Quantum Field Theory, Separation of Scales, and Beyond

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IAS

Classical Physics

Classical Mechanics

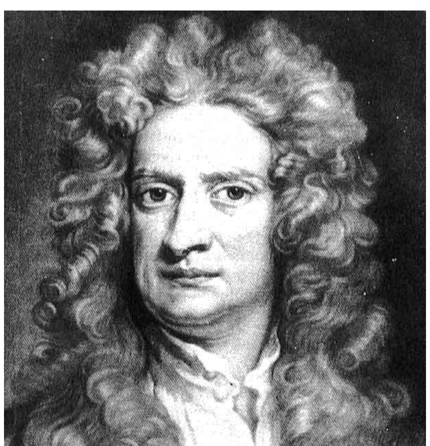
Time evolution of a finite number of particles

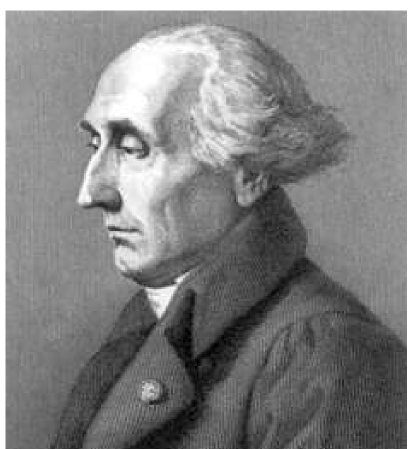
Ordinary differential equations

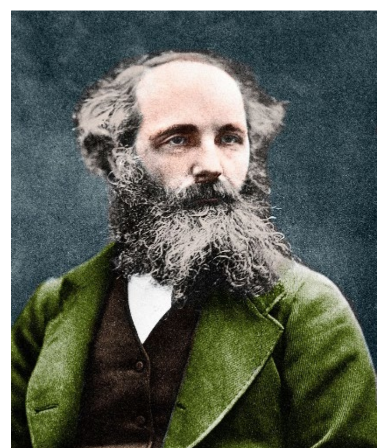
Classical Field Theory

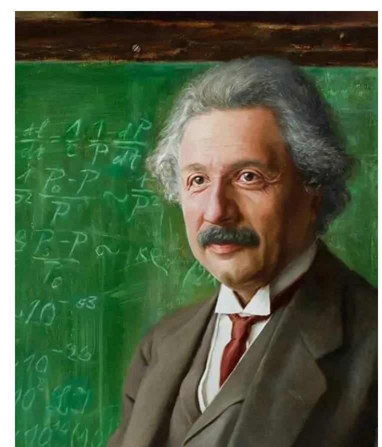
Time evolution of an infinite number (continuum) of degrees of freedom, e.g., electromagnetic field, velocity of fluid, metric

Partial differential equations









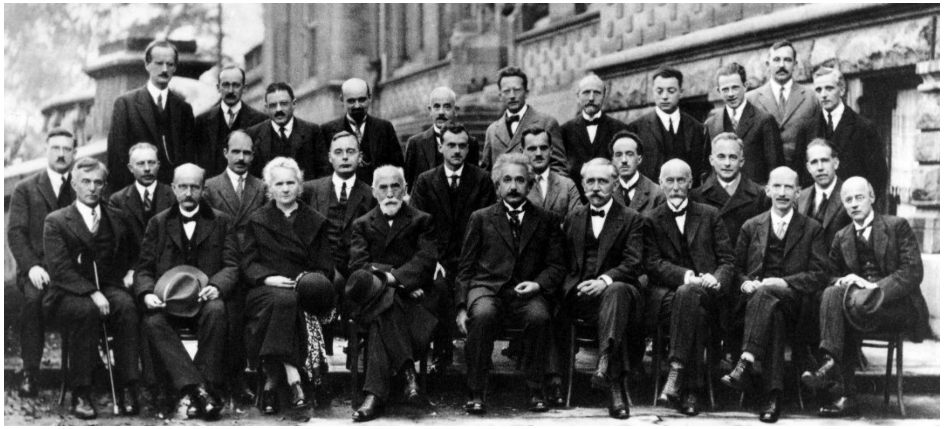
Quantum Physics

Quantum Mechanics

Time evolution of a finite number of quantum particles

Operators in a Hilbert space

Functional integral



Quantum Field Theory

Time evolution of an infinite number (continuum) of quantum degrees of freedom, e.g., electromagnetic field

A lot is known. Still very exciting progress.

