Bootstrapping Large N confining gauge theories

Jan Albert

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w/ Leonardo Rastelli

Bootstrap philosophy

UV: Confining gauge theory. $\{m_i^2, \lambda_{ijk}\}$

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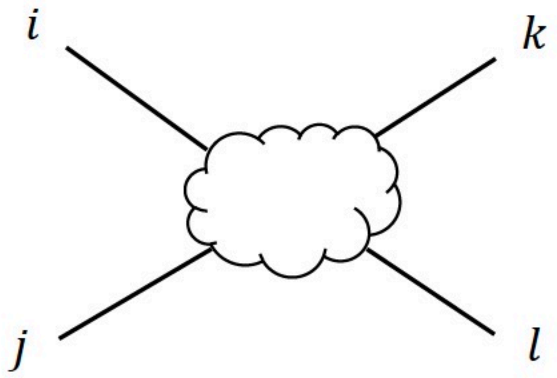


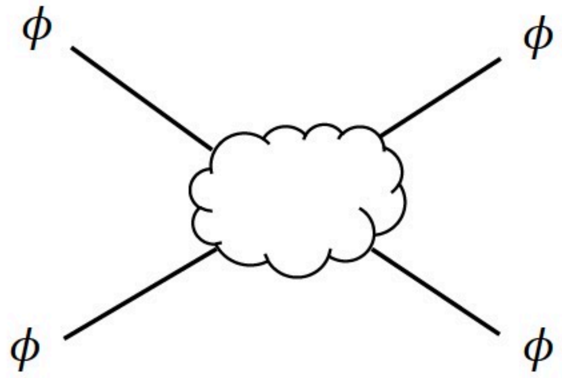
Bootstrap philosophy

UV: Confining gauge theory. $\{m_i^2, \lambda_{ijk}\}$

↓

IR: EFT for lowest excitations ϕ.

$$\mathcal{L}_{\text{EFT}} = -\frac{1}{2}(\partial\phi)^2 + g_0\phi^4 + g_1\phi^2(\partial\phi)^2 + g_2(\partial\phi)^4 + \cdots$$



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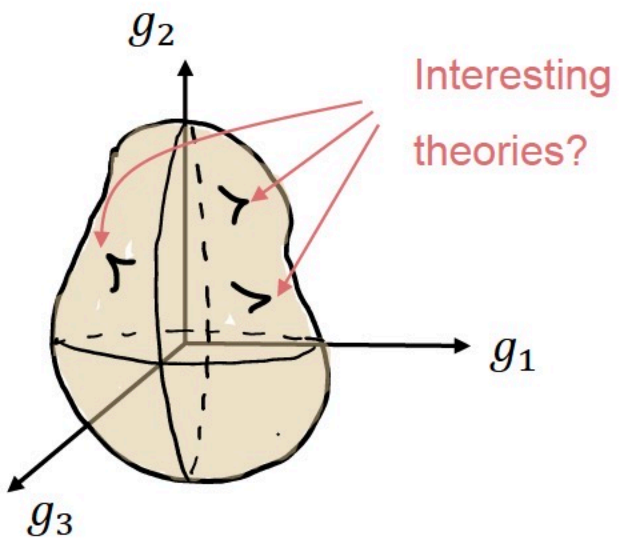

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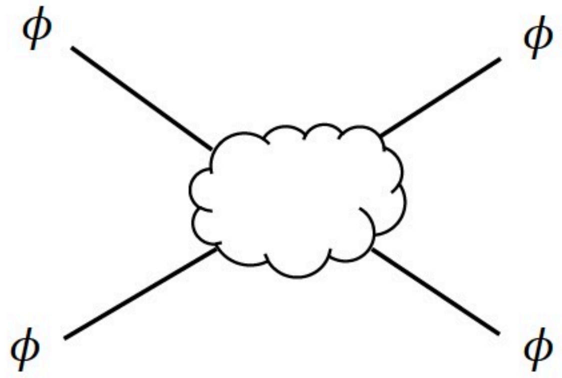

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Weakly coupled

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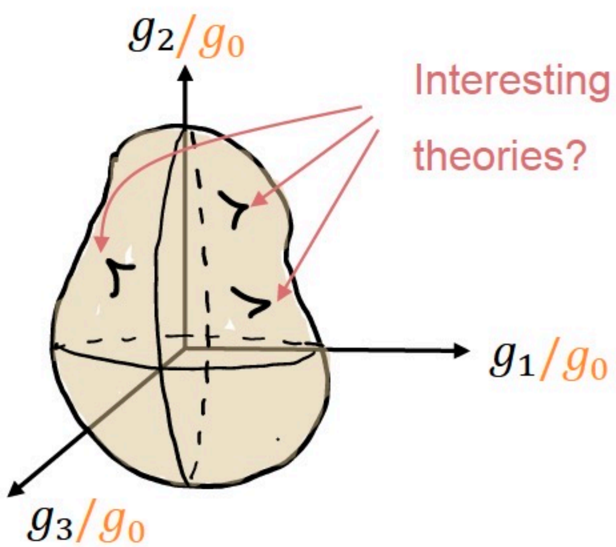

$\sim \frac{1}{N}$

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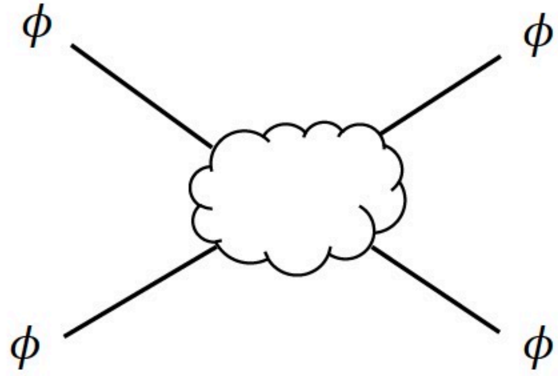

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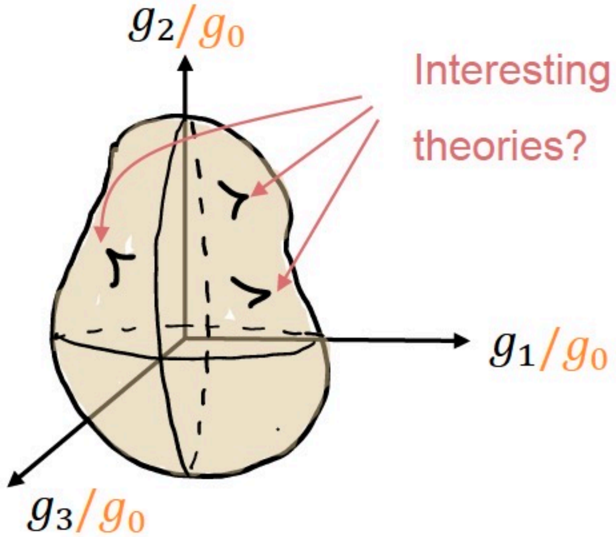

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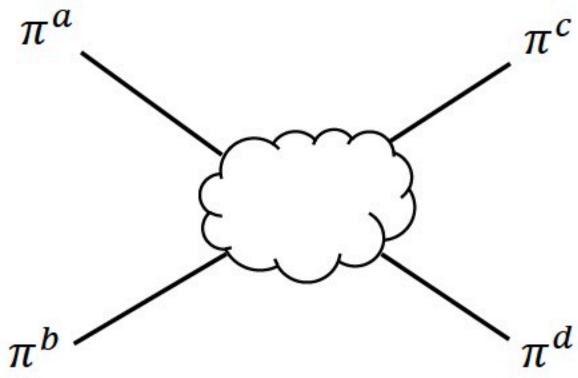
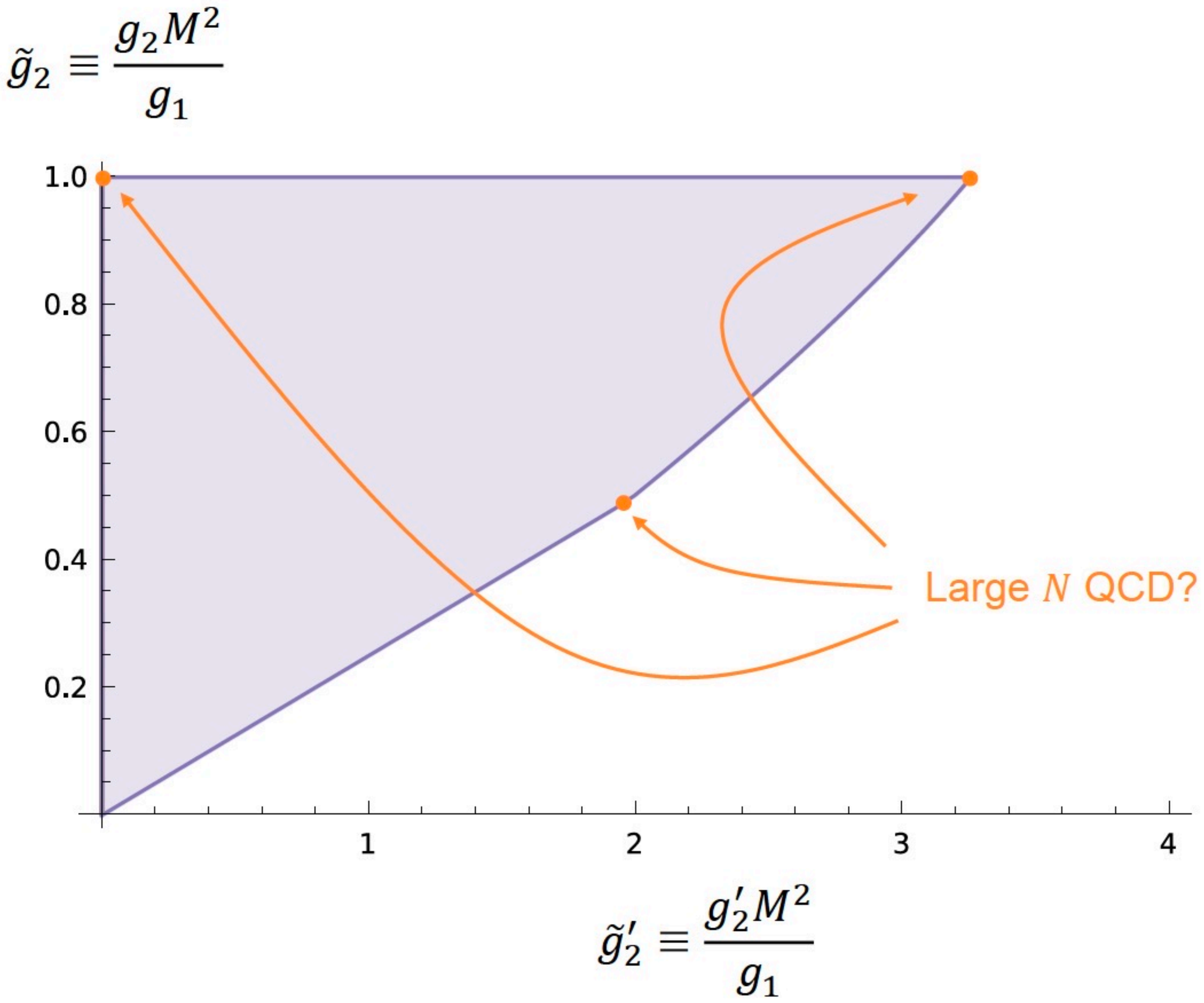
Target: Large N QCD.

