Symmetries, Duality, and the Unity of Physics

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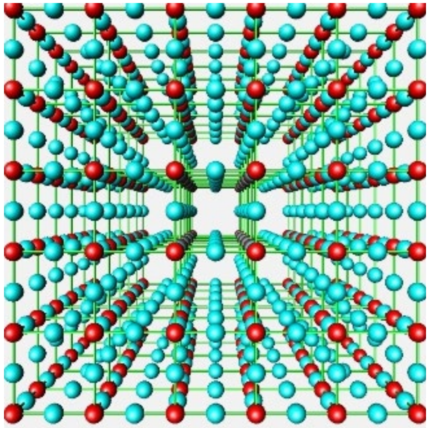
Symmetries

Physicists love symmetries



- Galileo, Lorentz, Poincare
- Crystallography
- Global symmetry (flavor)
  - e.g. Isospin $SU(2)$, $SU(3)$
- Gauge (local) symmetry
  - $A_\mu \to A_\mu + \partial_\mu \Lambda$ with $\Lambda(x)$ in Maxwell theory
  - $g_{\mu\nu} \to g_{\mu\nu} + \partial_\mu \xi^\rho g_{\rho\nu} + \partial_\nu \xi^\rho g_{\mu\rho} + \xi^\rho \partial_\rho g_{\mu\nu}$ with $\xi^\rho(x)$ in General Relativity
  - Color $SU(3)$, the standard model $SU(3) \times SU(2) \times U(1)$

Gauge symmetry is deep

- Largest symmetry (a group for each point in spacetime)
- Useful in making the theory manifestly Lorentz invariant, unitary, and local (and hence causal)
- Appears in
  - Maxwell theory, the Standard Model of particle physics
  - General Relativity
  - Many condensed matter systems
  - Deep mathematics (fiber bundles)

But

- Because of Gauss law the Hilbert space is gauge invariant.
- Hence: gauge symmetry is not a symmetry.
    - It does not act on anything.
    - All the operators are gauge invariant.

- A better phrase is gauge redundancy.

Gauge symmetry is not a symmetry

- Not surprising to mathematicians and some physicists
  - Manifestly true in many lattice constructions in condensed matter physics
  - Manifestly true in some low-dimensional continuum quantum field theories
- In particle physics, perhaps we should not look for a more complete theory with a larger gauge group

- Of course, this does not mean that gauge symmetry is not an extremely powerful concept and a useful tool.

Emergent symmetries

Global symmetries can emerge as accidental symmetries at long distances.

- Then they are approximate.
  - Continuous translations and rotations even though the microscopic theory is on a lattice.
  - Parity and time reversal symmetries are approximate symmetries at low energy, even though they are not symmetries of the standard model of particle physics
  - Approximate baryon number symmetry (and the related stability of the proton)

Emergent symmetries

Gauge symmetries (redundancies) can be emergent.

- They must be exact because they are not symmetries.
- First examples
  - lattice systems (mostly in condensed matter)
  - FQHE
  - continuum quantum field theory in low dimensions
    - These systems do not have a massless spin one particle reflecting the emergent gauge symmetry.
- We will soon discuss examples in higher dimensions

Symmetries in a theory of gravity

General considerations based on black holes strongly suggest that in any theory of quantum gravity

- There cannot be any exact global symmetry
  - Approximate, accidental global symmetries are possible.
- Gauge symmetries are possible.
  - The spectrum must include excitations with all electric and magnetic charges that are compatible with the symmetry.
    - Actually, this follows from the previous point.