My personal view: a new intellectual structure is needed – QFT

Quantum Field Theory is Everywhere

- Particle physics: the language of the Standard Model
  - Enormous success, e.g., the electron magnetic dipole moment is theoretically *1.001 159 652 18 …*
    experimentally *1.001 159 652 180...*
- Condensed matter
  - Description of the long-distance properties of materials: phases and the transitions between them
- Cosmology
  - Early Universe, inflation
- …

Quantum Field Theory is Everywhere

- String theory/quantum gravity
  - On the string world-sheet
  - In the low-energy approximation
  - The whole theory (gauge/gravity duality)
- Applications in mathematics especially in geometry and topology

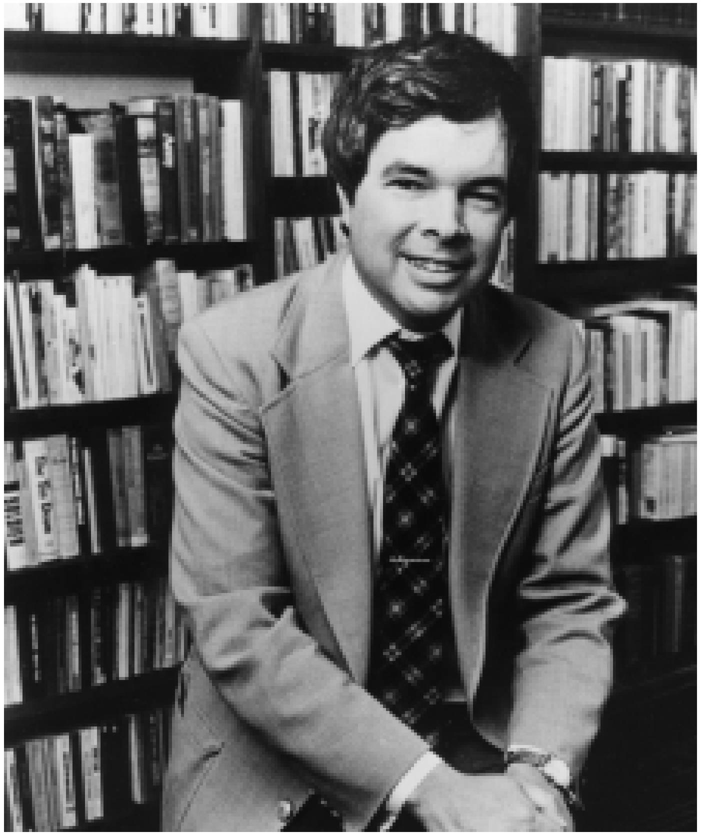
Separation of Scales

In physics (and in fact in all sciences, including social sciences), different effective descriptions at different scales.

- The long-distance phenomena are derived from short-distance rules – reductionism
- Examples
  - Thermodynamics
  - Hydrodynamics
  - Effective field theory and the renormalization group…
  - Many others
- Simplification. Independence of the details at other scales
- Nature is kind to us

Separation of Scales in QFT

Different field theories describe the phenomena at different length scales (or energy scales).



Different Theories at Short and Long Distances

- We formulate a problem at short distances and the answer is the long-distance behavior
- Possible phases are the possible long-distance behavior
- The IR theory is independent of most of the UV details – universality. Efficient, effective description.
- Even if the UV theory is on a lattice, the IR theory is in the continuum.
  - Approximating a sum by an integral
  - More about that, below.