$$U(N_f)_L \times U(N_f)_R \longrightarrow U(N_f)_V$$

N_f^2 GB $= \pi^a$

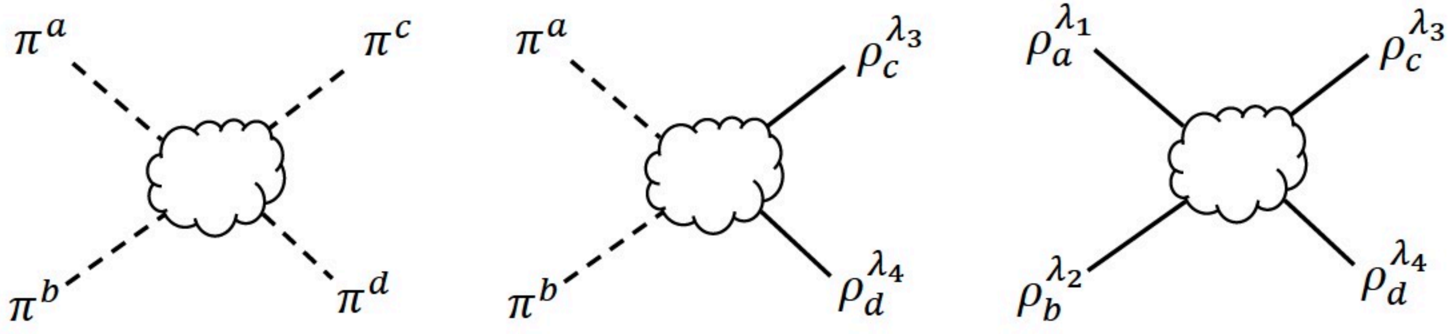


Exclusion plot



Beyond pions

External rho mesons:



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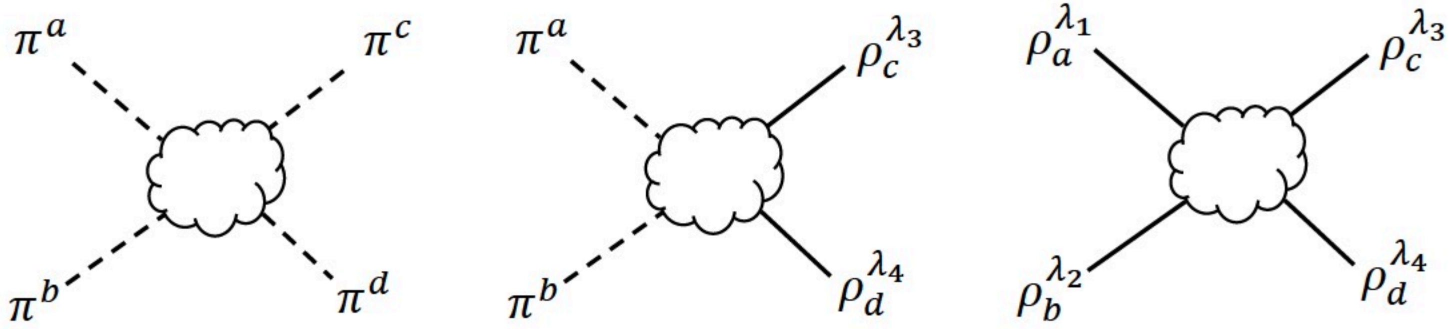
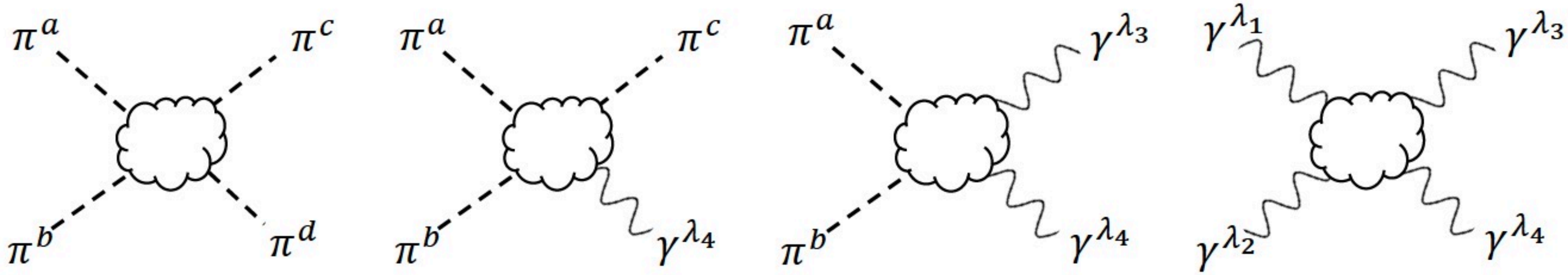
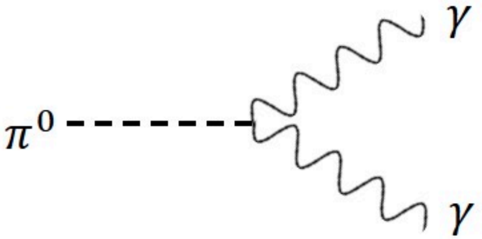


Probe photons:



Access the anomaly



Correlation Functions of Local Operators in the $T\bar{T}$-Theory from Topological Gravity

Presented by: Netanel Barel
Supervised by: Ofer Aharony

Weizmann Institute of Science

July 13, 2023

Overview

1. Introduction
2. Definitions
3. JT formalism

Introduction

- The first sentence of Leo Tolstoy's novel Anna Karenina is: "Happy families are all alike; every unhappy family is unhappy in its own way."

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- The first sentence of Leo Tolstoy's novel Anna Karenina is: "Happy families are all alike; every unhappy family is unhappy in its own way."
- The first sentence of a QFT course should be: "Local QFTs are all alike; every non local QFT is non local in its own way".

Introduction

- RG flow of Local QFTs: CFT in the UV to CFT in the IR.

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- RG flow of Local QFTs: CFT in the UV to CFT in the IR.
- $T\bar{T}$ deformation: CFT in the IR and non-local QFT in the UV.

Definitions

- Theory in 1+1d with energy-momentum tensor $T_{\alpha\beta}$, we define the composite operator:

Definition 1 - $\mathrm{T\bar{T}}$ Operator

$$\mathrm{T\bar{T}} \equiv \det\left(T_{\alpha\beta}\right)$$

Definitions

- We modify the Lagrangian:

Definition 2 - Deformation

$$\mathcal{L}^{(t+dt)} = \mathcal{L}^{(t)} + \mathrm{T}\bar{\mathrm{T}}^{(t)} dt,$$

$\mathrm{T}\bar{\mathrm{T}}^{(t)}$ is energy-momentum tensor determinant of $\mathcal{L}^{(t)}$.

Previous results

- Leading order correction:

$$C^t(q) = |q|^{2\Delta}\left(1 + \frac{tq^2}{\pi}\ln\left(\frac{|q|}{\mu}\right)\right)$$

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$$C^t(q) = |q|^{2\Delta}\left(\frac{|q|}{\mu}\right)^{\frac{tq^2}{\pi}}$$

- In a CFT the correlator looks like: $|q|^{2\alpha}$

JT formalism

- The undeformed theory coupled to JT gravity:

Definition 3 - JT Action

$$S_{\mathrm{T\bar{T}}}(\psi, e^a_\alpha, X^a) = S_0(\psi, g_{\alpha\beta}) + S_{JT}(X^a, e^a_\alpha)$$
$$S_{JT}(X^a, e^a_\alpha) \equiv -\frac{1}{2t}\int d^2\sigma \epsilon^{\alpha\beta}\epsilon_{ab}(\partial_\alpha X^a - e^a_\alpha)(\partial_\beta X^b - e^b_\beta).$$

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- Non-perturbative formulation of the $\mathrm{T}\bar{\mathrm{T}}$ deformation.

JT formalism

- local operators - deformations of an undeformed operators $O(\sigma)$ in position and momentum space are:

Definition 4 - Deformed Operator

$$\mathcal{O}(X_0) = \int d\sigma \sqrt{g(\sigma)} O(\sigma) \delta\left(X(\sigma) - X_0\right),$$

$$\mathcal{O}(q_0) = \int d\sigma \sqrt{g(\sigma)} O(\sigma) \exp\left(i q_0 X(\sigma)\right).$$

JT formalism 4

- The correlation function in momentum space:

Definition 5 - Correlator

$$\langle \mathcal{O}(q_1)\mathcal{O}(q_2)\rangle \equiv \frac{1}{Z_{\mathrm{T}\bar{\mathrm{T}}}} \int \frac{\mathcal{D}e\mathcal{D}X\mathcal{D}\psi}{V_{\mathrm{diff}}} \mathcal{O}(q_1)\mathcal{O}(q_2) e^{-S_{\mathrm{T}\bar{\mathrm{T}}}}$$

$$Z_{\mathrm{T}\bar{\mathrm{T}}} \equiv \int \frac{\mathcal{D}e\mathcal{D}X\mathcal{D}\psi}{V_{\mathrm{diff}}} e^{-S_{\mathrm{T}\bar{\mathrm{T}}}}.$$

Main Result

- High momentum limit (non-perturbative):

Main result

$$|q|^{2\Delta}\left(\frac{|q|}{\mu}\right)^{-\frac{t|q|^2}{\pi}}$$

- Remind high momentum limit of Cardy (perturbative):

$$|q|^{2\Delta}\left(\frac{|q|}{\mu}\right)^{+\frac{tq^2}{\pi}}$$

Generalized Symmetries and Noether's theorem in QFT

Valentin Benedetti

Based on work with Horacio Casini and Javier Magan

arXiv: 2205.03412, 2212.11291

When is there a well-defined Noether current for a continuous symmetry in QFT?

1. A priori **Noether's theorem** asserts the existence of a local conserved current when an action is invariant under a continuous symmetry group. However, a long-standing question **is to determine to what extent, or under what conditions, this theorem holds in QFT.**

2. Known **counter-examples** (the Noether current is not gauge invariant)
   - Duality Symmetry in Free Maxwell theory
   - **Weinberg Witten theorem** (for example, graviton stress-tensor)
   - Chiral symmetry and the ABJ anomaly
   - etc

3. What is peculiar about the symmetry in this cases?

4. What does this teach us about the completeness of the spectrum?

Given a theory with generalized symmetries and a global (0-form) symmetry described by a continuous symmetry group G :

If the **generalized symmetries are charged** under the action of the continuous symmetry group G

i.e. There are non-local operators that transform under the action of the group G.

$\Rightarrow$

$\Leftarrow$?

There is **no Noether current** for G
However, the symmetry is still implemented locally by twist operators
(Weak version of Noether's theorem)

Rederives Weinberg-Witten theorem

e.g. Generalized Symmetries of the free graviton are charged under space-time symmetries ($\Rightarrow$ no stress-tensor).
Same applies to all free massless particles of spin $\geq 3/2$.

