaker)

Gale duality, blowups and moduli spaces

CAROLINA ARAUJO

To understand the birational geometry of a projective variety X, one seeks to describe all rational contractions from X. From an algebraic perspective, information about all these contractions are encoded in the ring formed by all sections of all line bundles on X, the Cox ring of X. In this talk, we discuss the birational geometry and the Cox ring of blowups of projective spaces at points in general position. For that, we explore Gale duality, a correspondence between sets of $n = r+s+2$ points in projective spaces $\mathbb{P}^s$ and $\mathbb{P}^r$. For small values of s, this duality has a remarkable geometric manifestation: the blowup of $\mathbb{P}^r$ at n points can be realized as a moduli space of vector bundles on the blowup of $\mathbb{P}^s$ at the Gale dual points.

Emmy Noether's influence on physics

NIMA ARKANI-HAMED

Automorphisms of K3 surfaces and isometries of lattices

EVA BAYER-FLUCKIGER

The aim of this talk is to present some results on isometries of even, unimodular lattices and automorphisms of K3 surfaces.

Vanishing theorems in algebraic geometry

BHARGAV BHATT

Vanishing theorems in complex algebraic geometry, such as the Kodaira vanishing theorem, play an important role in understanding the structure of complex algebraic varieties. Ultimately, they rely on critical input from Hodge theory. After recalling some aspects of this story, I will discuss analogous results in mixed characteristic as well as applications; the key input now comes from p-adic Hodge theory.

Infinite rank twisted sheaves

AISE JOHAN DE JONG

Report on a paper written during the pandemic joint with Minseon Shin and Max Lieblich. Some key words: Brauer groups, nontorsion elements of $\mathrm{H}^2(X, \mathbf{G}_m)$, resolution property, Jouanolou devices, and derived categories. I will try to explain my motivation for this work and formulate some open questions.

The geometry of GL-varieties

JAN DRAISMA

A GL-variety X is an (infinite-dimensional) affine variety with an action of the infinite general linear group GL such that the coordinate ring of X is a polynomial GL-representation and generated by finitely many GL-orbits of elements.

In earlier work, we established that GL-varieties are topologically Noetherian: they satisfy the descending chain condition on closed subvarieties. The proof of this fact led to a coarse understanding of the geometry of GL-varieties, e.g. to a version of Chevalley's theorem on constructible sets and to the insight that GL-varieties are "unirational in the GL-direction": they admit a dominant GL-equivariant morphism from a GL-variety of the form $B \times A$ where B is a finite-dimensional affine variety with trivial GL-action and A is an affine space with linear GL-action.

In this talk I will present recent work with Bik, Eggermont, and Snowden on finer aspects of the geometry of GL-varieties. For instance, we show that any two points in an irreducible GL-variety can be joined by an (ordinary) curve, and we use this to establish uniformity results for limits of tensor decompositions.

Integrality properties of topological fundamental groups
HÉLÈNE ESNAULT

The talk is based on joint work with Michael Groechenig and Johan de Jong.

We single out integrality properties of the topological fundamental group of smooth quasi-projective complex varieties which rely on the Langlands program (both arithmetic and geometric) and yield an obstruction for a finitely presented group to be the fundamental group of a smooth quasi-projective variety.

The spectrum of elliptic Calabi-Yau threefolds
ANTONELLA GRASSI

I will discuss a dictionary between the quantities in the spectrum of the string theory compactified on the Calabi-Yau and object in the Calabi-Yau. I will also discuss some properties.

Species and dimension formulas for period spaces
ANNETTE HUBER-KLAWITTER

(joint work with Martin Kalck) Periods are complex numbers obtained by integrating algebraic differential forms over $\mathbf{Q}$ over a domain of algebraic nature. This includes numbers like π, $\log(2)$ or the values of the Riemann zeta function at integers. More conceptually, periods are the entries of the period pairing between the singular and de Rham realisation of a mixed motive over $\mathbf{Q}$. The period conjecture makes a sweeping qualitative prediction on the relations between period numbers, e.g., that π is transcendental.

We develop and apply the abstract theory of a very special class of Noetherian rings, namely finite dimensional $\mathbf{Q}$-algebras, in order to deduce formulas for the expected dimension of the space of periods of a single motive. This is of particular interest in the case of 1-motives (or periods of curves), where the linear version of the period conjecture is a theorem due to Huber and Wüstholz.

