Extremal Graph Theory - Problem Set 3

1. Show that there is a constant c depending on t such that $\mathrm{ex}(n, K_{t,t} - e) \leq cn^{2-1/(t-1)}$.

2. Show that there is a constant c such that the Ramsey number of the d-dimensional hypercube Q_d satisfies $r(Q_d) \leq 2^{cd}$.

3. Prove the general lower bound $\mathrm{ex}(n, H) \geq c_H n^{2-(v-2)/(e-1)}$ for H a graph with v vertices and e edges. Conclude that the extremal number of K_t', the 1-subdivision of K_t, is $n^{3/2-o_t(1)}$.

4. Show that there is a constant c such that the asymmetric extremal number $\mathrm{ex}(m, n, C_6)$, the largest number of edges in an $m \times n$ bipartite graph with no C_6, is at most $c(m^{2/3}n^{2/3} + m + n)$.