Loop Dynamics and a Geometric Solution of Planar QCD

Lecture III: The Hodge-Dual Surface and the Physical Vacuum

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Discussion map: choose a fork

In Lecture II, the 1D algebraic Master Field violently broke down at $\mathcal{W}^{(8)}$. This catastrophe mathematically mandates the introduction of 4D continuous geometry. Today we construct it:

Fork A: Loop Zero Modes

- Area derivative without divergences
- Exact factorization in MM equation
- The physical multi-instanton vacuum

Fork B: Topological Stability

- Self-intersecting loops
- Plateau vs. Goldschmidt
- The soap film phase transition

Fork C: Hodge-Dual Geometry

- The Holomorphic map to $\mathbb{C}^4$
- Virasoro constraint & uniformization
- Symmetrization by Parity

(Let's build the exact rigid 4D geometric background of the QCD string, step by step.)

The Big Picture: The Continuum Solution

- **The Goal:** A continuum framework for confining planar QCD ($N_c \to \infty$).
- **The Method:** Quantizing the Fermi-string (1981) degrees of freedom on the **rigid Hodge-dual minimal surface**.
- **Why a Rigid Surface?** The fermions are *not* placed on a random fluctuating surface. The bulk geometry is rigid and fixed holographically by the boundary loop.
- **The Payoff:** This completely avoids the Liouville instability of early random-surface models! There is no summation over fluctuating metrics.

The Singular Solution of the MLE and the QCD Vacuum

- In Lecture II, we found that the Momentum Loop Equation (MLE) does not allow a regular analytic solution expandable as a Taylor–Magnus series in P'.
- The Taylor–Magnus expansion emerged from iterations of the MLE starting with the **trivial zero mode**:

$$\mathcal{L}_\nu(W)[P] = \mathcal{O}(P^3) \quad \Longrightarrow \quad W[P] = \textit{ const } + \mathcal{O}(P^2) \tag{1}$$

- The mathematical alternative is a **singular solution** governed by the nontrivial zero-mode:

$$\mathcal{L}_\nu(W)[P] = 0 \quad \Longleftrightarrow \quad \mathcal{L}_\nu(W)[C] = 0 \tag{2}$$

- Physically, a nontrivial zero mode corresponds to a non-perturbative QCD vacuum—i.e., a background gauge field (Master Field) solving the classical Yang–Mills equations:

$$[D_\mu, F_{\mu\nu}] = 0 \tag{3}$$

- This vacuum state is notoriously difficult to build in the pure gauge-field representation, and equally hard in the momentum loop equation (MLE). However, as we shall see today, there is **an exact analytic solution** in coordinate loop space!

The Area Derivative and Loop Equation

- Historically, the area derivative was plagued by kinematic divergences. We have found a regular definition based on the discontinuity of the second variation of the area:

$$\frac{\delta W}{\delta \sigma_{\mu\nu}(\theta)} = \frac{\delta^2 W}{\delta C'_\mu(\theta - 0)\delta C'_\nu(\theta + 0)} - (\mu \leftrightarrow \nu) \tag{4}$$

- The planar loop equation for Yang–Mills gradient flow reduces to the loop space diffusion equation:

$$\mathcal{L}_\nu(W) = \left(\frac{\delta}{\delta C'_\mu(\theta + 0)} - \frac{\delta}{\delta C'_\mu(\theta - 0)} \right) \frac{\delta W}{\delta \sigma_{\mu\nu}(\theta)} = 0 \tag{5}$$

- **The Discovery:** This operator admits an exact solution (a zero mode) $W = \exp\left(-\kappa S[C]\right)$ provided $S[C]$ satisfies $\mathcal{L}_\nu(S) = 0$.

