What is Quantum Field Theory?

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Classical Physics

Classical Mechanics

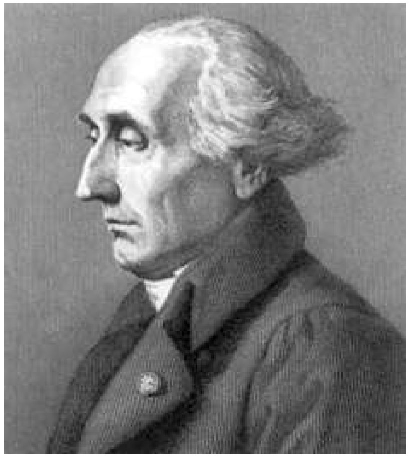
Time evolution of a finite number of particles

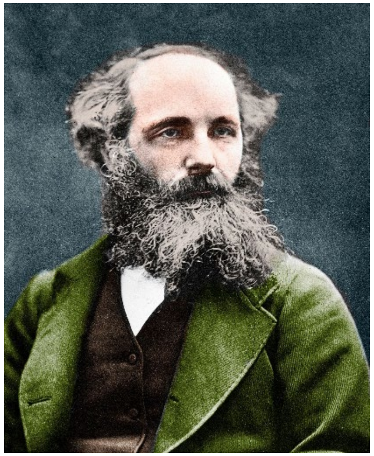
Ordinary differential equations

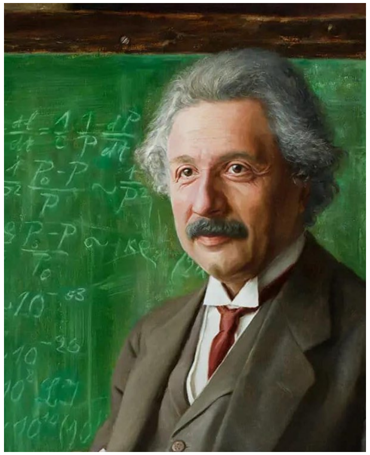
Classical Field Theory

Time evolution of an infinite number (continuum) of degrees of freedom, e.g., electromagnetic field, velocity of fluid, metric

Partial differential equations







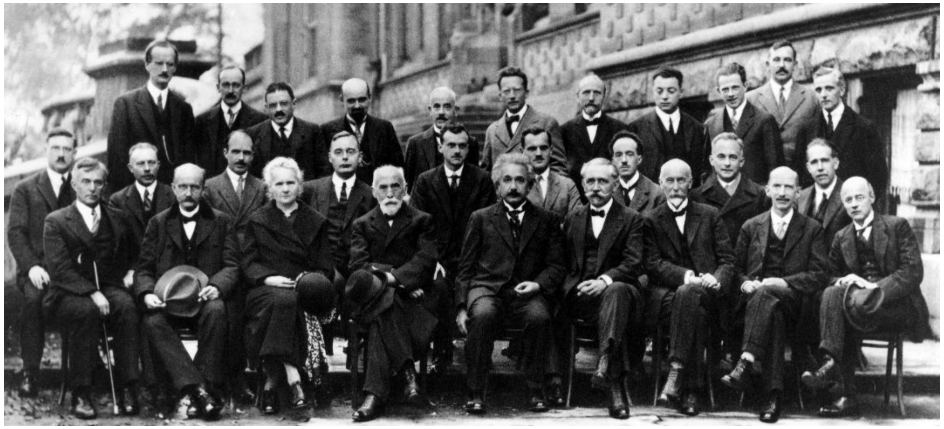
Quantum Physics

Quantum Mechanics

Time evolution of a finite number of quantum particles

Operators in a Hilbert space

Functional integral



Quantum Field Theory

Time evolution of an infinite number (continuum) of quantum degrees of freedom, e.g., electromagnetic field

A lot is known. Still very exciting progress.