$\Downarrow$

Non-local operators produce Haag duality violations, then **always come in dual classes assigned to complementary regions.**

In this case, they **both must form a continuum**

$\longrightarrow$

Haag duality defects form a **continuous non-compact group**

$\Downarrow$

QFT with a **free massless sector** or **the theory needs to be UV completed with charges that break all generalized symmetries**

$\Downarrow$

Charges in any UV completion of:
- neutral and non-linear electrodynamics
- interacting goldstone modes

$\downarrow$

Haag duality defects form a **continuous compact group**

e.g. **ABJ anomaly** and chiral symmetry

THANK YOU!

A brief review of Coon amplitudes

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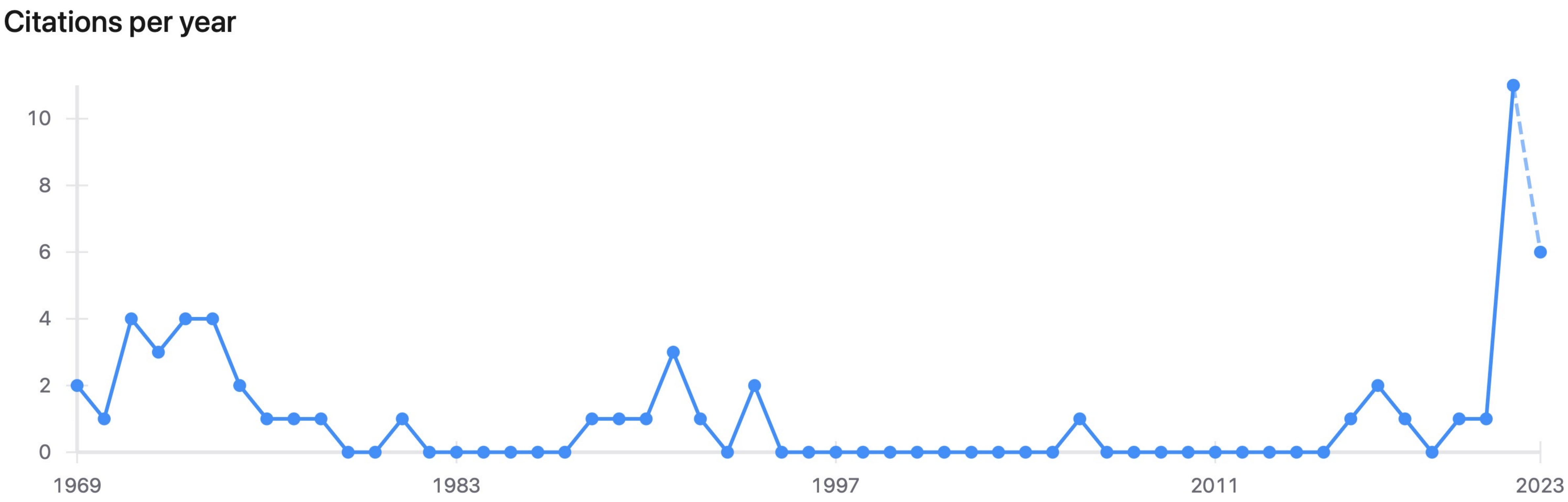
Princeton Institute for Advanced Study
PiTP 2023: Understanding Confinement
July 13, 2023

Coon amplitudes: old but back in style



[Coon 1969] [https://inspirehep.net/literature/56699] [slide inspired by Remmen 2023]

Coon amplitudes: long history & open problems

- 1968 : Veneziano discovery [Veneziano 1968]
- 1969 : Coon discovery [Coon 1969]
- 1970s : N-pt, operator formulation, pheno [Coon, Baker, ... 1969–1976]
- 1988 : independent rediscovery [Romans 1988]
- 2016 : S-matrix bootstrap counter-ex [Caron-Huot, Komargodski, Sever, Zhiboedov 2016]
- 2022– : **physical realization?** (*open strings on D-branes in AdS*) [Maldacena, Remmen 2022]
- 2022– : **generalizations?** [Geiser, Lindwasser 2022] [Cheung, Remmen 2022–2023]
- 2022– : **unitarity?** [Figueroa, Tourkine 2022] [Geiser, Lindwasser 2022] [ICTS group 2022] [Brown group 2022] [Jepsen 2023]

Coon amplitudes: family of ampl's parametrized by $q \geq 0$

- 4-pt, q-gamma func's

$$\mathcal{A}_q^{(4)}(s,t) = q^{\alpha_q(s)\alpha_q(t)} \frac{\Gamma_q(-\alpha_q(s))\Gamma_q(-\alpha_q(t))}{\Gamma_q\big(-\alpha_q(s)-\alpha_q(t)\big)} \quad \xrightarrow[q\to 1]{} \quad \mathcal{A}_{\text{Ven}}^{(4)}(s,t)$$

- Regge traj. $\alpha_q(s) = \log_q[1+(q-1)(s/\mu^2-\delta)]$
- spectrum $m_n^2 = \mu^2(\frac{1-q^n}{1-q}+\delta)$
  - $q = 0, \infty \implies$ scalars
  - $q = 1 \implies$ strings (super $\delta = 0$, bosonic $\delta = -1$)
  - $q < 1 \implies$ accumulation pt $(m_\infty^2 = \mu^2(\frac{1}{1-q}+\delta))$ & non-meromorphic
  - $q \geq 1 \implies$ unbounded $(m_\infty^2 = \infty)$ & meromorphic
- unitarity $\implies$ $q \leq 1$ & $-1 \leq \delta \leq \frac{1}{3}$ [Figueroa, Tourkine 2022]

Coon amplitudes: problems at higher points

- 5-pt, q-hypergeometric func's [Baker, Coon 1970] [Romans 1988] [Geiser *to appear*]

$$\mathcal{A}_q^{(5)} = \mathcal{A}_q^{(4)}(s_{12}, s_{23})\, \mathcal{A}_q^{(4)}(s_{34}, s_{45}) \qquad \xrightarrow[q\to 1]{} \qquad \mathcal{A}^{(5)}_{\text{string}}$$

$$\times\, {}_3\Phi_2\left[\begin{matrix} q^{\alpha_q(s_{12})} & q^{\alpha_q(s_{45})} & q^{\alpha_q(s_{23})+\alpha_q(s_{34})-\alpha_q(s_{51})} \\ & q^{\alpha_q(s_{12})+\alpha_q(s_{23})} & q^{\alpha_q(s_{34})+\alpha_q(s_{45})} \end{matrix}\, ; q^{-1}\, ; q^{\alpha_q(s_{51})}\right]$$

- $q \leq 1$ & special kinematics $\implies$ $\mathcal{A}_{q,r}^{(4)}$ from [Cheung, Remmen 2023]
- $q \geq 1$ $\implies$ factorization in all channels
- $q < 1$ $\implies$ factorization in subset of channels
- N-pt generalization for $q > 1$ (non-unitary)

Thank you!

Questions?

On Unitarity Of The Coon Amplitude

Rishabh Bhardwaj & Shounak De
(Brown University)

Based on:
- 2212.00764 (RB, SD, M. Spradlin, A. Volovich)
- 2208.xxxxx (RB, SD - to appear)

Veneziano amplitude (1968):

$$\mathcal{A}^V(s,t) = \frac{\Gamma(-\alpha_0 - s)\,\Gamma(-\alpha_0 - t)}{\Gamma(-2\alpha_0 - s - t)}$$

i) crossing symmetric in $s \leftrightarrow t$

ii) polynomial residues

iii) meromorphic

1969: Worldsheet realization as a dual resonance model → 50+ years of ST!

(Susskind, Nambu, Goto, ...)

Question: Is $\mathcal{A}^V(s,t)$ unique?

Coon amplitude (1969):

$$\mathcal{A}^C(s,t) = q^{\alpha_q(s)\alpha_q(t)-\alpha_q(s)-\alpha_q(t)} \frac{\Gamma_q(-\alpha_q(s))\,\Gamma_q(-\alpha_q(t))}{\Gamma_q(1-\alpha_q(s)-\alpha_q(t))}$$

where $\alpha_q(s) = \dfrac{\ln(1-(1-q)(\alpha_0+s))}{\ln q}$.

i) $\lim_{q\to 1} \mathcal{A}^C(s,t) = \mathcal{A}^V(s,t)$

ii) $\lim_{q\to 0} \mathcal{A}^C(s,t) = \dfrac{1}{s+\alpha_0} + \dfrac{1}{t+\alpha_0} + 1$

Question: Is $A^c(s,t)$ unitary ?

Answer: Yes, in some regimes !

$$A^c(s,t) \longrightarrow \frac{1}{s-[n]_q} \times R^c_{[n]_q}(t)$$

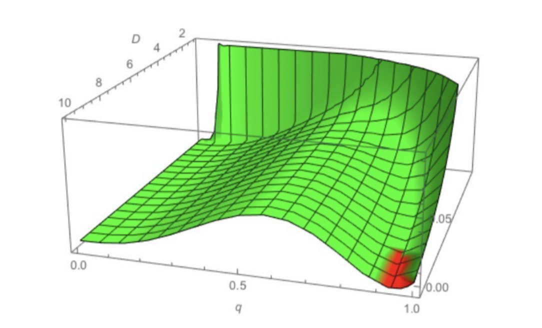
$R^c_{[n]_q}(t)$ → residue polynomials

Condition of unitarity: $R^c_{[n]_q}(t) = \sum_j B^{(D)}_{n,j}(q)\, G^{(D)}_j(t)$

$G^{(D)}_j(t)$: D-dim'l Gegenbauer polynomials

with $B^{(D)}_{n,j} \geq 0$

(2212.00764)
(RB, S.De, M.Spradlin, A.Volovich)



(a) The coefficient $B^D_{3,0}(q)$

Question: What are the ST origins of $\mathcal{A}^c(s,t)$? (if any)

Answer: Maybe none (wip - RB, SD)

$SL(2,\mathbb{C}) \xrightarrow{\text{q-def.}} SL(2,\mathbb{C})_q \longrightarrow$ construct qCFTs

$\downarrow$ q-Ward identities

q-deformed amplitudes $\longleftarrow$ q-deformed $\langle\ \rangle$

$\downarrow$

NO observables that resemble $\mathcal{A}^c(s,t)$!

Quasinormal modes of the D0 brane black hole

Anna Biggs

Princeton University

July 2023

Dp brane holography

Near-horizon limit of $D3$ branes $\rightarrow$ $AdS_5 \times S^5 =$ SYM in $4d$

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There is an analogous duality for $p \neq 3$, but the Yang-Mills theory is not conformally invariant. Itzhaki, Maldacena, Sonnenschein, Yankielowicz 1998

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Supergravity geometry: conformal to $AdS_{2+p} \times S^{8-p}$

$$ds^2_{string} \propto z^{\frac{3-p}{5-p}} \left(\frac{-dt^2 + dx_p^2 + dz^2}{z^2} + \mathcal{R}^2 d\Omega^2_{8-p} \right), \qquad \mathcal{R} = \frac{5-p}{2}$$

$$e^{\phi} \propto z^{\frac{(7-p)(3-p)}{2(5-p)}}, \qquad A_{0\ldots p} \propto z^{-\frac{2(7-p)}{5-p}}$$

A black hole described by a quantum mechanics

today: $p = 0$ case

near-horizon geometry of charged BH	$=$	BFSS matrix model
(D0 brane supergravity solution)		(low temp regime)

Scaling similarity

The $AdS_{2+p} \times S^{8-p}$ supergravity geometries

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have the property that, under a coordinate rescaling

$$t \to \gamma t, \quad z \to \gamma z, \quad x \to \gamma x$$

the action gets rescaled:

$$S \to \gamma^{-\bar{\theta}} S, \qquad \bar{\theta} = \frac{(3-p)^2}{5-p}$$

Not a symmetry, but a "similarity" Landau, Lifschitz 1982

Perturbations of D0 black hole

How does the system respond to perturbations?