Self-Duality in Loop Space

- **The Golden Key:** The loop space diffusion equation is identically satisfied for any functional $S[C]$ with a **Self-Dual (SD) or Anti-Self-Dual (ASD) area derivative**:

$$\star \frac{\delta S_\pm}{\delta \sigma_{\mu\nu}(\theta)} = \pm \frac{\delta S_\pm}{\delta \sigma_{\mu\nu}(\theta)} \tag{6}$$

- Why? Due to the kinematic **Bianchi identity** (a Jacobi identity for a triple commutator), the divergence of an (anti)self-dual area derivative identically vanishes:

$$\epsilon_{\alpha\mu\nu\lambda} \left(\frac{\delta}{\delta C'_\alpha(\theta+0)} - \frac{\delta}{\delta C'_\alpha(\theta-0)} \right) \frac{\delta S_\pm}{\delta \sigma_{\mu\nu}(\theta)} \equiv 0 \tag{7}$$

- This is the exact same mathematical reason why the self-dual gauge field solves the Yang–Mills equations of motion: the Jacobi identity for the double commutator

$$\epsilon_{\alpha\beta\gamma\delta} D_\alpha D_\beta D_\delta \equiv 0 \tag{8}$$

Compatibility with the Makeenko–Migdal Equation

- Due to the **Leibniz property** of the loop differential operator, we can dress the fluctuative part of the Wilson loop with this zero mode:

$$W[C] = \exp\left(-\kappa S_{\pm}[C]\right) W_{\mathsf{fluct}}[C] \tag{9}$$

- In general, the zero mode factor corresponds to a superposition (with unknown weights w_i) of the (anti)self-dual gauge fields $A^{(i)}$ with fixed Hodge chirality χ

$$\exp\left(-\kappa S_{\chi}[C]\right) = \sum_{\star F^{(i)} = \chi F^{(i)}} w_i \operatorname{tr} \hat{T} \exp\left(\oint_C A^{(i)}_{\mu}(x) \mathrm{d}x_{\mu}\right) \tag{10}$$

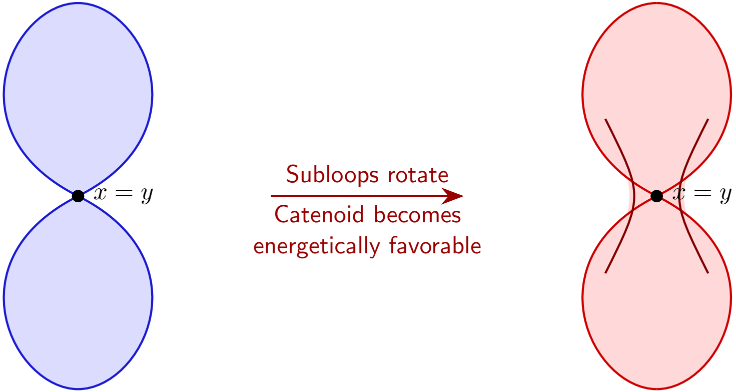
- The full Makeenko–Migdal equation has a singular integral term at self-intersections $x = y$:

$$\mathcal{L}_{\nu}(W[C]) = \lambda \int_C dy_{\nu} \delta^4(x-y) W[C_{xy}] W[C_{yx}] \tag{11}$$

Topological Stability: The Area Functional at Intersections

- **Crucial Additivity Requirement:** This dressing **only** works if the surface area is **strictly additive** at self-intersections ($S[C] = S[C_{xy}] + S[C_{yx}]$). Only then does the confining factor exactly factor out of the RHS integral and cancel!
- Consider a self-intersecting loop $C = C_{xy} \circ C_{yx}$ (e.g., a figure-8).
- There are two competing topological classes of minimal surfaces bounding this loop:
  1. **Two Disks (The Goldschmidt Solution):** The surface pinches off completely at the intersection. The area is strictly additive: $S[C] = S[C_{xy}] + S[C_{yx}]$.
  2. **A Connected Surface (Plateau Minimum):** A catenoid-like cylindrical surface bridging the gap between the two sub-loops.
- **The Phase Transition:** As the sub-loops rotate around the intersection point to get closer, the connected cylindrical shape may become energetically favorable (the absolute global minimum).
- Does the QCD vacuum undergo this phase transition?