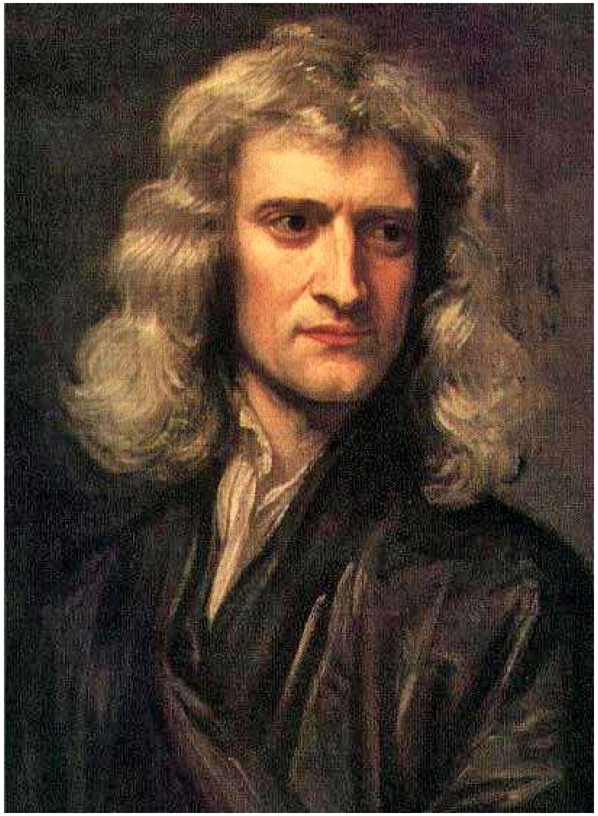
My personal view: a new intellectual structure is needed – QFT

Quantum Field Theory

- Quantum field theory is the natural language of physics:
  - Particle physics
  - Condensed matter
  - Cosmology
  - String theory/quantum gravity
- Applications in mathematics especially in geometry and topology
- Quantum field theory is the modern calculus
  - Natural language for describing diverse phenomena
- Enormous progress over the past decades, still continuing
- Indications that it should be reformulated…

Calculus vs. Quantum Field Theory

- New mathematics
- Motivated by physics (motion of bodies)
- Many applications in mathematics, physics, and other branches of science and engineering
- Sign that this is a deep idea
- Calculus is a mature field.
  Streamlined – most books and courses more or less the same

