

Extremal Graph Theory - Problem Set 1

1. Show that if a graph G with n vertices has minimum degree $d \geq 3$, then it has at least $nd^3/12$ paths of length 3.
2. Show that any C_6 -free graph G has a subgraph which is also C_4 -free with at least $\frac{1}{2}e(G)$ edges.
3. Show that $\text{ex}(n, C_6) \leq Cn^{4/3}$ for some constant C (by a result of Erdős and Simonovits, you may assume that any extremal graph G has the property that $\Delta(G) \leq K\delta(G)$ for some positive constant K).
4. Show that $\text{ex}(n, Q_3) \leq Cn^{8/5}$ for some constant C , where Q_3 is the skeleton of the 3-dimensional cube.