UV theory at short distances

↓

IR theory at long distances

The Phases of QFT

Consider the theory in large but finite volume. The spectrum of the Hamiltonian can be

- Gapped/massive, e.g., free massive particles
  - Unique ground state
  - Several states become degenerate as the volume goes to infinity. Topological FT

- Gapless/massless
  - Free massless particles, e.g., photons
  - Interacting massless particles, notion of particle is ill-defined. Conformal FT

- More refined classification depending on more details

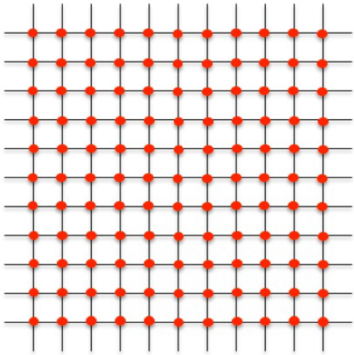
Different Theories at Short and Long Distances

- Continuum QFT classifies and organizes the possible phases and the transitions between them.
- Interesting new phases teach us about QFT.
- The IR theory is expected to be scale invariant
  - Conformal field theories (CFT)
  - $\supset$ Topological field theories (TQFT)
  - $\supset$ Invertible TQFT
- Interesting flow from the UV to the IR
  - Given the question in the UV, determine the answer in the IR

UV theory at short distances

IR theory at long distances

Concrete example: lattice models



Approximate space (or spacetime) by a lattice, (e.g., a cubic lattice).

Place degrees of freedom at the sites (vertices), links (edges), etc.

Postulate short-range interactions (e.g., coupling between nearest neighbors).

Typically, the low-energy limit of such a system is described by a continuum QFT – the lattice becomes a continuum.

- In condensed matter physics, the lattice is physical.
- In high-energy physics, it is a tool helping us to define the continuum theory (and to perform numerical calculations).

UV/IR mixing – counterexamples to the separation of scales dogma

UV/IR mixing – no separation of scales – long-distance/low-energy phenomena reflect high-energy physics.

- Common in gravity:
  - High energy in a small volume leads to a large black hole, hiding the short-distance physics
  - Dualities relate small ↔ large
  - Many questions and confusions in quantum gravity circle around this issue
- String theory with vanishing Newton constant is a non-gravitational theory. Typically, it is a QFT. But certain peculiar examples exhibit UV/IR mixing…

UV/IR mixing – counterexamples to the separation of scales dogma

- Examples based on limits of string theory
  - Little string theory
    - No local operators
    - Duality: short and long distances are indistinguishable
  - Field theory on a non-commutative space [Minwalla, Van Raamsdonk, NS]
    - dipole with high momentum is large in space along another direction
    - comparing with the same theory on a commutative space, fewer UV divergences and instead new IR divergences

Exotic lattice models (including models of fractons) also lead to UV/IR mixing…

Exotic lattice models

Many exotic models of various kinds (soon we will review them)

- Lifshitz theory [many references]
- XY-plaquette model [Paramekanti, Balents, Fisher; …]
- Fracton models [Chamon; Haah; Vijay, Haah, and Fu; …]
- Many others

They do not have a standard continuum limit.

This challenge is related to their peculiar properties…

Exotic models – peculiar properties

- Unusual symmetries (position dependent global symmetries)
- Excitations with restricted mobility (particles restricted to a point, line, etc.)
- Large ground state degeneracy, e.g., $2^{k(L^x,L^y,L^z)}$, where L^i are the number of lattice sites in direction i. The limit $L^i \to \infty$ is not well-defined and often infinite.
- UV/IR mixing (long-distance/low-energy phenomena are sensitive to short-distance details)
- Others

Exotic models – questions

Are these peculiarities related?

(Not all examples have all these peculiarities.)

How should we think of such theories?

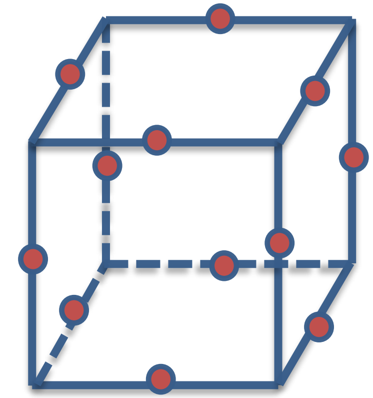
How should we organize them?