In the bulk: black hole quasinormal modes, $\omega_i \in \mathbb{C}$

In the quantum system: thermalization timescale

A simple observable in the matrix model we can predict using gravity.

Perturbations of D0 black hole

Finding wave equation for linearized fluctuations around $D0$ brane background = hard

Perturbations of D0 black hole

Finding wave equation for linearized fluctuations around $D0$ brane background = hard

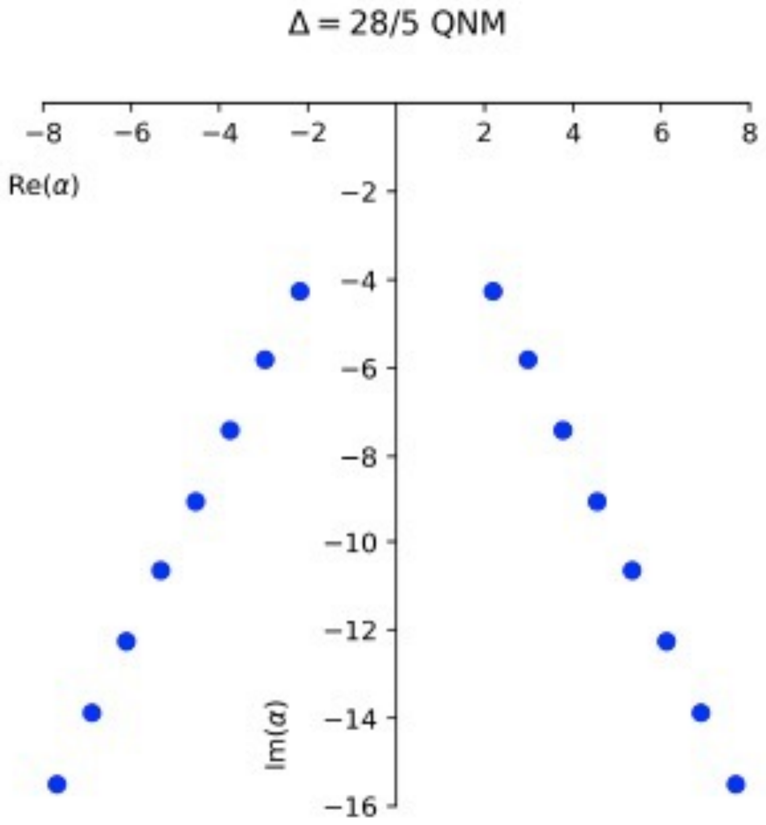
Tricks:

1) View Dp brane geometry as the dimensional reduction of $AdS_{2+p+\bar{\theta}} \times S^8$. Kantischeider, Skenderis 2009

2) Masses of perturbations can be classified using the scaling similarity.

(Computed previously by Sekino, Yoneya 2000)

Quasinormal mode spectrum



$$\omega_n = 2\pi T \alpha_n$$

Entanglement entropy from non-equilibrium lattice simulations

Andrea Bulgarelli

Università degli Studi di Torino and Istituto Nazionale di Fisica Nucleare

Based on: A. Bulgarelli and M.Panero JHEP 06 (2023) 030

UNIVERSITÀ DI TORINO

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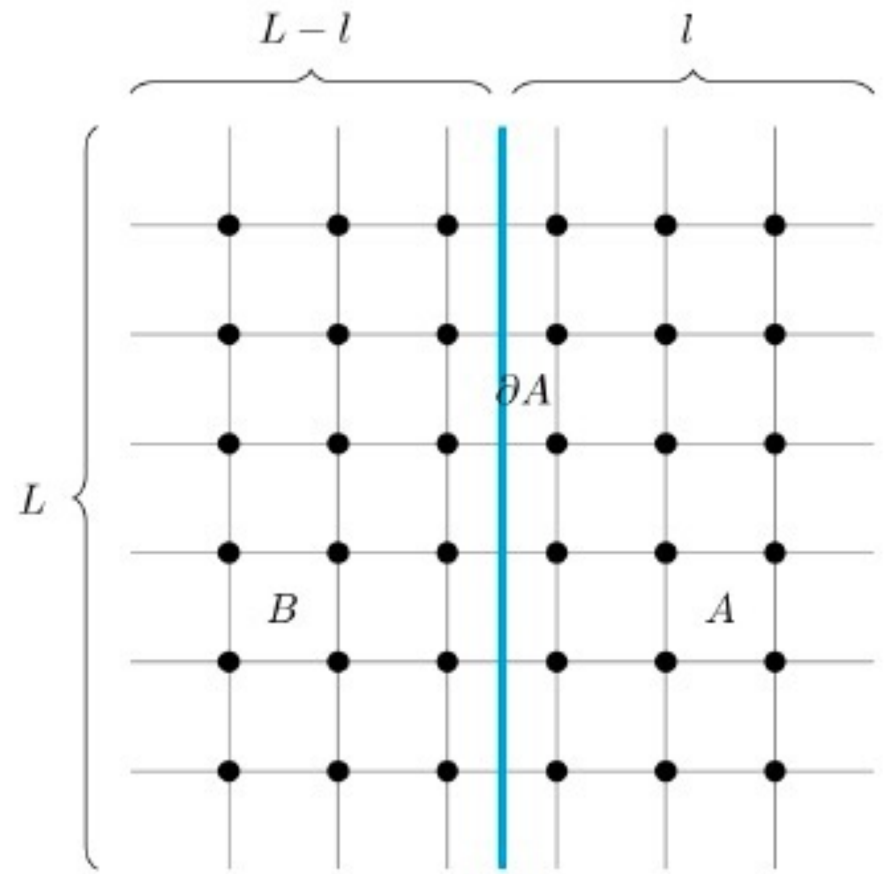
Entanglement in (L)QFT



$$S(A) = -\operatorname{Tr}(\rho_A \log \rho_A) \qquad S_n(A) = \frac{1}{1-n} \log \operatorname{Tr} \rho_A^n$$

Replica trick

- A common way to calculate Rényi entropies and other entanglement measurements is to exploit the replica trick [Calabrese, Cardy; 2004]

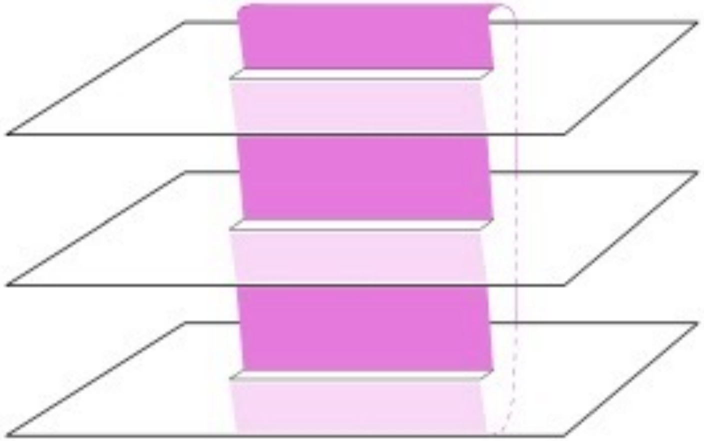


Image taken from [Cardy *et. al.*; 2007].

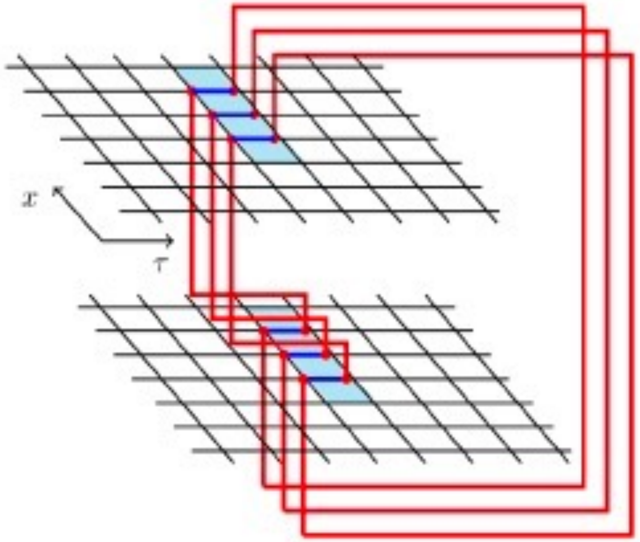


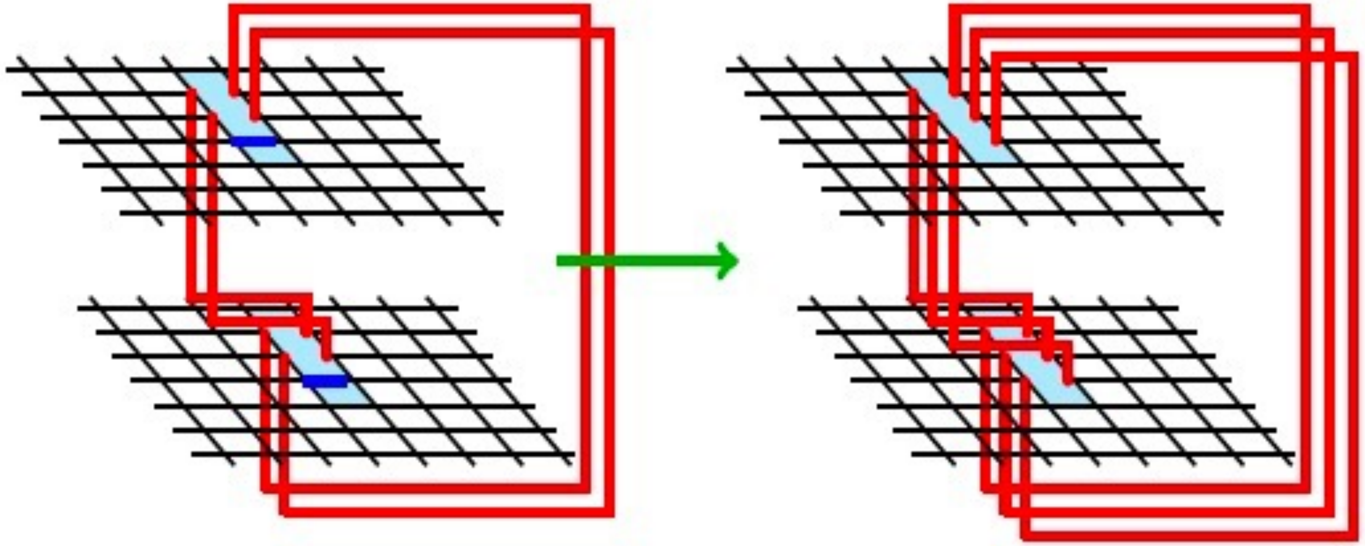
Image adapted from [Alba; 2016].