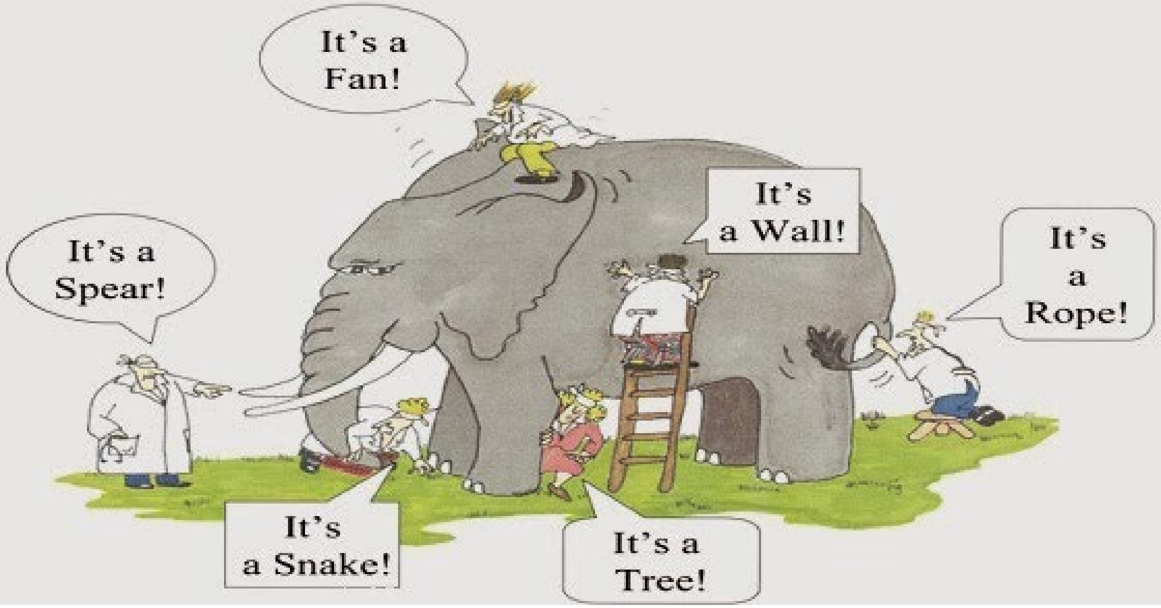


Calculus vs. Quantum Field Theory

- New mathematics (in fact, not yet rigorous)
- Motivated by physics (particle physics, condensed matter)
- Many applications in mathematics and physics
- Sign that this is a deep idea
- QFT is not yet mature – books and courses are very different (different perspective, order of presentation)
- Indications that we are still missing big things – perhaps QFT should be reformulated

How should we think about QFT?

Many starting points/views. None of them covers the whole thing.



Hence, the question in the title of this talk. The answer will likely lead to deep advances and insights.

Presentations of quantum field theory

- Traditional. Use a Lagrangian…
- More abstractly, operators and their correlation functions (expectation value of their product). They are constrained by some axioms.
  - In the traditional Lagrangian approach this is the outcome
- Others?

Abstract presentation of QFT

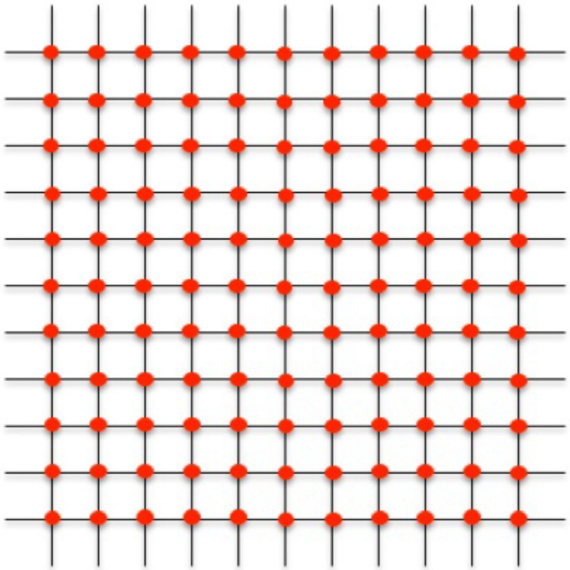
Use a collection of operators and their correlation functions. These include point operators (local operators), line operators, surface operators, etc.

- We do not distinguish between operators/observables/ defects (this depends on their orientation in spacetime)
- Their correlation functions should be well defined – mutually local

- Place the theory on various manifolds
  - This can lead to more choices (parameters)
  - More consistency conditions
  - Can we recover this information from local measurements?

Lagrangian

- Natural starting point – quantize a classical system
  - Canonical quantization
  - Functional integral
  - Others
- To make it meaningful, need to regularize (e.g., a lattice). Then need to prove the existence of
  - the large volume limit
  - the continuum limit



Lagrangian

- Pick "matter fields" (commuting or anticommuting) ϕ – map from spacetime to a target space
  - Non-derivative couplings, e.g., a potential $V(\phi)$
  - Derivative terms, e.g., kinetic terms $\sum_{ij\mu} g_{ij}\partial_\mu\phi^i\partial^\mu\phi^j$, ($g_{ij}$ is the pullback of the metric on the target space to spacetime)
  - Higher derivative terms
- Pick a gauge group. It can act on the matter fields.
  - Kinetic term for the gauge fields
  - Various topological terms

Key point: locality – the action satisfies certain gluing conditions.