- Classification
- More examples
- Continuum field theory description?

The X-cube Model [Vijay, Haah, and Fu]

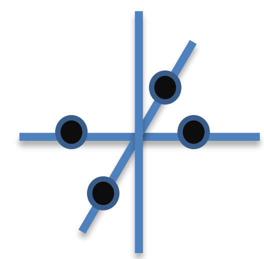
- $\mathbb{Z}_2$ spin on every spatial link (qubit on every link)
- The Hamiltonian has two kinds of terms

$$H = -\sum_c B_c - \sum_s \left(A_s^x + A_s^y + A_s^z\right)$$



  – Cube interaction

$$B_c = \prod_{\text{links around } c} \sigma^1$$



  – Site interactions

$$A_s^z = \prod_{\text{links at } s \perp \text{ to } z} \sigma^3$$

- Innocent-looking local Hamiltonian

The X-cube Model [Vijay, Haah, and Fu]

- On a lattice with $L^x \times L^y \times L^z$ sites with periodic boundary conditions, huge ground state degeneracy

$$2^{2(L^x+L^y+L^z)-3}$$

  - Depends on the number of lattice sites
  - Entropy $2(L^x + L^y + L^z) - 3$ not proportional to the volume (sub-extensive)
  - Infinite in the continuum limit: lattice spacing $a \to 0$, $L^i \to \infty$ with fixed size $\ell^i = aL^i$
- The ground states reflect short distance physics – high momentum modes (of lattice scale) have zero energy – UV/IR mixing.

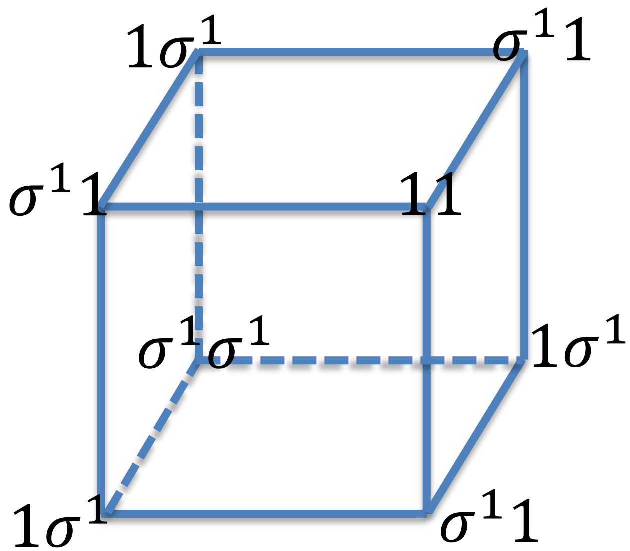
The X-cube Model [Vijay, Haah, and Fu]

- Gap in the spectrum.
- Small deformations of the Hamiltonian do not change the low-energy physics.
- Localized excitations with restricted mobility:
  – Fractons are fixed at a point
  – Lineons fixed to move on a line (x, y, or z)
  – Planon fixed to move on a plane (xy, yz, or xz)

Haah Code [Haah]

Two qubits at every site.

Two kinds of terms in the Hamiltonian



$1\sigma^1$ means the action of $1 \otimes \sigma^1$ on the two qubits.

Innocent-looking local Hamiltonian.

Haah Code [Haah]

Excitations with restricted mobility.

The ground state degeneracy is $2^{k(L^x,L^y,L^z)}$ with $k(L^x, L^y, L^z)$

- a complicated number-theoretic function of L^x, L^y, L^z
- complicated even in the special case $L = L^x = L^y = L^z$
- not monotonic in L
- bounded by $\sim L$
- for some sequence of $L \to \infty$, $k(L) \to$ finite

The limit $L^i \to \infty$ is ambiguous. Is there a continuum limit?

