On dynamics of $\mathscr{B}$-free systems generated by Behrend sets.

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This is a joint work with Mariusz Lemańczyk and Sebastian Zuniga Alterman. Given a set $\mathscr{B}$ of natural numbers, not containing 1, we denote by $\mathcal{F}_{\mathscr{B}}$ the set of $\mathscr{B}$-free numbers, that is, $\mathcal{F}_{\mathscr{B}} = \mathbb{Z} \setminus \bigcup_{b\in\mathscr{B}} b\mathbb{Z}$. Let X_η be the $\mathscr{B}$-free subshifts, that is the subshift induced by η, where η denotes the characteristic function of $\mathcal{F}_{\mathscr{B}}$. That means, X_η is the closure of the set of all shifts of η in the space $\{0, 1\}^{\mathbb{Z}}$ equipped with the product topology. We are interested in the case when $\mathscr{B}$ is a Behrend set, that is, when the set of $\mathscr{B}$-free numbers has zero density. It turns out that this is the case precisely when X_η is proximal and has zero entropy. We prove that the complexity of X_η, with $\mathscr{B}$ being a Behrend set, can achieve any subexponential growth. Together with the $\mathscr{B}$-free shift we investigate the $\mathscr{B}$-admissible shift $X_{\mathscr{B}}$ and we show that it is transitive if and only if the set $\mathscr{B}$ is pairwise coprime, which allows one to characterize dynamically the subshifts generated by the Erdös sets (infinite, coprime and not Behrend). We also estimate the complexity for some classical subshifts (the subshift of primes or semi-primes). The lower estimates are obtained conditionally on Hardy-Littlewood Conjecture or Dickson's Conjecture. We remark on a recent result of Tao and Ziegler (not assuming the conjectures) that the shift of primes is uncountable.