Global Existence of Smooth Solutions to a Barely Supercritical Wave Equation

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Abstract

In this talk I will discuss the global existence of H^2 solutions of a barely energy supercritical equation, namely a loglog defocusing quitic wave equation. The first step is to control the $L_t^4 L_x^{12}$ norm of the solution on an arbitrary long-time interval $[0, T]$ by an a priori bound of its $L_t^\infty H^2([0, T])$ norm. To do that we divide the interval into subintervals on which the $L_t^4 L_x^{12}$ norm of the solution is substantial. We prove that the length of these subintervals is also substantial. Then, by using a variant of a Morawetz-type estimate from Shatah-Struwe (the variant we use is due to Bahouri-Gerard), we can control the length of large sequences of these subintervals. Eventually we can estimate their number. The second estimate is to estimate a posteriori the $L_t^\infty H^2([0, T])$ norm of the solution. This is done by a connectedness-in-time argument, using the Strichartz estimates and the control of the $L_t^4 L_x^{12}$ norm.