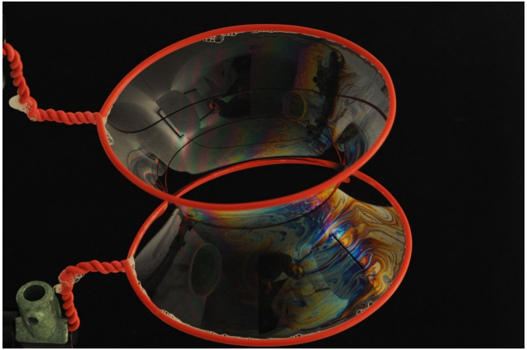
Visualizing the Topological Phase Transition



Goldschmidt Solution
Two Disks (Local Min)
$S_{add} = S[C_{xy}] + S[C_{yx}]$

Plateau Solution
Connected Cylinder (Global Min)
$S_{conn} < S[C_{xy}] + S[C_{yx}]$

Classical Physics vs. Planar QCD: The Catenoid



A classical soap film finds the absolute global minimum (the Plateau solution).

- **Classical Physics (Soap Films):** Nature minimizes physical energy. When two loops are close, the film dynamically transitions into this connected **catenoid**.
- **The QCD String:** This beautiful classical solution is **strictly forbidden** as the vacuum geometry of Planar QCD!
- **Why?** A connected cylinder destroys exact planar factorization. The RHS of the MM loop equation requires the amplitude to factorize into two disjoint pieces: $W[C] \to W[C_{xy}]W[C_{yx}]$.
- Therefore, the QCD vacuum geometry must remain trapped in the additive **Goldschmidt branch** (two separate disks), even when the loops are arbitrarily close.

Mathematical Stability of the Goldschmidt Branch

The Loop Equation dictates the Topology

We are not solving the absolute Plateau problem; we are solving the QCD equations of motion. We **must** select the additive **Goldschmidt branch** (disjoint disks) over any complex topology.

- **But is it mathematically stable?** Yes.
  - **Douglas & Radó:** A disconnected solution always exists for separated/touching boundaries.
  - **Gulliver's Barrier:** A topological barrier ensures the additive Goldschmidt branch remains a stable *local minimum* in functional space.
  - **White's Bridge Principle:** The limit of an infinitesimal neck smoothly converges to the touching disks in the varifold sense, maintaining a continuous, divergence-free loop calculus and avoiding "catenoid collapse".

The Physical Vacuum: Multi-Instanton Resummation

What is the physical origin of this rigid background?

The MM equations are exact equations of motion. They possess an inherent ambiguity corresponding to the choice of the **physical vacuum**. Multiplying by a zero mode factor is equivalent to specifying this vacuum.

- In standard gauge theory, the non-perturbative QCD vacuum is a complex mixed state of multi-instantons.
- A single instanton explicitly breaks macroscopic translation invariance. Restoring Poincaré symmetry requires a highly complex integration over the entire multi-instanton moduli space.
- **Loop Space Magic:** Just as instantons have $F_{\mu\nu} = \pm \star F_{\mu\nu}$, our geometric zero mode has $\frac{\delta S}{\delta \sigma_{\mu\nu}} = \pm \star \frac{\delta S}{\delta \sigma_{\mu\nu}}$.
- Because our minimal area functional $S_\chi[C]$ depends only on the relative geometry of the loop and is invariant under rigid translations, it **is the exact, translation-invariant holographic representation** of the implicitly resummed multi-instanton vacuum!