Global vs. Local Symmetries

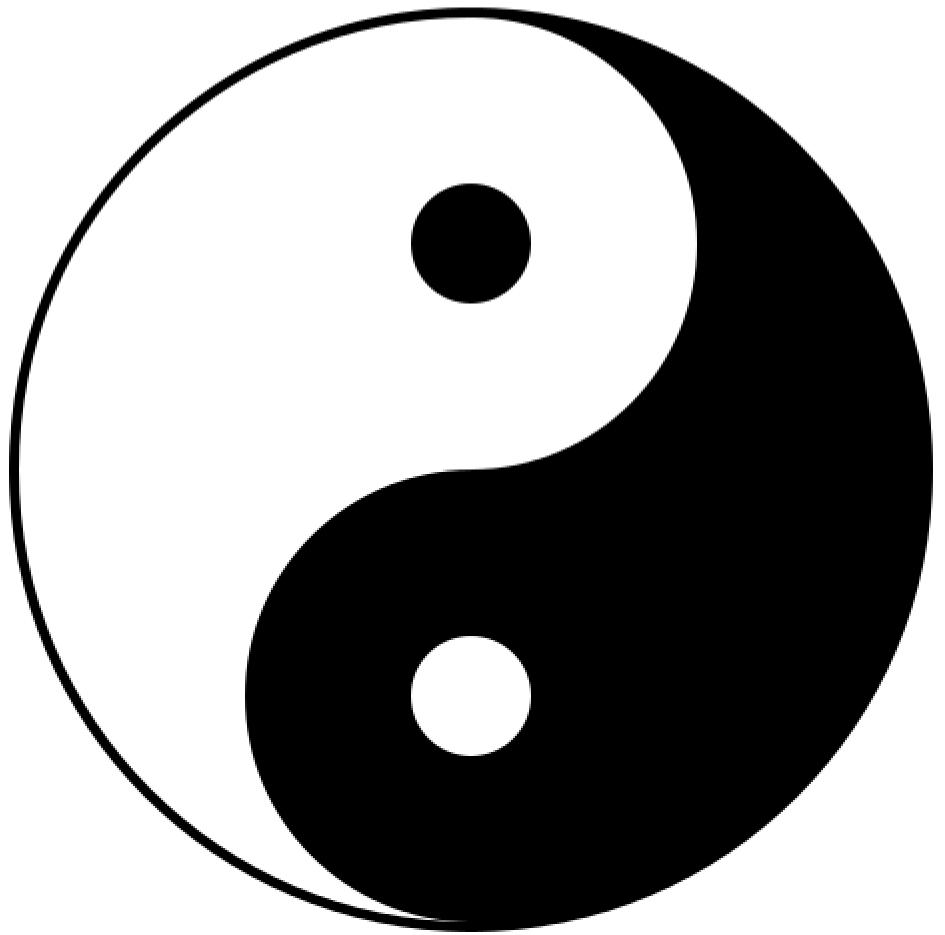
Global

- Intrinsic
- Can be accidental at long distances – approximate
- Classify operators
- Can be spontaneously broken
- If unbroken can classify states
- Useful in classifying phases
- ‘t Hooft anomalies
- Not present in a theory of gravity

Local (gauge)

- Ambiguous – duality
- Can emerge at long distances – exact
- All operators are invariant
- Not really a symmetry
- Hence it cannot be broken (Higgs description meaningful only at weak coupling)
- Cannot be anomalous
- Appears essential in formulating the Standard Model and in Gravity

Duality



Simple examples of duality

Simple dualities in free (solvable) theories (easy to establish)

Harmonic oscillator

$$H = \frac{1}{2m}p^2 + \frac{k}{2}q^2$$

Map H to itself:

- Classically, a canonical transformation ($q \to \frac{p}{\sqrt{mk}}$, $p \to -\sqrt{mk}q$)
- Quantum mechanically, a Fourier transform

Duality and emergent gauge symmetry

Again, easy to establish (essentially Fourier transform):

A free scalar field in *2+1d* is dual to Maxwell theory

$$F_{\mu\nu} \sim \epsilon_{\mu\nu\rho}\partial^{\rho}\phi$$

- The vector potential A_{μ} is related in a non-local way to ϕ.
- No gauge symmetry in the description using ϕ
- The equation of motion of $F_{\mu\nu}$ is trivial in terms of ϕ.
- The Bianchi identity of $F_{\mu\nu}$ is the equation of motion of ϕ.