Gapless Models [Pretko]

Motivated by earlier models of symmetric tensor gauge theories in the continuum and the lattice [Xu; …]

$$A_0 \to A_0 + \partial_0 \alpha$$
$$A_{ij} \to A_{ij} + \partial_i \partial_j \alpha$$

Gauge invariant electric and magnetic fields

$$E_{ij} = \partial_0 A_{ij} - \partial_i \partial_j A_0$$
$$B_{[ij]k} = \partial_i A_{jk} - \partial_j A_{ik}$$

Lagrangian $\mathcal{L} = E^2 - B^2$

Gapless/massless "photon"

Gapless Models [Pretko]

$$A_0 \to A_0 + \partial_0 \alpha$$
$$A_{ij} \to A_{ij} + \partial_i \partial_j \alpha$$

Gauss law with matter $\sum_{ij} \partial_i \partial_j E_{ij} = \rho$

Conserved charge $\int d^3x \, \rho$

Conserved dipole charge $\int d^3x \, x^i \rho$

Restricted mobility because of the conservations – fractons

Questions

- More examples, perhaps with more exotic phenomena?
- What is the underlying reason for the bizarre behavior?
- What are the possible theories/phases?
  - Organization/classification
  - Is there a sensible continuum quantum field theory for the long-distance behavior?
- Many more

Continuum Lagrangians

The naïve continuum limit of these models leads to unusual Lagrangians.

Schematically,

- $\mathcal{L} = (\partial_t \phi)^2 - \sum_{ij} \left(\partial_i \partial_j \phi\right)^2$ [Henley; …; Chen, Huang; …; Radičević; …; NS, Shao; …]
  - Lifshitz theory: sum over all ij
  - XY-plaquette model [Paramekanti, Balents, Fisher; …]: sum only over $i \neq j$
- $\mathcal{L} = E^2 - B^2$ [Xu; Xu, Wu; Pretko; … NS, Shao; …]

$$E_{ij} = \partial_t A_{ij} - \partial_i \partial_j A_t \quad , \qquad B_{ijk} = \partial_i A_{jk} - \partial_j A_{ik}$$

$$A_t \to A_t + \partial_t \alpha \quad , \qquad A_{ij} \to A_{ij} + \partial_i \partial_j \alpha$$

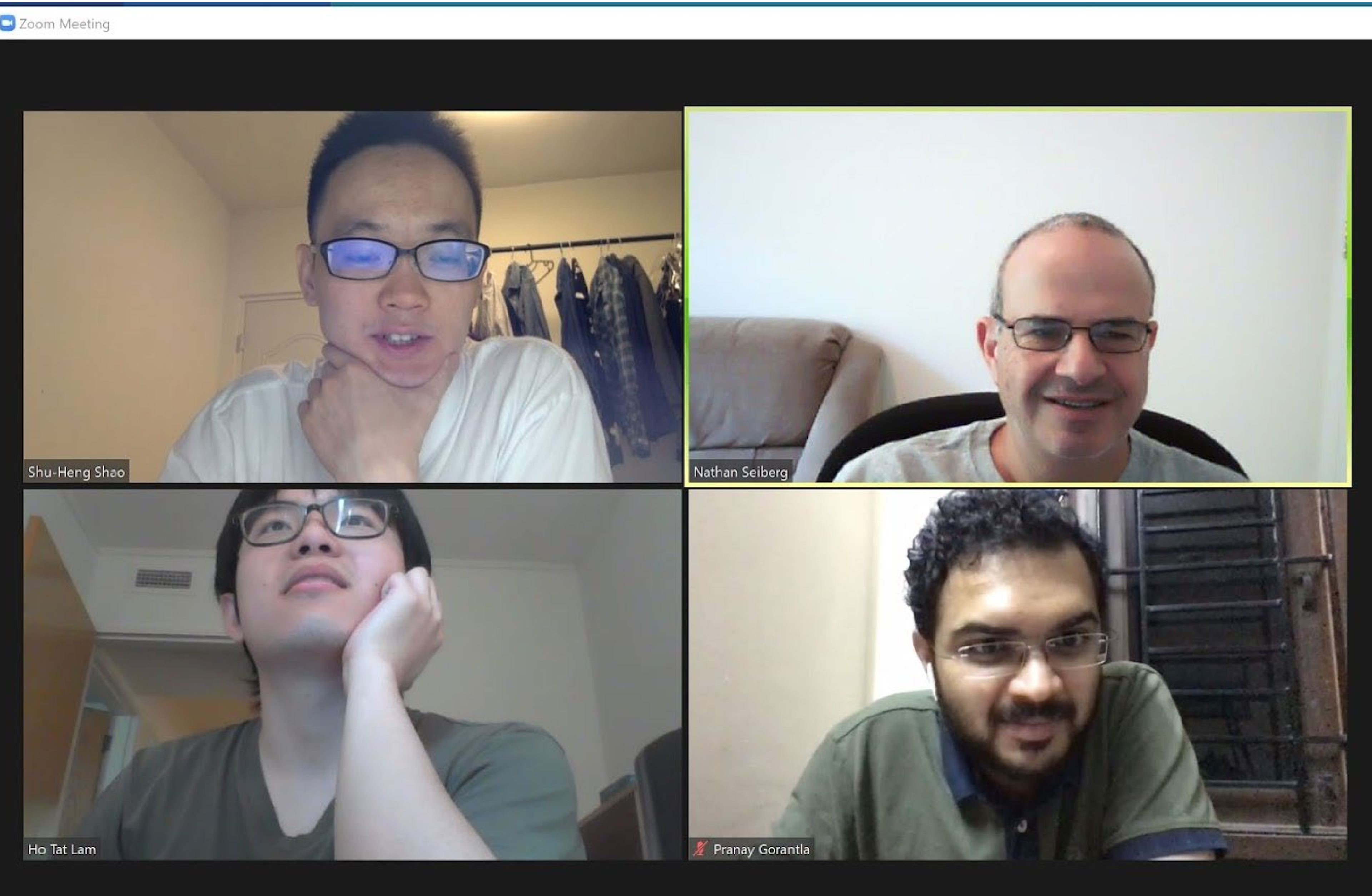
  - sum over all ij [Xu; Pretko; …]
  - sum only over $i \neq j$ [Xu, Wu; Ma, Hermele, Chen…]