Constructing the Hodge-Dual Minimal Surface

- **We define a surface** mapping the unit disk $\mathcal{D}$ to an extended space $\mathbb{R}^3 \otimes \mathbb{R}^4$ via coordinates $X^i_\mu(\xi), i = 1, 2, 3$.
- **The area is defined** as $S_\chi[C] = \min_X \int_\mathcal{D} d^2\xi \sqrt{\Sigma^2_{\mu\nu}/2},\ \Sigma_{\mu\nu} = e_{ab}\partial_a X^i_\mu \partial_b X^i_\nu$.
- **Hodge duality constraint**: $\star\Sigma_{\mu\nu} = \chi\Sigma_{\mu\nu}$
- The Hodge chirality $\chi = \pm 1$ enters exactly through 't Hooft matrices $\eta^{\chi,i}_{\mu\nu}$ in the **boundary conditions**:

$$X^i_\mu(\partial\mathcal{D}) = \eta^{\chi,i}_{\mu\nu} C_\nu, \qquad \eta^{\chi,i}_{\mu\nu} = \delta_{i\mu}\delta_{\nu 4} - \delta_{i\nu}\delta_{\mu 4} + \chi\epsilon_{i\mu\nu 4} \tag{12}$$

- **The Holomorphic Ansatz:** We resolve the duality constraint exactly by mapping to $\mathbb{C}^4$:

$$X^i_\mu = \eta^{\chi,i}_{\mu\nu}(f_\nu(z) + \bar{f}_\nu(\bar{z})), \quad z = \xi_1 + \mathrm{i}\xi_2 \tag{13}$$

Boundary Conditions and Area Duality

- The boundary conditions for the Hodge-dual surface reduce to a conventional Dirichlet problem in $\mathbb{C}^4$:

$$2\,\mathrm{Re}\, f_\mu(e^{\mathrm{i}\theta}) = C_\mu(\theta) \tag{14}$$

- Our surface area reduces to the induced metric in this space:

$$\Sigma_{\mu\nu} = 2(F_{\mu\nu} + \chi \star F_{\mu\nu}), \qquad F_{\mu\nu} = \mathrm{i}(f'_\mu \bar{f}'_\nu - \bar{f}'_\mu f'_\nu) \tag{15}$$

- The area element is automatically Hodge-dual for *any* arbitrary holomorphic function $f_\mu(z)$:

$$\star\Sigma_{\mu\nu} = \chi\Sigma_{\mu\nu} \tag{16}$$

- This guarantees the self-duality of the area derivative, making the Hodge-dual minimal area an exact zero mode of the loop diffusion operator.

The Virasoro Constraint & Uniformization

- The geometric area functional $S = \int \sqrt{-\det g}\, d^2 z$ possesses full reparametrization invariance $z \to w(z, \bar{z})$.
- By the **Uniformization Theorem**, we can always choose isothermal (conformal) coordinates where the induced metric is diagonal: $g_{zz} = 0$.
- For our specific holomorphic ansatz, this is precisely the **Virasoro null constraint**:

$$(f'_\mu(z))^2 = 0 \tag{17}$$

- **The Magic of the Conformal Gauge:** The highly non-linear geometric Nambu-Goto area density strictly collapses to an exact **quadratic Lagrangian**:

$$\mathcal{L} = \sqrt{(g_{z\bar{z}})^2 - |g_{zz}|^2} \xrightarrow{(f')^2=0} g_{z\bar{z}} \propto |f'_\mu|^2 \tag{18}$$

Exact Zero Modes and the Hilbert Transform

- The linear boundary problem $2\,\mathrm{Re}\, f_\mu = C_\mu$ is explicitly solved by the **Hilbert Transform** ($\mathcal{H}$):

$$f_\mu = \frac{1}{2}(1 + \mathrm{i}\mathcal{H})C_\mu; \qquad \mathcal{H}[u](\theta) = \frac{1}{2\pi} P.V. \int_0^{2\pi} d\theta' \cot\left(\frac{\theta - \theta'}{2}\right) u(\theta') \tag{19}$$

- In the holomorphic gauge, the minimal value of BOTH S_+ and S_- reduces identically to the standard, chirality-independent Dirichlet functional (the Goldschmidt area): $S_+[C] = S_-[C] \propto \int_D |f'(z)|^2 d^2 z$.
- Because their area derivatives are strictly self-dual and anti-self-dual, the kinematic **Bianchi identity** applies to each term individually.

$$\mathcal{L}_\nu(S_+) = 0 \qquad \text{and} \qquad \mathcal{L}_\nu(S_-) = 0 \tag{20}$$

- Thus, each chiral surface S_+ and S_- solves the exact Makeenko-Migdal equation **completely on its own**!