Duality and emergent gauge symmetry

Again, easy to establish (essentially Fourier transform):

Maxwell theory in *3+1d* is dual to a magnetic Maxwell theory ($E \sim \tilde{B},\ B \sim -\tilde{E}$)

$$F_{\mu\nu} \sim \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\tilde{F}^{\rho\sigma}$$

- The equation of motion of $F_{\mu\nu}$ is the Bianchi identity of $\tilde{F}_{\mu\nu}$ and vice versa.
- The new vector potential $\tilde{A}_\mu$ is related in a non-local way to the original vector potential A_μ.
- Its gauge symmetry is emergent (not present in the original formulation).

Simple examples of duality

In all these examples duality is achieved by a Fourier transform.

Small fluctuations in q $\leftrightarrow$ large fluctuations in p

Small fluctuations in p $\leftrightarrow$ large fluctuations in q

Both descriptions are correct, but depending on the problem one description can be more useful than another.

More examples of duality

These examples involve more subtle (interacting) theories

- Kramers–Wannier duality in the Ising model and its generalizations in higher dimensions including gauge systems
- Many examples in *1+1d*, especially in the context of bosonization, conformal field theory, etc.

As the previous examples, these can be established rigorously.

More examples of duality

In the simple examples

large fluctuations $\leftrightarrow$ small fluctuations

In the more subtle (interacting) examples

strong coupling $\leftrightarrow$ weak coupling

large $\hbar$ $\leftrightarrow$ small $\hbar$

- Useful in solving complicated systems
- Duality is an intrinsically quantum mechanical phenomenon.

Duality in *3+1d* $\mathcal{N}=4$ supersymmetry

- This is a scale invariant interacting theory of gluons characterized by a gauge group G and coupling constant α (like the fine structure constant)
- The theory with G and α is equivalent to the theory with another group $\widetilde{\mathrm{G}}$ and $\frac{1}{\alpha}$
  - Exchanges strong and week coupling – large and small fluctuations
  - The gluons of $\tilde{G}$ are magnetic monopoles of G. As above, the map between them is non-local.

Duality in *3+1d* $\mathcal{N} = 4$ supersymmetry

$$G, \qquad \alpha \quad \leftrightarrow \quad \tilde{G}, \qquad \frac{1}{\alpha}$$

- The duality is an exact equivalence of theories. Like the Fourier transform above, but cannot be performed explicitly (cannot prove it)
  - Same spectrum of states
  - Same spectrum of operators
  - Same correlation functions

Duality in *3+1d* $\mathcal{N} = 4$ supersymmetry

- The gauge symmetry of the dual description is emergent!
- Which of the two gauge symmetries is fundamental?
- Perhaps neither gauge symmetry is fundamental.
- Which set of gluons is elementary?
- Notion of “elementary particle” is ill-defined.

Another notion of duality

The previous examples are exact dualities – two different descriptions of the same physics.

An alternate notion: two theories with the same long distance (low energy) physics

Hamiltonian at short distances (e.g. many electrons with some interactions) H_{UV}

$$\downarrow$$

Effective Hamiltonian describing the long distance physics (typically different degrees of freedom) H_{IR}

Another notion of duality

Many known examples

- Sometimes we can identify the long distance degrees of freedom using the short distance variables; e.g.
  - Ginzburg–Landau theory of superconductivity
  - QCD at short distances; pions at long distances
- Sometimes the relation is not so easy – it might be non-local
  - The examples of emergent gauge symmetry
  - Particle-vortex duality in *2+1d*
  - Emergent gauge fields in the FQHE

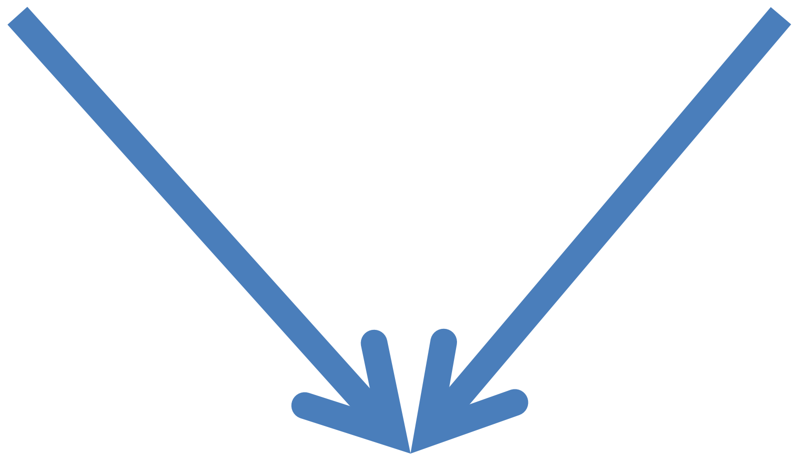
Duality in interacting field theories
$\mathcal{N} = 1$ supersymmetry