Continuum Lagrangians

- Chern-Simons-type Lagrangians of exotic gauge theories lead to gapped theories including the X-cube model [Slagle, Kim; NS, Shao; …]
- Many others

The higher derivative terms lead to important subtleties.

Gorantla, Lam, NS, Shao



The questions we studied
[Gorantla, Lam, NS, Shao]

- What do continuum field theories based on such Lagrangians mean?
  - Need to study discontinuous field configurations
  - Understand the global structure – fluxes, operators, defects.
- What is the relation between these continuum theories and the underlying lattice models?

Many results about many models…

Some of our conclusions

[Gorantla, Lam, NS, Shao]

- Typically, the continuum theories have more symmetries than the underlying lattice models.
  - New lattice models exhibit these symmetries. They are closer to the continuum theories.
- Starting with a lattice, we can study two limits:
  - Continuum limit: $a \to 0, L \to \infty$ with fixed length $\ell = aL$ – leading to continuum field theory in finite volume.
  - Thermodynamic limit: $L \to \infty$ with fixed a (and hence, $\ell \to \infty$)

  They do not commute!
- Can also study other limits, leading to different low-energy/continuum theories

UV/IR mixing

Summary

- Quantum field theory is everywhere
  - It appears to be the language of physics
- A fundamental property of quantum field theory is the phenomenon of separation of scales – different effective theories at different scales – reductionism
  - Most of the details of the short-distance UV theory do not affect its long-distance IR description.
- Certain string theory constructions and some lattice models lead to peculiar systems outside the framework of standard QFT – UV/IR mixing

Summary

- Exotic systems, including fractons, exhibit interesting properties
  - UV/IR mixing:
    - Large ground state degeneracy. Sometimes there is no well defined limit as $L \to \infty$
    - Observables change at the lattice scale – discontinuous in the continuum limit.
  - Excitations with restricted mobility
- These seem incompatible with the framework of continuum QFT.

Summary

- We analyzed nonstandard continuum QFTs for these systems
  - They capture the universal properties of the lattice models. They reproduce their long-distance physics and the properties of probe particles.
- Many puzzles remain.

Thank you