Spinor Factorization and Gauss Maps

- The twistor factorization of the null tangent vector $f'_{a\dot{b}} = \lambda_a \mu_{\dot{b}}$ leads to an exact factorization of the induced metric on the worldsheet:

$$e(z, \bar{z}) = 2\sqrt{2} f'_\mu \bar{f}'_\mu = \sqrt{2}(\bar{\lambda}\lambda)(\bar{\mu}\mu)$$

- In this spinor representation, the area derivative simplifies beautifully:

$$\frac{\delta S_\pm}{\delta \sigma_{\alpha\beta}} = -n_i^\pm \eta^{i\pm}_{\alpha\beta}$$

- Here, the maps $n_i^\pm$ are identified with the **Left and Right Gauss maps** of the minimal surface:

$$n_i^+ = \frac{\bar{\lambda}\sigma_i\lambda}{\bar{\lambda}\lambda}, \quad n_i^- = \frac{\bar{\mu}\sigma_i\mu}{\bar{\mu}\mu}$$

- For the Hodge-dual minimal surface S_χ, the area element $\Sigma_{\mu\nu}$ transforms under $SO(4)$ strictly as a single Gauss map, corresponding to its chirality.

The True Difference: Active vs. Frozen Twistors

- **The crucial point:** The Hodge-dual solutions **ARE** different from the ordinary minimal area! The difference manifests directly in the holomorphic twistor parametrization $f'_\alpha(z) = \lambda(z)\sigma_\alpha\mu(z)$.

- **Self-Dual (SD) Solution:** The left $SU(2)$ is active. $\lambda(z)$ is given by a set of poles, while $\mu(z)$ is **constant**, leading to an active left Gauss map n_i^+.
- **Anti-Self-Dual (ASD) Solution:** The right $SU(2)$ is active. $\mu(z)$ has poles, while $\lambda(z)$ is **constant**, leading to an active right Gauss map n_i^-.

- The matrix $\lambda \otimes \mu$ transforms exclusively by either left or right $SU(2)$ multiplications. This permanent freezing of one chiral sector makes these solutions fundamentally distinct from the usual, generic minimal surface.
- The symmetric combination (required by parity) will later be used as the classical geometric background determining the mass spectrum of QCD, leading to observable predictions supporting the Hodge-dual surface!

Parity Restoration & Why Not Nambu-Goto?

- **Restoring Parity:** If S_+ and S_- independently solve the loop equations, why symmetrize?
- A single Hodge-dual surface explicitly breaks macroscopic Parity ($\chi = \pm 1$). To preserve the exact parity invariance of the physical QCD vacuum, we take the symmetric combination:

$$S[C] = \frac{1}{2}(S_+[C] + S_-[C]) \tag{21}$$

- **Why not just use Nambu-Goto?** The conventional Nambu-Goto area lacks this chiral decomposition, so the Bianchi identity does not apply.
- The NG area produces $\frac{|\delta x|}{(\delta y)^2 + (\delta x)^2}$ contact singularities in the loop diffusion operator $\mathcal{L}$. The limit depends on the relation between the normal δy and the tangent to the curve δx distances from the point on a boundary of the disk, where the $\mathcal{L}$ operator is applied.
- Our chiral solution uniquely bypasses this singularity and resolves this ambiguity, similar to the invariant point splitting resolving anomalies in QFT.