Electric theory

G

Magnetic theory

$\tilde{G}$



Non-trivial fixed point at long distances

Duality in interacting field theories $\mathcal{N} = 1$ supersymmetry

Another option:

Electric theory

Based on G

↓

Approximately free theory (IR free)

Based on $\tilde{G}$

Duality in interacting field theories
$\mathcal{N} = 1$ supersymmetry

At short distances an asymptotically free theory based on G

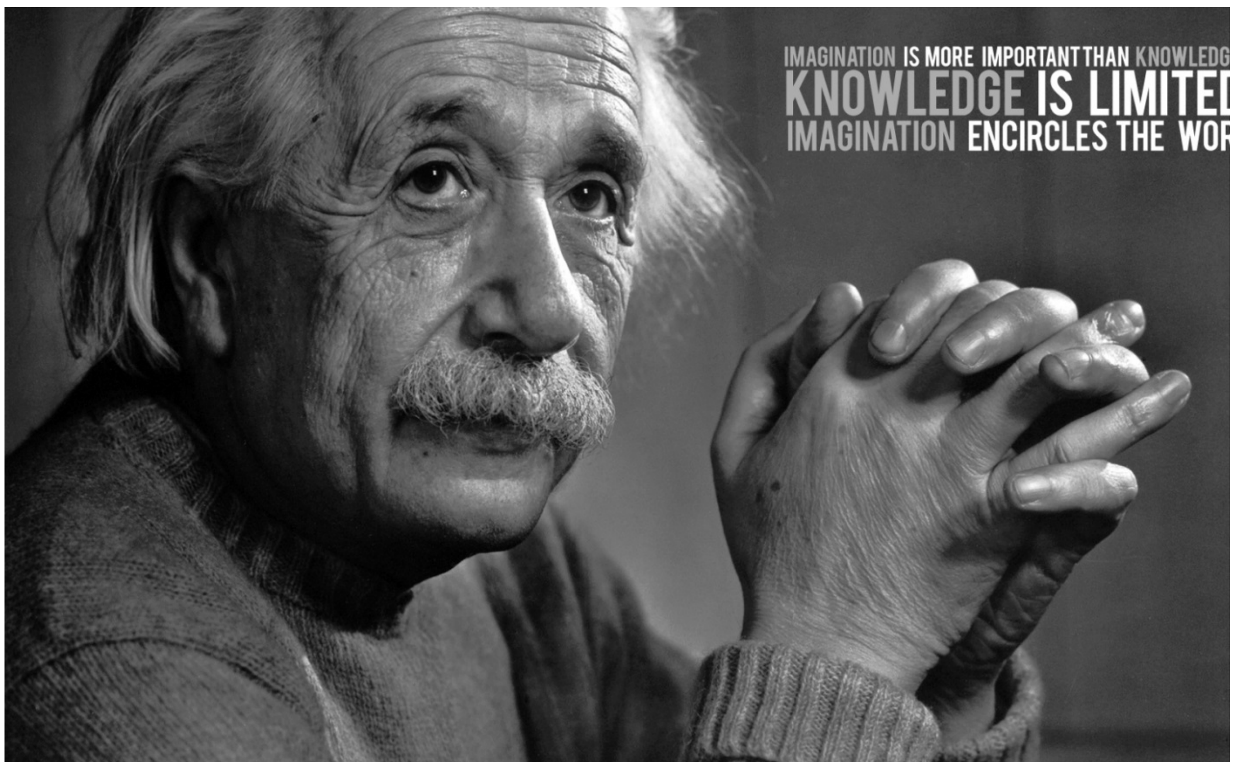
At long distances an almost free theory based on $\tilde{G}$

Unlike ordinary QCD, at long distances this theory has a non-Abelian gauge theory – $\widetilde{QCD}$.