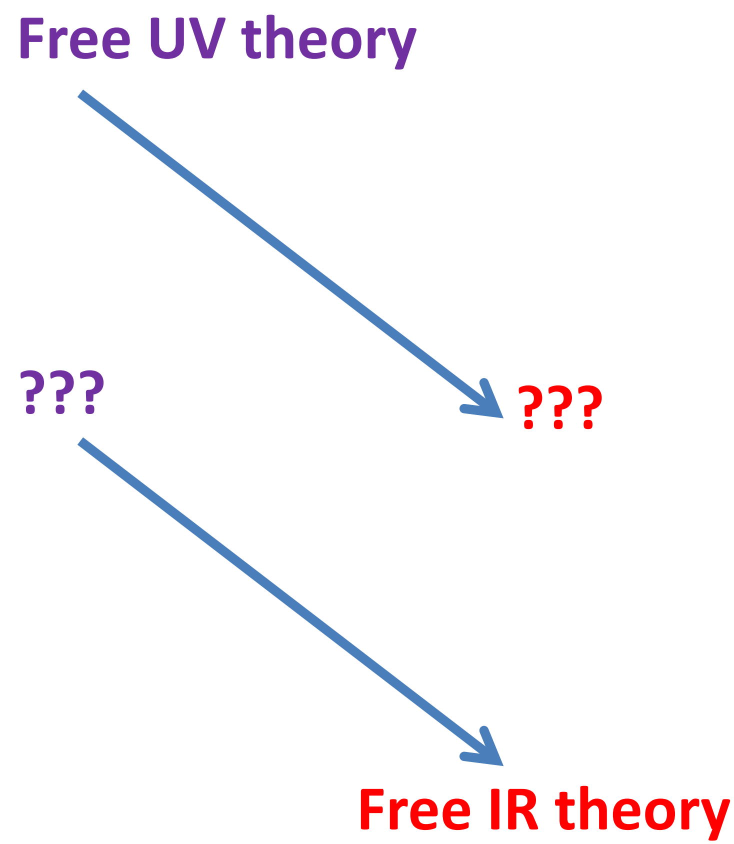
Lagrangian

Questions:

- Do we know all such constructions?
- Do we know all consistency conditions?
- Duality: When do different such constructions lead to the same physics?
- More below

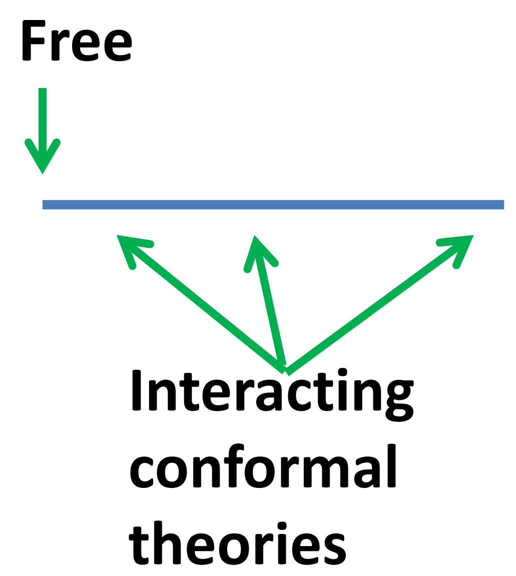
Lagrangians are meaningful and useful when they are weakly-coupled

- Free theory (quadratic) at short distances (UV). Perturb to find a new theory at long distances (IR), e.g.,
  - Asymptotically free 4d gauge theory (e.g. QCD)
  - . . .
- Free theory at long distances (IR) arising from another theory at short distances
  - 4d QED
  - Theory of pions
  - . . .



Lagrangians are meaningful and useful when they are weakly-coupled

- Family of conformally invariant theories connected to a free theory, e.g.,
  - 4d N=4 super Yang-Milles
  - 3d Chern-Simons theory (large level)
  - 2d sigma model with Calabi-Yau target space
  - . . .