Singularities of pairs in positive characteristic and in characteristic zero
SHIHOKO ISHII

For studies on singularities over the field of characteristic zero, we can use many convenient tools: resolutions of singularities, Bertini's theorem (generic smoothness), cohomology vanishing of Kodaira type, and so on. However, in positive characteristic cases, these are not available. Therefore many good properties obtained in characteristic zero using these tools are not accessible in positive characteristic cases. My talk focuses on an invariant of singularities mld (minimal log discrepancy) and lct (log canonical threshold) and makes a bridge between positive characteristic and characteristic zero in the discussions of mld and lct. By the bridge, we obtain the following in the positive characteristic case:

(1) the discreteness of log discrepancies.
(2) Existence of prime divisor computing mld.
(3) lct and accumulation points of lct are rational numbers.
(4) ACC holds for lct of ideals on a smooth variety.

Abstract Algebra in Homotopy-Coherent Mathematics
JACOB LURIE

Emmy Noether was a central figure in the development of abstract algebra in the early 20th century. Her ideas were profoundly influential, touching nearly every corner of mathematics. In this talk, I'll discuss how those ideas have taken new shape in the setting of stable homotopy theory, and describe some curious phenomena which emerge in that context.

The log canonical threshold revisited
JAMES MCKERNAN

The log canonical threshold plays a fundamental role in algebraic geometry, especially birational geometry and Mori theory. Recently the problem of classifying foliations on algebraic varieties has been revolutionized by introducing ideas from Mori theory.

We introduce a version of the log canonical threshold in this context. We conjecture that this satisfies the ascending chain condition and we check that this conjecture holds in dimension two. On the way we introduce an interpolated canonical divisor and we show that we can run the MMP for such divisors.

A Formalization Experiment
SOPHIE MOREL

A few months ago, I decide to attempt to formalize one of my own papers using the Lean4 theorem prover. I will report on this experiment. (The Lean code can be found there:
`https://github.com/smorel394/TS1`)

Matrix Factorizations and Complete Intersection Rings
IRENA PEEVA

Motivated by applications in Invariant Theory, Hilbert introduced an approach to describe the structure of modules by free resolutions. Hilbert's Syzygy Theorem shows that minimal free resolutions over a polynomial ring are finite. Most minimal free resolutions over quotient rings are infinite. We will discuss the properties of such resolutions. The Serre-Kaplansky Problem was a central question in Commutative Algebra for many years; it asked whether the generating function of the minimal free resolution of the residue field of a commutative noetherian local ring is always rational. Anick constructed the first example providing a negative answer and a strong indication that the structure of these resolutions can be quite complex.

The concept of matrix factorization was introduced by Eisenbud 35 years ago, and it describes completely the asymptotic structure of minimal free resolutions over a hypersurface. Matrix factorizations have applications in many fields of mathematics: for the study of cluster algebras, Cohen-Macaulay modules, knot theory, moduli of curves, quiver and group representations, and singularity theory. Starting with Kapustin and Li, physicists discovered amazing connections with string theory. The talk will focus on the concept of matrix factorization for complete intersection rings, which we introduced and showed that it suffices to describe the asymptotic structure of minimal free resolutions over complete intersections.

Integral points on curves via Baker's method and finite étale covers
BJORN POONEN

We prove results in the direction of showing that for some affine curves, Baker's method applied to finite étale covers is insufficient to determine the integral points.

Finite quotients of 3-manifold fundamental groups, and unramified extensions of global fields
MELANIE MATCHETT WOOD

It is well-known that for any finite group G, there exists a closed 3-manifold M with G as a quotient of the fundamental group of M. However, we can ask more detailed questions about the possible finite quotients of 3-manifold groups, e.g. for G and $H_1, \dots, H_n$ finite groups, does there exist a 3-manifold group with G as a quotient but no H_i as a quotient? We give an answer to all such questions, and explain how this is related to our work on the analogous problem of whether there exist number fields or function fields with an unramified G extension but no unramified H_i extension. This talk is on joint work with Will Sawin, and also includes joint work with Yuan Liu and David Zureick-Brown.

An irregular Deligne–Simpson problem
ZHIWEI YUN

The Deligne–Simpson problem asks for a criterion of the existence of connections on an algebraic curve with prescribed singularities at punctures. We give a solution to a generalization of this problem to G-connections on $\mathbb{P}^1$ with a regular singularity and an irregular singularity (satisfying a condition called isoclinic). Here G can be any complex reductive group. Perhaps surprisingly, the solution can be expressed in terms of representations of rational Cherednik algebras. This is joint work with Konstantin Jakob.