- The quarks and gluons of $\widetilde{QCD}$ are composite. They can be thought of as magnetic monopoles of G
- Their gauge symmetry is emergent.

Many more examples. Duality and emergent gauge symmetries are ubiquitous.

Other Dualities and the Unity of Physics



Emergent general covariance and emergent spacetime

- So far we discussed duality between two field theories
- String-string duality
  - T-duality
  - S-duality
  - U-duality
- String-fields duality
  - Matrix models for low dimensional string theories
  - BFSS M(atrix) model
  - AdS/CFT
  - More generally gauge-gravity duality

Simple Boson/Fermion dualities

- Bosnization in *1+1d*
  - A theory of fermions is equivalent to a theory of bosons
  - No notion of spin of particles in *1+1d* (only spin of operators)
- Spin and statistical transmutation in *2+1d.* Flux attachment in the FQHE
  - Massive particles coupled to gauge fields with special interactions (Chern-Simons coupling)
    bosons $\leftrightarrow$ fermions or bosons $\leftrightarrow$ bosons'
    or fermions $\leftrightarrow$ fermions'

Dualities in 2+1 dimensions

- Boson ↔ boson coupled to a gauge field – Particle/vortex duality [Peskin; Dasgupta, Halperin]
- New boson ↔ fermion dualities [… Chen, Fisher, Wu; Barkeshli, McGreevy; Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin; Aharony; Karch, Tong; NS, Senthil, Wang, Witten; Hsin, NS; Aharony, Benini, Hsin, NS; …]
- New fermion ↔ fermion dualities [… Son; NS, Senthil, Wang, Witten; Xu, You; Hsin, NS; Gomis, Komargodski, NS; …]

New dualities in 2+1 dimensions

Many *2+1d* examples of interacting theories of massless matter coupled to gauge fields

Bosons and/or fermions
+ gauge fields

Bosons and/or fermions
+ gauge fields

Non-trivial fixed point at long distances

New dualities in 2+1 dimensions

Many *2+1d* examples of interacting theories of bosons and/or fermions at short distances flowing to free massless Fermions

Bosons and/or fermions + gauge fields

↓

Free Fermions

Motivation (unity of physics)

- Well established particle/vortex dualities
- Previously found dualities in supersymmetric theories
  - Many tests
- String duality
  - Many tests
- AdS/CFT and large N
  - Same bulk theory is dual to different boundary theories
  - Checks at large N
- Some suggestions in the condensed matter literature

Checks and relations

- Can use these new dualities to derive well established dualities
- Relation to other conjectured dualities, which were subjected to many checks
- Relation to mathematics
  - Mirror symmetry
  - Level/rank duality
- Not independent: assuming some of these dualities are right we can derive others – a web of dualities

Applications in Condensed Matter Physics

- Fractional Quantum Hall Effect
- Physics of first Landau level at half filling
- Gapped phases of topological insulators and topological superconductors
- …

Symmetries in Quantum Gravity and in Condensed Matter Physics

- No exact global symmetries
  - In gravity because of the physics of black holes
  - In condensed matter physics, if no exact global symmetry at short distances, all global symmetries at long distances are approximate
- Can have emergent gauge symmetries with emergent gauge fields.
  - Then there must be excitations with all allowed charges.

Conclusions

- Symmetries are common in physics
- Both in a theory of gravity and in condensed matter physics (unity of physics)
  - No exact global symmetries
  - Exact gauge symmetries can be emergent (all charges must be present)
- Dualities are common in physics: two (or more) different descriptions of the same physics

Conclusions

- The unity of physics. Insights from different branches of physics have recently converged leading to new dualities in *2+1d.*
  - Better understanding of known phenomena and mechanisms.
  - New phenomena and mechanisms.

Conclusions

- Gauge symmetry is not fundamental.
  - It is often convenient to use it to make the description manifestly Lorentz invariant, unitary, and local.
  - But there can be different such (dual) descriptions.
- Look for a reformulation of quantum field theory, which makes the duality manifest.
  - We should not be surprised by duality!