$$C_n = \frac{l^{D-2}}{|\partial A|}\frac{\partial S_n}{\partial l} = \frac{l^{D-2}}{|\partial A|}\frac{1}{1-n}\frac{1}{a}\log\frac{Z_n(l+a)}{Z_n(l)}$$

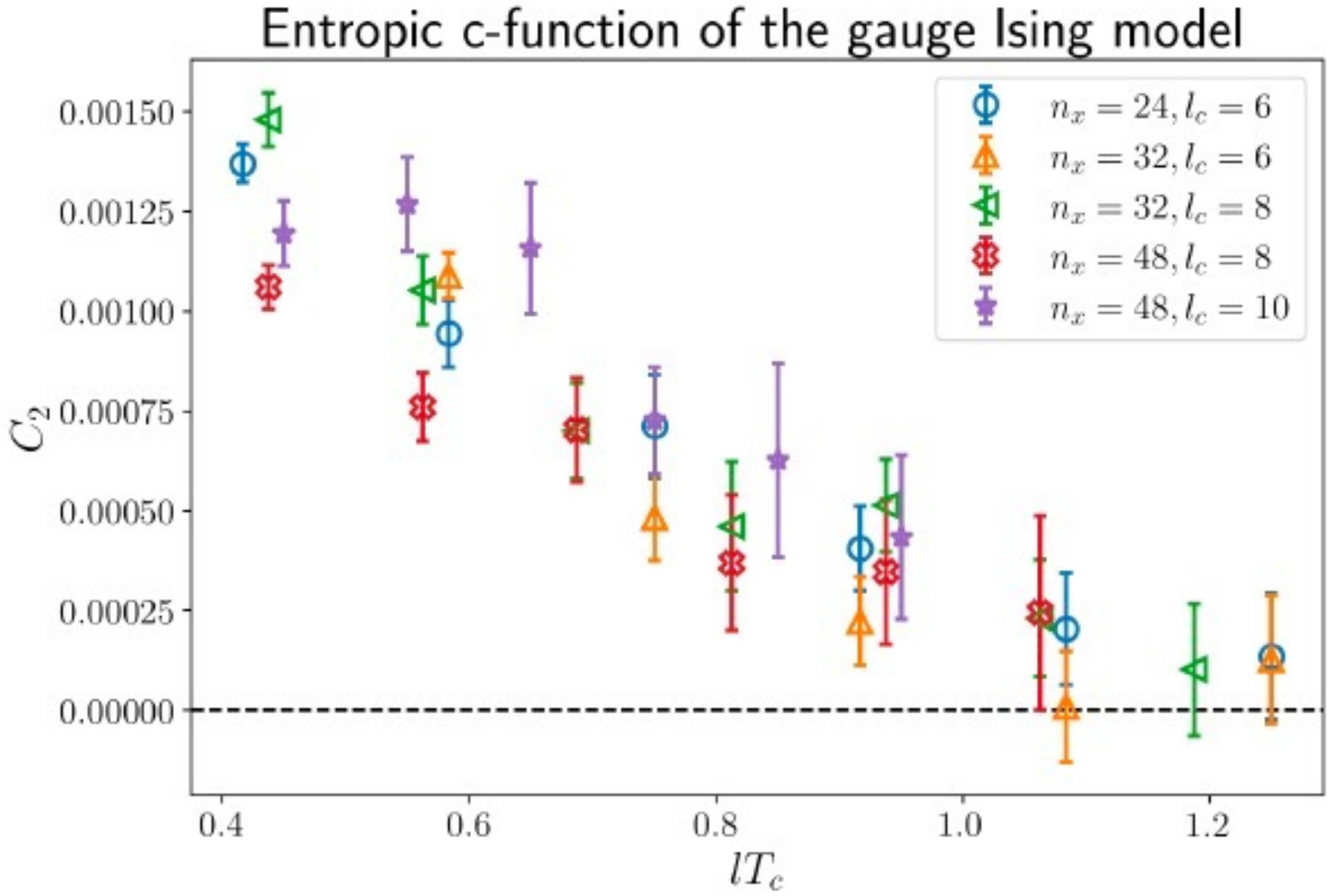
Jarzynski's theorem for lattice simulations

- Jarzynski's theorem [Jarzynski; 1996] is an exact result that connects averages of out-of-equilibrium trajectories of a statistical system to equilibrium free energies.

$$\frac{Z_n(l+a)}{Z_n(l)} = \left\langle \exp\left(-\int \beta\delta W\right)\right\rangle$$



Future work: $D = 3$ gauge $\mathbb{Z}_2$ and gauge-Higgs $\mathbb{Z}_2$



CP-broken Deconfined Phase at $\theta=\pi$

Shi Chen

Department of Physics, the University of Tokyo

4D SU(N) Yang-Mills (see Nati's lecture) (pure, with adjoint matter, etc.)

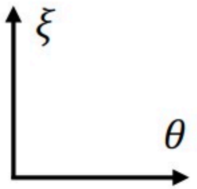
- $\mathbb{Z}_N^{[1]}$ symmetry
  [Gaiotto, Kapustin, Seiberg, Willett, 2014]
  - Unbroken ➡ Confinement
  - Spontaneously broken ➡ Deconfinement/Higgsing

Couple background $\mathbb{Z}_N$ 2-form gauge field $w \in H^2(-, \mathbb{Z}_N)$

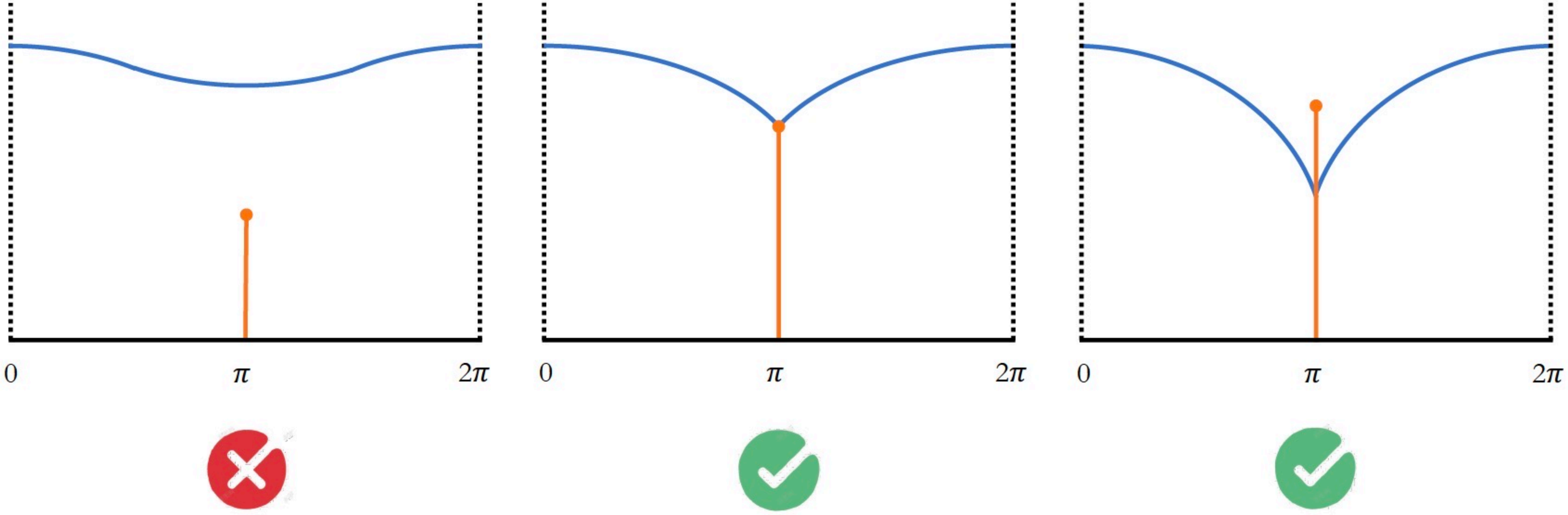
- Instanton charge fractionalization $\int c_2(a) + \frac{1}{N}\int P(w) = 0 \mod 1$
  - N odd $P(w) \equiv w \cup w$
  - N even $P(w) \equiv \frac{1}{2}\mathcal{P}(w)$

Mixed 't Hooft anomaly

- Between $\mathbb{Z}_N^{[1]}$ and θ ($U(1)^{[-1]}$ symmetry) ➡ ~~$\mathbb{Z}_N^{[1]}$-symmetric θ-uniform phase~~
  [Córdova, Kapustin, Lam, Seiberg, 2019]
- At $\theta = \pi$: between $\mathbb{Z}_N^{[1]}$ symmetry and CP ($\mathbb{Z}_2^{[0]}$ symmetry) ➡ ~~$\mathbb{Z}_N^{[1]}$-symmetric CP-symmetric phase~~
  [Gaiotto, Kapustin, Komargodski, Seiberg, 2017]



ξ is a parameter that causes a confinement-deconfinement phase transition



[Gaiotto, Kapustin, Komargodski, Seiberg, 2017]

$\mathcal{N}=1$ SU(N) super Yang-Mills

- on $\mathbb{R}^3 \times S^1$ with susy boundary condition — L: the size of S^1
- softly susy-breaking by a light massive gluino — m: the mass of gluino

Semiclassical computation (see Mithat's lecture) [Davies, Hollowood, Khoze, 2000] [Poppotz, Schäfer, Ünsal, 2012] [Chen, Fukushima, Nishimura, Tanizaki, 2020]

$$(\phi,\sigma) \in \frac{\frac{\mathbb{R}^r}{2\pi\Lambda_r^\vee} \times \frac{\mathbb{R}^r}{2\pi\Lambda_w}}{W_g}$$