String Scale and the Operator Product Expansion

- The string tension parameter κ in $W = W_{pert} \exp\left(-\kappa S[C]\right)$ is not fixed by the MM equation; it is a non-perturbative property of the QCD vacuum.
- To find its physical origin, we match the geometric vacuum to the Operator Product Expansion (OPE) at short distances.
- Evaluate the 2nd area derivative for a small circular loop at opposite points ($|x_{12}| = 2r$). The perturbative OPE gives:

$$\frac{\delta^2 W_{pert}}{\delta\sigma_{\mu\nu}(1)\delta\sigma_{\lambda\rho}(2)} \propto \langle \operatorname{tr} F_{\mu\nu}(x_1) U_{1,2} F_{\lambda\rho}(x_2) U_{2,1} \rangle \tag{22}$$

- The leading short-distance conformal term scales as $|x_{12}|^{-4}$:

$$\text{OPE} \to \frac{\alpha_{eff}}{4\pi} \frac{I_{\mu\lambda} I_{\nu\rho} - I_{\nu\lambda} I_{\mu\rho}}{|x_{12}|^4} W_{pert}[0], \qquad I_{\mu\lambda} = \delta_{\mu\lambda} - 2\hat{x}_\mu \hat{x}_\lambda \tag{23}$$

- We isolate the non-perturbative dimension-4 contribution by contracting with the antisymmetric projector $\Pi_{\mu\nu\lambda\rho} = \frac{1}{2}(\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\nu\lambda}\delta_{\mu\rho})$.

The OPE Cancellation at $D = 4$ and the Gluon Condensate

- **A Dimensional Miracle:** Contracting the projector with the leading conformal tensor structure in d dimensions yields exactly:

$$S(d) = \Pi_{\mu\nu\lambda\rho}(I_{\mu\lambda}I_{\nu\rho} - I_{\nu\lambda}I_{\mu\rho}) = (\mathrm{tr}\, I)^2 - \mathrm{tr}(I^2) = (d-1)(d-4) \tag{24}$$

- **In exactly $D = 4$, this is strictly zero!** The leading UV conformal divergence drops out completely under this projection.
- The surviving dimension-4 operator is the **Gluon Condensate**:

$$\Pi_{\mu\nu\lambda\rho} \langle \mathrm{tr}\, F(1)UF(2)U \rangle \to -\frac{g^2}{2N_c} \left\langle (G^a_{\mu\nu})^2 \right\rangle W_{pert}[0] \tag{25}$$

- Matching this to the second area derivative of our geometric zero mode $(W = W_{pert} \exp{(-\kappa S[C])})$ on a small circle yields $16\kappa^2 W_{pert}[0]$.
- **The Result:** The macroscopic string tension κ is generated directly by the microscopic vacuum condensate!

$$\kappa^2 = \frac{\pi^2}{8N_c} \left\langle \frac{\alpha_s}{\pi} (G^a_{\mu\nu})^2 \right\rangle \tag{26}$$

The Physical Vacuum and the MLE Singularity

- **A Non-local Zero Mode in the MLE:** What does this geometric dressing factor imply for the momentum loop equation, and how is it related to the non-analyticity?
- Because this factor is a local multiplication in coordinate loop space, it becomes a **convolution** in momentum loop space:

$$W[P] = \int \mathcal{D}Q\, \mathcal{G}[P - Q] W_{\text{fluct}}[Q] \tag{27}$$

$$\mathcal{G}[Q] = \int \mathcal{D}C\, \exp\left(-\kappa S[C] + \mathrm{i} \oint d\theta\, C'_\mu(\theta) Q_\mu(\theta) \right), \quad \mathcal{L}_\nu(\mathcal{G})[Q] = 0 \tag{28}$$

- This representation is parametrically invariant, but generally **not analytic**. One cannot expand $\exp\left(\mathrm{i} \oint d\theta\, C'_\mu(\theta)(P_\mu(\theta) - Q_\mu(\theta))\right)$ in a functional Taylor series in $P(\theta)$, because the path-integral moments of $C'(\theta)$ **diverge for Brownian paths!**
- This convolution provides an explicit, non-local, and singular resolution to the $\mathcal{W}^{(8)}$ algebraic contradiction we discovered in Lecture II.
- The existence of an explicit analytic solution for the Hodge-dual minimal surface is **another mathematical miracle of 4D geometry, akin to instantons and twistors.**

