1. Show that the two formulae $f_1(n,d)$ and $f_2(n,d)$ given by

$$f_1(n,d) := \frac{(dn)!}{(dn/2)!2^{dn/2}d!^n} \exp\left(-\frac{d^2-1}{4} + O\Big(\frac{d^3}{n}\Big)\right) \text{ and } f_2(n,d) := \frac{\binom{\binom{n}{2}}{dn/2}\binom{n-1}{d}^n}{\binom{n(n-1)}{dn}} e^{1/4+o(1)}$$

are asymptotically equal, i.e. $f_1 = f_2(1+o(1))$, under the assumptions that $dn \to \infty$ and $d = o(n^{1/3})$.

2. Prove Lemma 5.1 from the lecture notes: Let $X = (x_1,\ldots,x_N)$ and $Y = (y_1,\ldots,y_N)$ be sets of non-negative real numbers, and let $\xi = \xi(N) > 0$. If $\sum_i x_i = (1+O(\xi))\sum_i y_i$, and $x_i/x_j = (1+O(\xi))y_i/y_j$ uniformly for all $i,j \in [N]$, then $x_i = y_i(1+O(\xi))$ uniformly for all $i \in [N]$.

3. Recall the concentration inequality for the hypergeometric distribution in Lemma 5.2. Use it to prove Corollary 5.3, that is, assume that $n\log n \ll m \le n/2$ and show that in both models $D(G_{n,m})$ and in $\mathcal{B}_m(n)$, with probability $1-o(n^{-23})$ we have that $|d_i - d| \le 10\sqrt{d\log n}$ for all $i \in [n]$, where $d = 2m/n$.

4. Show that the local change graph S as defined in the lecture is connected and has diameter at most $20\sqrt{d\log n}$.

5. Argue carefully why $\mathrm{Bad}_{(a,b)}(\mathbf{d}) = \frac{1}{d_a}\Big(P_{ab}(\mathbf{d}) + \sum_{v\neq a,b} Y_{avb}(\mathbf{d})\Big)$ for any graphical sequence $\mathbf{d}$.

6. *Bipartite model.*

   Let A and V be disjoint sets of size n each. For two sequences $\mathbf{s}$ and $\mathbf{t}$, each of length n, denote by $\mathcal{G}(\mathbf{s},\mathbf{t})$ the set of bipartite graphs G with bipartition (A,V) that realise the bi-degree sequence $(\mathbf{s},\mathbf{t})$, that is, vertex $a \in A$ has degree s_a and vertex $v \in V$ has degree t_v. Note that a necessary condition for $(\mathbf{s},\mathbf{t})$ to be "bi-graphical" is that $m = \sum_a s_a = \sum_v t_v$. Fix now two sequences $\mathbf{s}$ and $\mathbf{t}$ of length n each such that $\sum_a s_a = 1 + \sum_v t_v$.

   (a) For $a,b \in A$, mimic the degree switching argument from class to find a formula of $R_{(a,b)}(\mathbf{s},\mathbf{t}) := \frac{|\mathcal{G}(\mathbf{s}-\mathbf{e}_a,\mathbf{t})|}{|\mathcal{G}(\mathbf{s}-\mathbf{e}_b,\mathbf{t})|}$ in terms of $\mathrm{Bad}_{(a,b)}$.

   (b) How does the formula for $\mathrm{Bad}_{(a,b)}$ in Lemma 5.10 change in this bipartite case?

   (c) Can we define $R_{(a,v)}(\mathbf{s},\mathbf{t})$ for $a \in A$ and $v \in V$? Why or why not?