Extremal Graph Theory - Problem Set 2

1. Show that for every t there exists a constant C and a subset of $\mathbb{F}_q^t$ of size $\Omega(q^{t-1})$ which meets every line in at most C points.

2. Show that for every t there exists a constant C and a subset of $\mathbb{F}_q^t$ of size $\Omega(q)$ which meets every hyperplane in at most C points.

3. Verify that if H_1 and H_2 are the 3-uniform hypergraphs with edge sets $E(H_1) = \{abc, abd\}$ and $E(H_2) = \{abd, bce, caf\}$, then $\mathrm{ex}(n, H_1) = \Theta(n^2)$ and $\mathrm{ex}(n, H_2) = \Theta(n^2)$.

4. The triangle removal lemma states that every n-vertex graph with $o(n^3)$ triangles can be made triangle-free by removing $o(n^2)$ edges. Show that this implies that any n-vertex graph where every edge is contained in a unique triangle has $o(n^2)$ edges. Deduce that $\mathrm{ex}(n, \{H_1, H_2\}) = o(n^2)$.