Helicoid of the rigid string

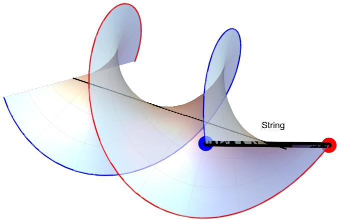


Figure: The helicoid spanned by rotating $\bar{q}q$ pair connected by a rigid stick (string). This minimal surface bounded by a double helix, was discovered by Meusnier in 1785.
In Minkowski space:

$$X(\tau, r) = (r \cos \omega\tau,\, r \sin \omega\tau,\, 0,\, \tau) \tag{29}$$

$$g_{ab} = \mathrm{diag}(1 - \omega^2 r^2,\, 1) \tag{30}$$

The holographic string and Regge trajectories

- This holographic area law offers the same phenomenological confinement mechanism via the linear potential $V = \sigma R$, with $\sigma = 2\sqrt{2}\kappa$, as the old string model.
- This Area in Minkowski space provides an effective action for the quark loop in the QCD vacuum:

$$S = -2\sigma \int_0^R dr \sqrt{1 - \omega^2 r^2} - 2m_q \sqrt{1 - \omega^2 R^2} \tag{31}$$

- The classical Lagrangian (with velocity $v = \omega R$) and angular momentum $J = \partial L / \partial \omega$ evaluate to:

$$L(v, \omega) = -\frac{\sigma}{\omega} \left(v\sqrt{1 - v^2} + \arcsin v \right) - 2m_q \sqrt{1 - v^2} \tag{32}$$

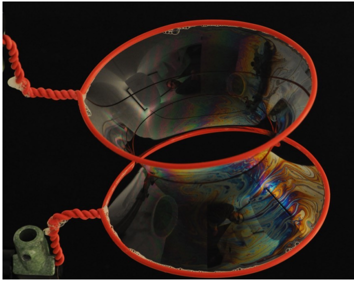
- Minimizing and resolving for R, v, we find that at large J and zero quark mass $m_q = 0$, the velocity $v \to 1$, and the **Regge trajectory is strictly linear**:

$$E^2 \to 2\pi\sigma J \tag{33}$$

- **The Crucial Distinction: There are no vibrational modes, this is a rigid stick rotation.**

Philosophical Discussion: Quantum Physics vs. Classical Models

- **The Classical Illusion:** Look at this catenoid. A classical soap film will always transition to a connected topology because it dynamically minimizes its physical surface energy.
- **The Quantum Reality:** But we must remember the fundamental physics—there is no literal "string" and no "surface energy" in QCD! Those were phenomenological models built decades ago to mimic confinement.
- **No Bulk Fluctuations:** Our physical requirement is minimizing the vacuum energy of the quantum gauge field. In our exact holographic solution, there are **no fluctuating surface degrees of freedom** in the bulk. There is no physical membrane exploring a configuration space to find a global minimum with cylindrical topology.



A classical mirage.

The Core Principle

Summary of Lecture III

1. **The Zero Mode:** The multi-instanton QCD vacuum is geometrically represented by a rigid surface with a (anti)self-dual area derivative.
2. **Goldschmidt Additivity:** To satisfy the non-linear factorization of the MM hierarchy, the functional must remain strictly additive at self-intersections (no topological phase transitions).
3. **Hodge-Dual Geometry:** We constructed this exactly by embedding a holomorphic map in $\mathbb{R}^3 \otimes \mathbb{R}^4$ via 't Hooft matrices.
4. **Active vs. Frozen Twistors:** The Hodge-dual solutions are fundamentally distinct from generic minimal surfaces due to the permanent freezing of one chiral twistor sector.
5. **The Gluon Condensate:** The $D = 4$ conformal OPE singularity exactly vanishes, tying the geometric string tension κ directly to the physical SVZ gluon condensate.

Next Lecture:

How to place Majorana fermions on this rigid surface to cancel non-planar intersections via the Pauli Principle and recover asymptotically free QCD.