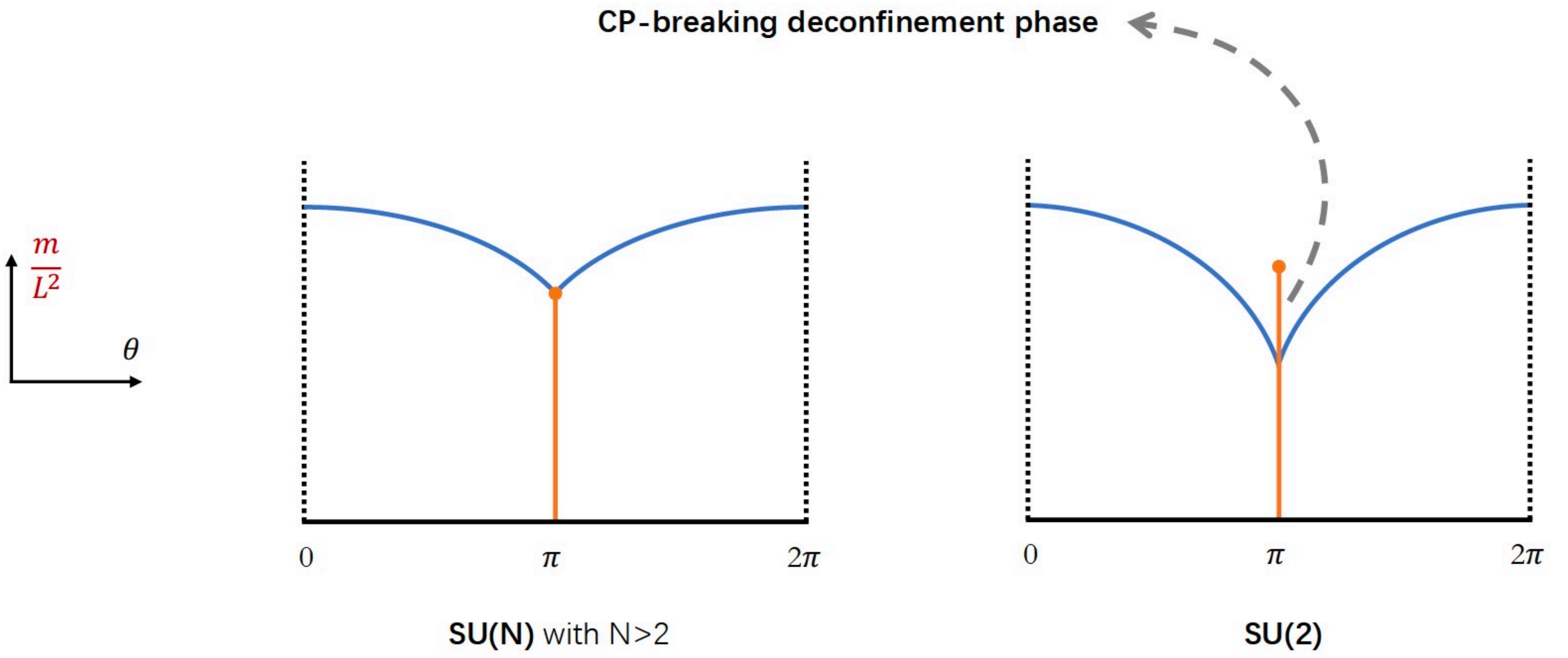
$$(\phi,\sigma) \xrightarrow{\mathbb{Z}_N^{[1]}} (\phi + 2\pi\boldsymbol{\mu}_c^\vee, \sigma) \qquad \boldsymbol{\mu}_c^\vee \in \Lambda_w^\vee/\Lambda_r^\vee \simeq \mathbb{Z}_N$$

$$\begin{cases} \theta = 0: & (\phi,\sigma) \xrightarrow{\mathrm{CP}} (\phi, -\sigma) \\ \theta = \pi: & (\phi,\sigma) \xrightarrow{\mathrm{CP}} (\phi, -\sigma-\phi) \end{cases}$$

$$\boxed{\mathcal{V}(\phi,\sigma) \propto \sum_{i=0}^{r}\sum_{j=0}^{r} M_i^*(\boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_j) M_j - \# \frac{m}{L^2} \sum_{i=0}^{r} \frac{M_i^* + M_i}{|\boldsymbol{\alpha}_i|^2}\left(1 - \frac{c_2 g^2}{8\pi^2}\ln|M_i|\right)}$$

$$\boldsymbol{z} \equiv i\left(\sigma + \frac{\theta}{2\pi}\phi\right) - \frac{4\pi}{g^2}\phi$$

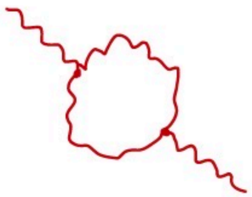
$$\begin{cases} M_i \equiv \exp\left\{\boldsymbol{\alpha}_i^\vee \cdot \boldsymbol{z} + \dfrac{8\pi^2}{c_2 g^2}\right\} \text{ for } i = 1,2,\cdots,r \\ M_0 \equiv \exp\left\{\boldsymbol{\alpha}_0^\vee \cdot \boldsymbol{z} + \dfrac{8\pi^2(1-c_2)}{c_2 g^2} + i\theta\right\} \end{cases}$$

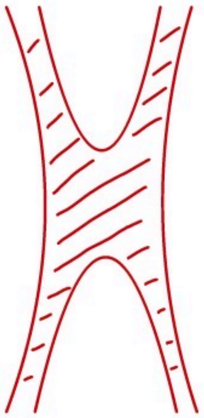


[Chen, Fukushima, Nishimura, Tanizaki, 2020]

Thank you for your attention!

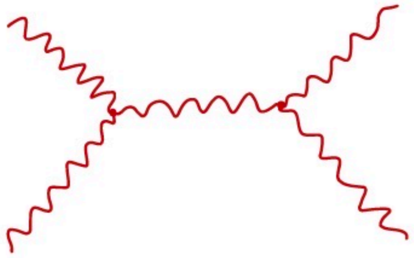
Leading Singularities of Gluon Amplitudes from the Bosonic String.





Carolina Figueiredo

July 2023



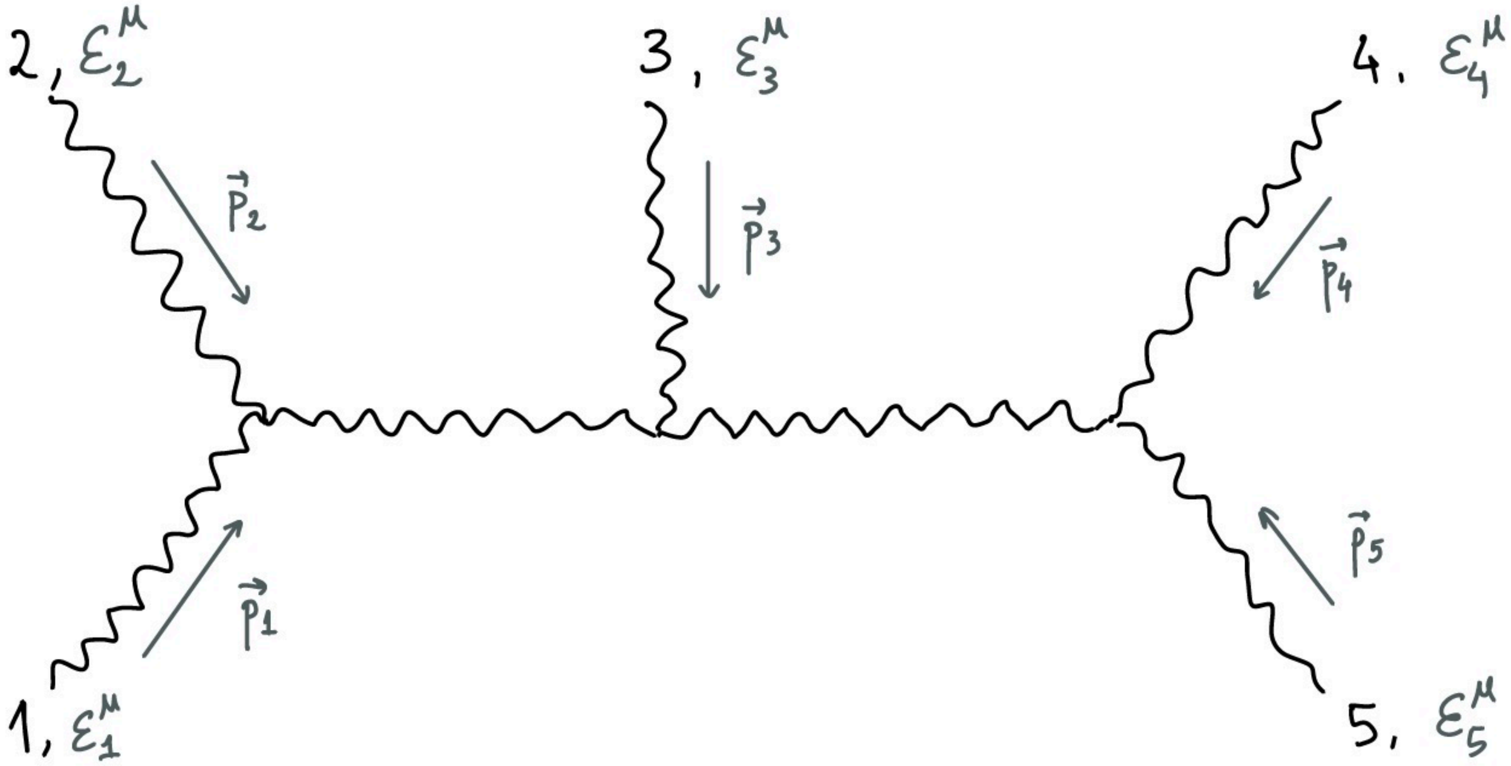
w/ Nima Arkani-Hamed

Qu Cao, Jin Dong

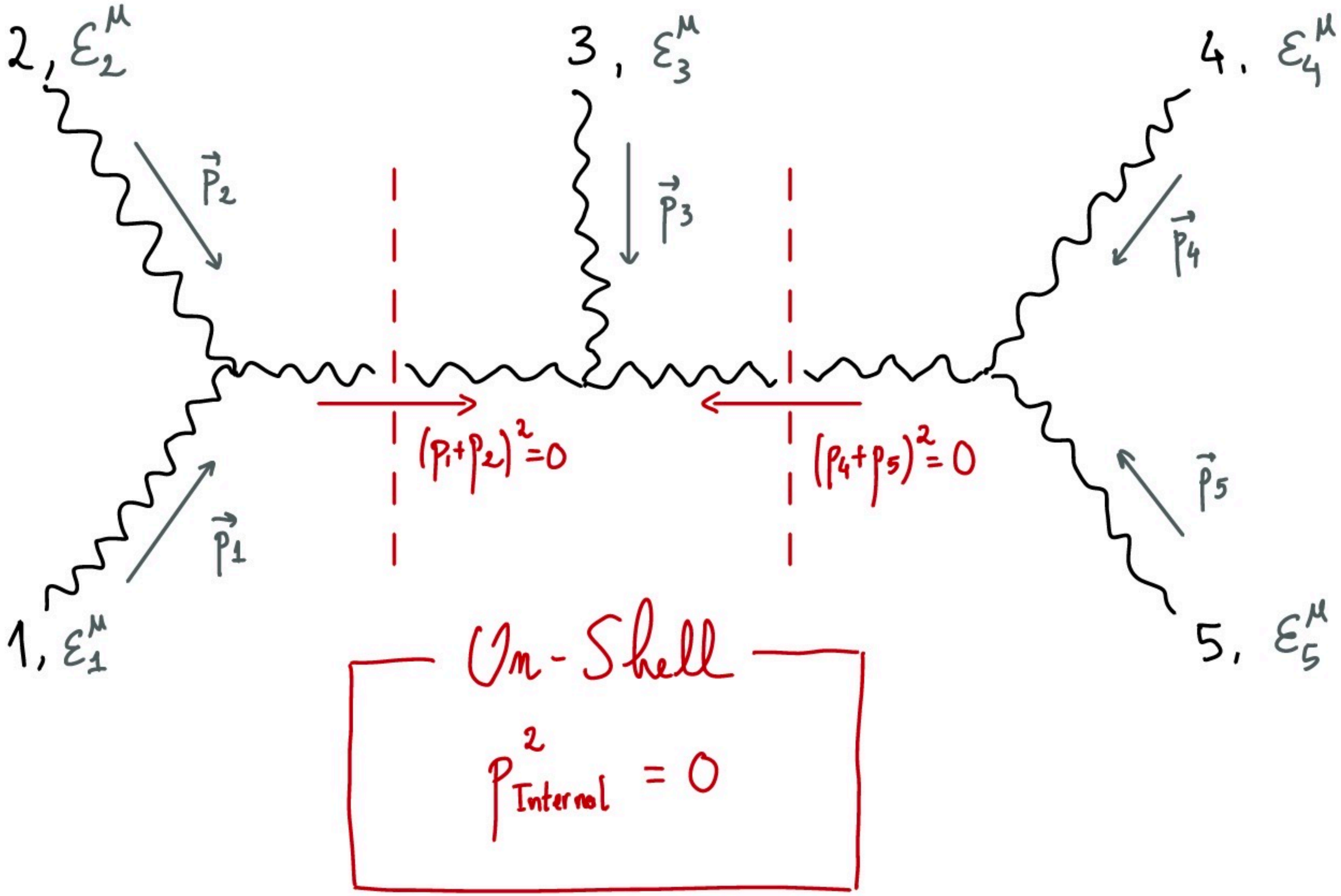
and Song He.