Lagrangians are not good enough

- Strong coupling
- Not used in exact solutions
- Duality
- Theories without Lagrangian

Lagrangians are not good enough

Strong coupling

Given a weakly-coupled theory, described by a Lagrangian, there is no clear recipe to analyze it at strong coupling.

This is one of the main challenges in Quantum Field Theories.

Examples:

- Gapped systems, e.g., confinement in 4d gauge theory
- Interacting Conformal Field Theories
- Strongly-coupled Topological Field Theories like 3d Chern-Simons theory with small level, k
- Many problems in condensed matter physics
- . . .

Lagrangians are not good enough

Not used in exact solutions

Rely on the more abstract presentation.

Use consistency to constrain the answer:

- Bootstrap methods in Conformal Field Theories use associativity of the operators…
- Integrable systems are solved using a large symmetry.
- Rational conformal field theories are solved using the combination of a large symmetry and consistency.
- . . .

Lagrangians are not good enough

Not used in exact solutions

Rely on the more abstract presentation.

Use consistency to constrain the answer:

- Supersymmetric theories are analyzed using holomorphy/BPS/chiral/topological observables.
- Some Topological Quantum Field Theories are solved using simple rules (not the Lagrangian).
- Some field theories are analyzed by embedding them in String Theory.
- Scattering amplitudes are analyzed using consistency like unitarity.

Lagrangians are not good enough

Duality

Duality: several distinct Lagrangians leading to the same physics.

Duality is intrinsically quantum mechanical.

- Various free field theories
- Interacting conformal theories labeled by a dimensionless parameter. Here, duality relates weak (described by a Lagrangian) and strong coupling, e.g.,
  - Maximally supersymmetric 4d Yang-Mills theory
  - Various 3d Chern-Simons theories
  - 2d models associated with mirror Calabi-Yau target spaces
- …

Lagrangians are not good enough

Duality

Duality: several distinct Lagrangians leading to the same physics.

Duality is intrinsically quantum mechanical.

- Two different weakly-coupled theories become the same at long distances
- One weakly-coupled theory at short distances becomes another weakly-coupled theory at long distances
- Examples: some 4d supersymmetric gauge theories (some of these are relevant to 4d topology)

Lagrangians are not good enough

Theories without Lagrangians

- Theories without (Lorentz invariant) Lagrangians
  - Chiral fermions
  - Selfdual bosons (even free) – intrinsically quantum mechanical
- Theories that are not in the strong coupling limit of a weakly-coupled theory (which can be described by a Lagrangian) – no semi-classical limit
  - Some nontrivial conformal theories in 4d, 5d, and 6d

As the list of such theories keeps growing and their marvelous properties are being uncovered, it is wrong to dismiss them.

Suggest that QFT should be reformulated

- Lagrangians are not good enough
  - Not useful at strong coupling
  - Not used in exact solutions
  - Duality – more than one Lagrangian of the same theory
  - Sometimes no Lagrangian
- Not mathematically rigorous
- Extensions of traditional local QFT
  - Field theory on a non-commutative space
  - Little string theory
  - The low-energy limit of certain lattice models
  - Others?

How should we think about it?

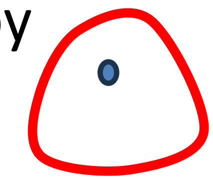
Of course, I do not know!

- A good place to start is Topological Quantum Field Theory
  - It is simple
  - Enormous progress during the recent decades
- But what about the more generic, non-topological theories (i.e., theories with local degrees of freedom)?
- Following [A. Kapustin, NS, arXiv:1401.0740; D. Gaiotto, A. Kapustin, NS, B. Willett, arXiv:1412.5148], study topological operators in a non-topological field theory
  - Theories with global symmetries have topological observables/operators/defects.

