(1) In class, we discussed the metric 
\[ ds^2 = -2e^q dv du + g_{AB}(dx^A + c^A dv)(dx^B + c^B dv), \]
where \( q \) is independent of \( u \) at \( v = 0 \), but otherwise the functions \( q \), \( g_{AB} \), and \( c^A \) depend on all coordinates \( u, v, x^A \). As discussed in class, this is a canonical form for the metric near a null hypersurface \( Y \) that is swept out by a family of orthogonal null geodesics from a codimension 2 spacelike submanifold \( W \). \( W \) is the hypersurface \( u = v = 0 \) and \( Y \) is the hypersurface \( v = 0 \).

Calculate \( R_{uu} \) along \( Y \), that is at \( v = 0 \). As explained in class, this step leads to Raychaudhuri’s equation.

(2) Consider the metric 
\[ ds^2 = (t^2 - 1) \left( -dt^2 + d\tilde{x}^2 \right), \]
where \( \tilde{x} = (x^1, \ldots, x^{D-1}) \).

In this spacetime, consider the codimension 2 spacelike hypersurface \( W \) defined by \( t = t_0, \ |\tilde{x}| = R \), with constants \( t_0, R \). What is the condition for \( W \) to be a trapped surface?

(3) In a spacetime \( M \), let \( S \) be a spacelike hypersurface (dimension \( D - 1 \) if \( M \) has dimension \( D \)) and let \( Q \subset S \) be a manifold with boundary, also of dimension \( D - 1 \); let \( \partial Q \) be the boundary of \( Q \). For example, in Minkowski space, \( Q \) might be a closed ball and then its boundary \( \partial Q \) is a sphere. As usual, we let \( J^+(Q) \) be the causal future of \( Q \) and \( J^+(\partial Q) \) be the causal future of \( \partial Q \). The boundaries of these sets are \( \partial J^+(Q) \) and \( \partial J^+(\partial Q) \).

Show that any point in \( \partial J^+(Q) \) that is not in \( Q \) itself (in other words, any point that is strictly to the future of \( Q \) is in \( \partial J^+(\partial Q) \). This fact will be useful in discussing black holes.