PITP

What are leading singularities?

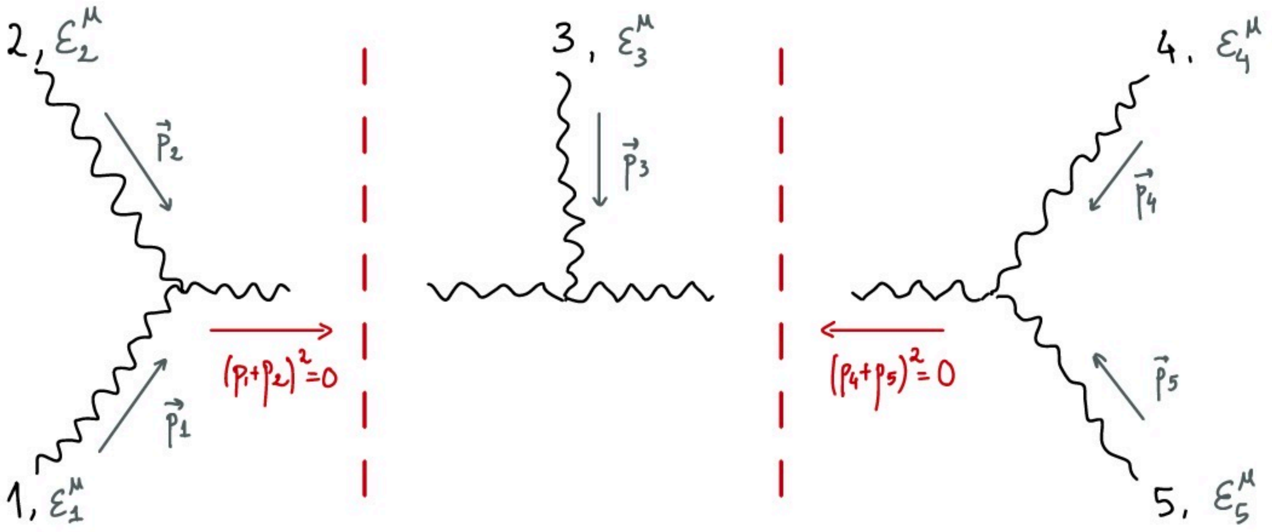


What are leading singularities?



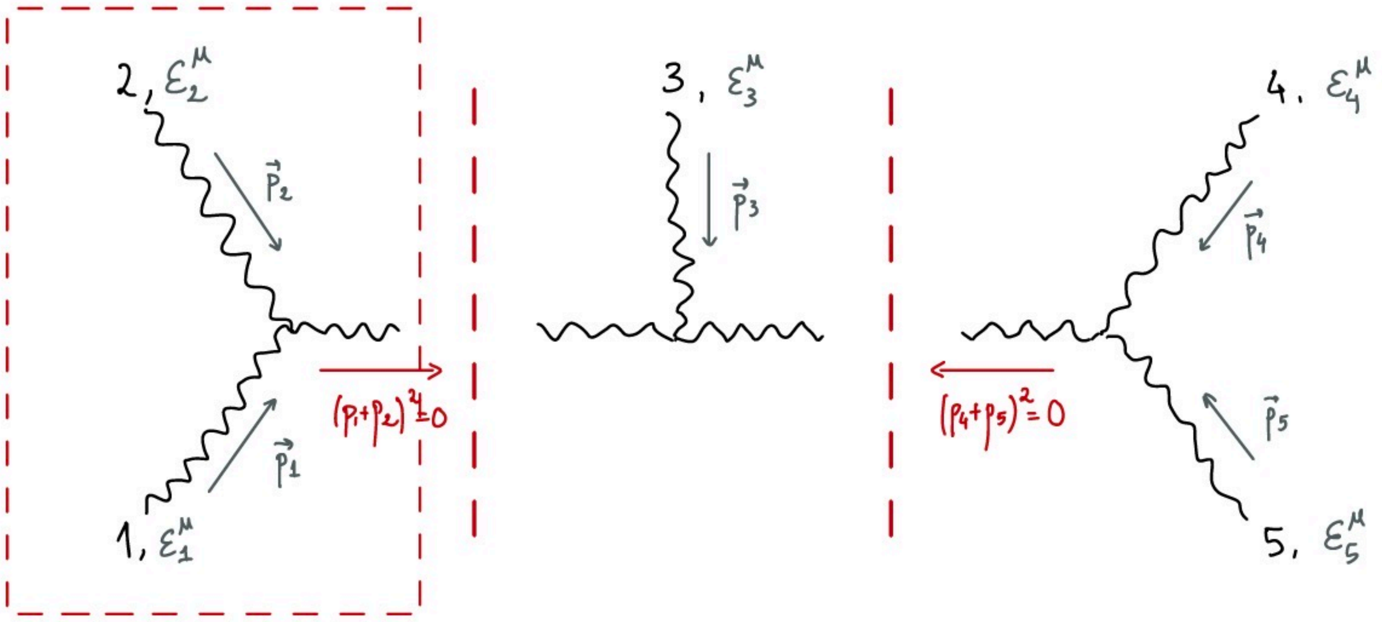
How to compute them?

FACTORIZATION:

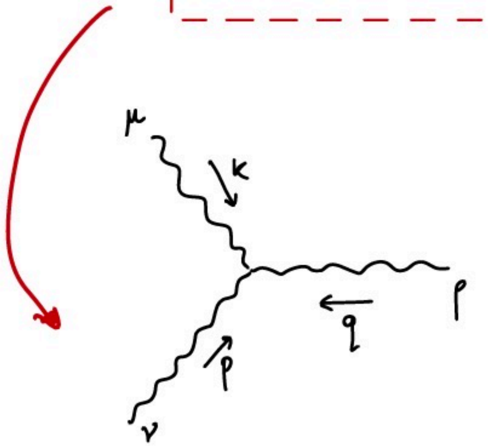


How to compute them?

FACTORIZATION:

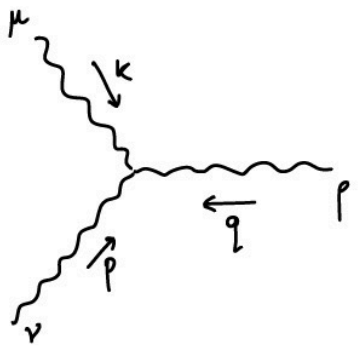


3-point vertex



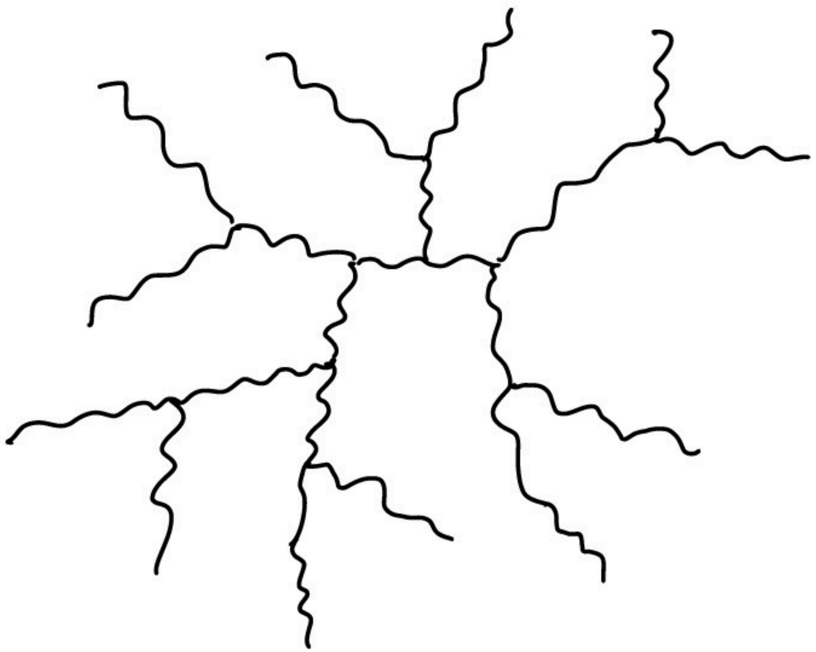
$$\propto g_{YM}\left[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu \right]$$

3-point vertex

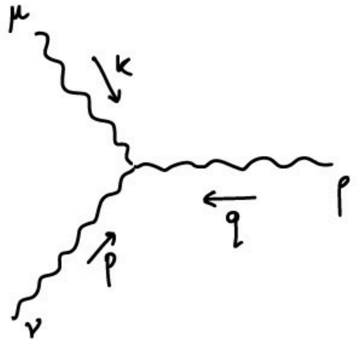


$$\propto g_{YM}\left[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{\rho\mu}(q-k)^{\nu} \right]$$

Increasing number of external gluons :

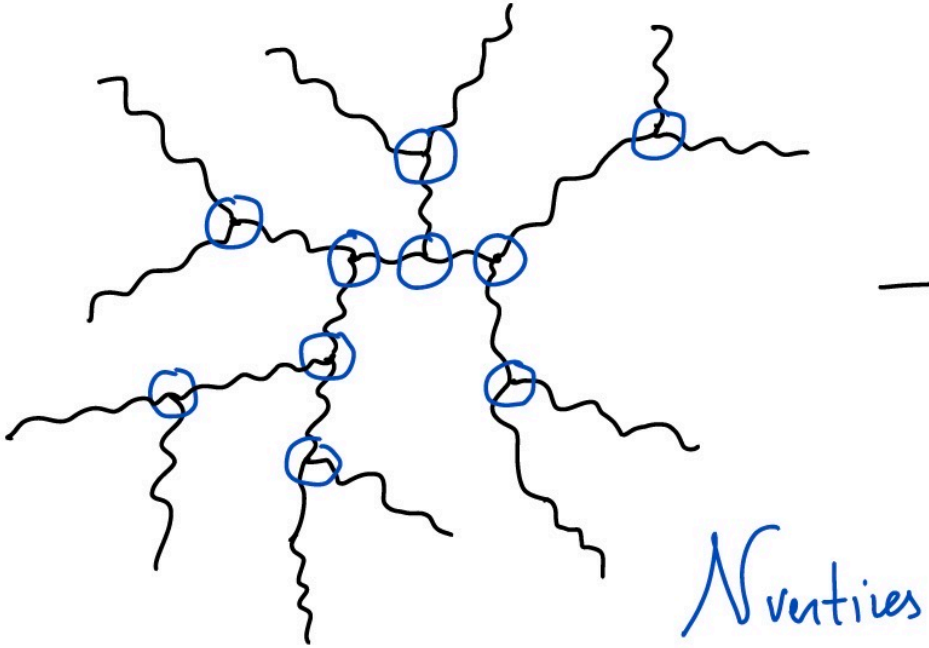


3-point vertex



$$\propto g_{YM}\left[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{\rho\mu}(q-k)^{\nu} \right]$$