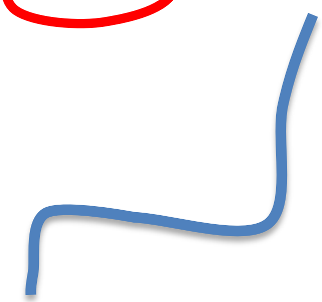
Generalized global symmetries

- Ordinary global symmetries
  - Act on local operators
  - The charged states are particles

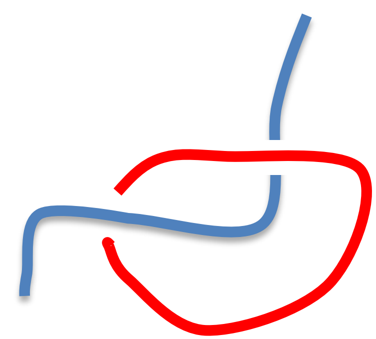
  - The symmetry operator is a topological operator with co-dimension one in spacetime. It acts by surrounding the charged local operators.



- Generalized global symmetries
  - The charged operators are lines, surfaces, etc.
  - The charged objects are strings, domain walls, etc.



  - The symmetry operators are of higher co-dimension

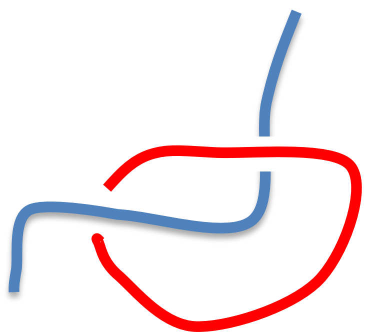


- Repeat all the things that are always done with ordinary symmetries.

Generalized global symmetries

Terminology:

- q-form global symmetries act on q-dimensional operators/defects, by a codimension $q + 1$ operator in spacetime.
- Ordinary symmetry: 0-form global symmetry



No need for a Lagrangian

Exist abstractly, also in theories without a Lagrangian

Generalized global symmetries

As with ordinary symmetries:

- Selection rules on expectation values of operators
- Dual theories often have different gauge symmetries. But the global symmetries must be the same
  - Non-trivial tests of duality
- Couple to a classical background gauge field (twisted boundary conditions)
  - Anomalies and more tests of duality
- Gauging by summing over the classical background gauge fields (summing over twisted boundary conditions)
- The symmetry might or might not be spontaneously broken. This can be used to classify the possible phases…

Generalized global symmetries
Characterizing phases

Consider an $SU(N)$ gauge theory.

It has a one-form symmetry – shift the gauge field by a flat gauge field (modulo gauge transformation).

The holonomy of the gauge group (Wilson loop) W transforms as

$$W \to e^{\frac{2\pi i}{N}} W$$

- In a confining phase, this one-form symmetry is unbroken.
  - The confining strings are charged and are classified by the unbroken symmetry.
  - $\langle W \rangle \sim e^{-\sigma \,\mathrm{Area}}$ with σ the string tension. It vanishes when the loop is large – symmetry unbroken.

Generalized global symmetries
Characterizing phases

- In a Higgs or Coulomb phase the one-form symmetry is spontaneously broken.
  - The large size limit of $\langle W \rangle$ is nonzero – vev "breaks the symmetry."
  - No strings
  - Massless photon is the Goldstone boson of spontaneously broken one-form symmetry. (Earlier discussion in [Rosenstein, Kovner, 1990]; He, Mitra, Porfyriadis, Strominger, 2014])
- In a TQFT, a discrete broken symmetry.

Summary

- Quantum field theory is everywhere
- It appears to be the language of physics
- There has been a lot of progress, but there are also big open questions
  - New phenomena are constantly being discovered
- Despite the progress, there is no satisfactory presentation of it.
- We focused on symmetries of quantum field theories.
  - Characterized by topological observables and hence they are under more control than the full theory
  - Generalize the notion of symmetry
  - Later, these ideas have been generalized further (non-invertible symmetries, …)

Thank you