1. Assume that $n \log^3 n \leq m = o(n^2)$ and let $\mu = 5m/\binom{n}{2}$.

   (a) Let (P, Y) be the pair of "real" edge and 2-path probabilities in, that is, the pair of functions $(\mathbf{p}, \mathbf{y})$ defined by $\mathbf{p}(a, v, \mathbf{d}) = P_{av}(\mathbf{d})$ and $\mathbf{y}(a, v, b, \mathbf{d}) = Y_{avb}(\mathbf{d})$.
   Show that (P, Y) is μ-probability-like on $\mathcal{B}_{16 \log n}(\mathcal{D}_m)$.

   (b) Recall the guessed approximations P^g and Y^g defined by

   $$P^g_{av}(\mathbf{d}) = \frac{d_a d_v}{d(n-1)} \left(1 - \frac{(d_a - d)(d_v - d)}{d(n-1-d)}\right) \text{ and } Y_{avb}(\mathbf{d}) = P^g_{av}(\mathbf{d}) P^g_{bv}(\mathbf{d} - \mathbf{e}_a - \mathbf{e}_v) \left(1 + \frac{1}{n}\right).$$

   Convince yourself that the pair (P^g, Y^g) is also μ-probability like on $\mathcal{B}_{16 \log n}(\mathcal{D}_m)$.

2. Prove part (a) of the Contraction Lemma: If $(\mathbf{p}, \mathbf{y})$ and $(\mathbf{p}', \mathbf{y}')$ are μ-probability like on some set $\mathcal{B}_1(\mathcal{D}_0)$, and $\xi > 0$ such that $\mathbf{p} \equiv \mathbf{p}'(1 \pm \xi)$ and $\mathbf{y} \equiv \mathbf{y}'(1 \pm \xi)$ on $\mathcal{B}_1(\mathcal{D}_0)$ then

   $$\mathcal{R}(\mathbf{p}, \mathbf{y}) = \mathcal{R}(\mathbf{p}', \mathbf{y}')(1 + O(\mu\xi)) \text{ on } \mathcal{D}_0.$$

3. Let $n = 4$. Expand $\prod_{1 \leq i < j \leq 4}(1 + x_i x_j)$ and list all monomials of the form $x_1^2 x_2^2 x_3^2 x_4^2$.

   (a) How many are there?

   (b) What does each monomial correspond to?

   (c) Conclude that the number of 2-regular graphs on four vertices is equal to that number.