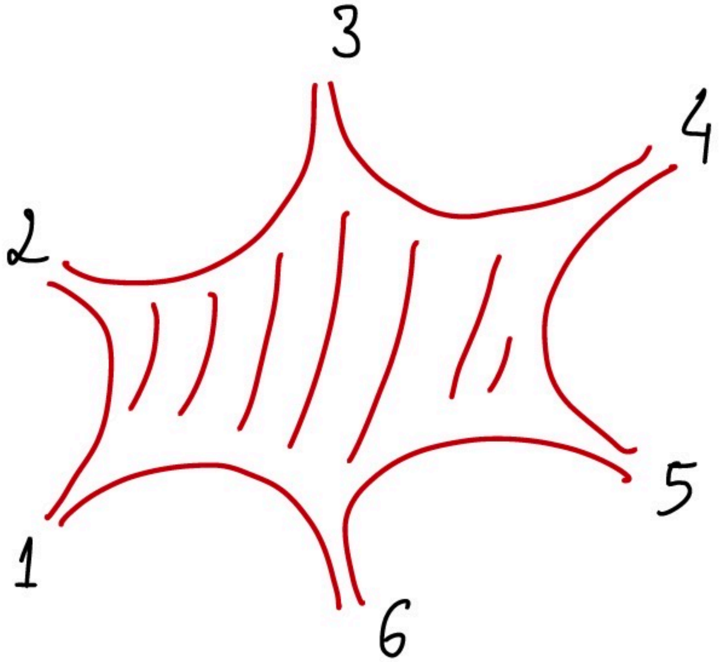
Increasing number of external gluons:



$$\longrightarrow 3^{N_{\text{vertices}}}$$

What else can we do?

Bosonic String.

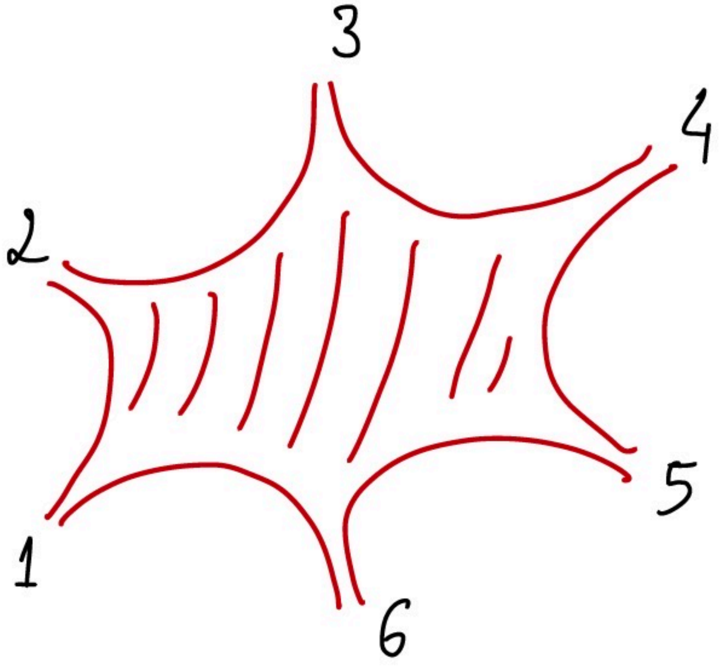


[color-ordered amplitudes on disk]

$$A_6^{\text{tree}} = \int_{\mathcal{M}} \Omega$$

What else can we do?

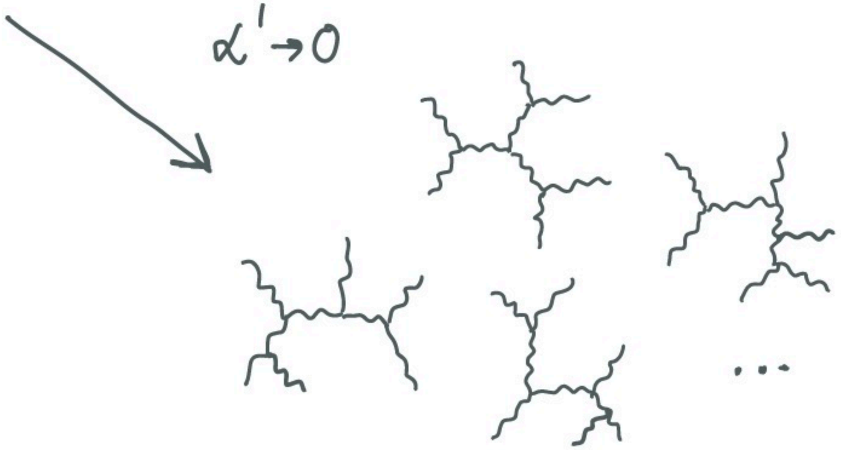
Bosonic String.



[color-ordered amplitudes on disk]

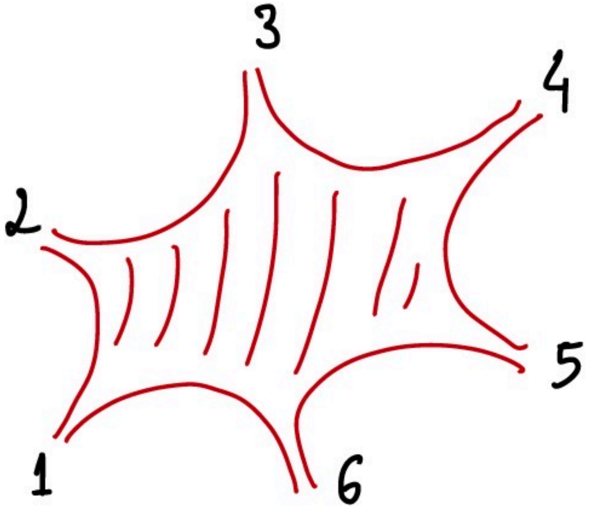
$$A_6^{\text{tree}}$$

$\alpha' \to 0$

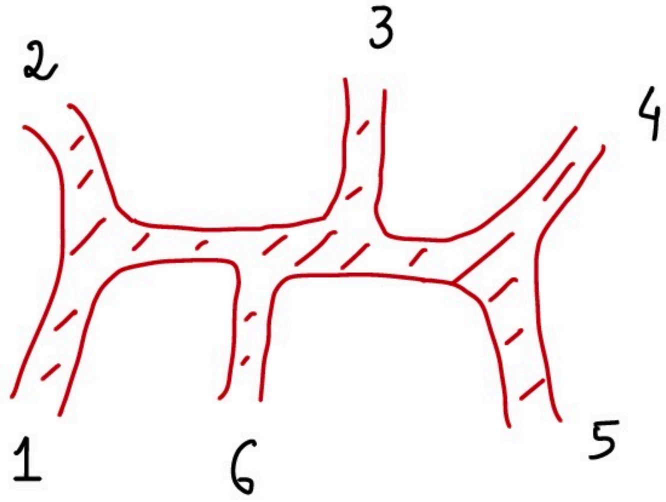


What else can we do?

Bosonic String.



→ Residues



$$A_6^{tree} = \int_{\mathcal{R}} \Omega$$

Leading Singularity

$$\text{Res}_{z_{ij}\to 0}\, \Omega$$

What else can we do?

Bosonic String.

$$A_n^{\text{tree}}(1,2,\ldots,n) = \int \frac{dz_1 \ldots dz_n}{SL(2,\mathbb{R})} \times \prod_{i<j} z_{ij}^{2\alpha' p_i \cdot p_j} \exp\left\{ -\sum_{i \neq j} \left(\frac{\sqrt{\alpha'}\, \varepsilon_i \cdot p_j}{z_{ij}} - 2\, \frac{\varepsilon_i \cdot \varepsilon_j}{z_{ij}^2} \right) \right\} \Bigg|_{\text{multilinear in } \varepsilon_i}$$

w/ $z_{ij} = z_i - z_j$

What else can we do?

Bosonic String.

$$A_n^{\text{tree}}(1,2,\dots,n) = \int \frac{dz_1 \dots dz_n}{SL(2,\mathbb{R})} \times \prod_{i<j} z_{ij}^{2\alpha' p_i \cdot p_j} \exp\left\{-\sum_{i\neq j}\left(\frac{\sqrt{\alpha'}\,\varepsilon_i \cdot p_j}{z_{ij}} - 2\,\frac{\varepsilon_i \cdot \varepsilon_j}{z_{ij}^2}\right)\right\}\Bigg|_{\text{multilinear in } \varepsilon_i}$$

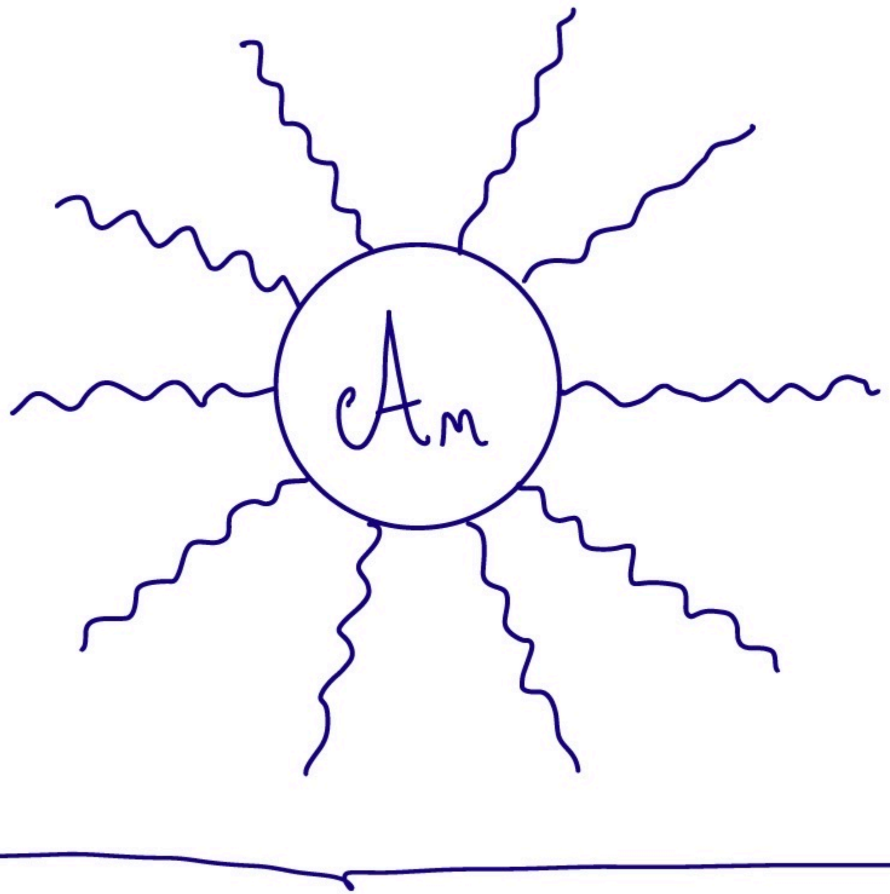
sum of lots of terms!

↓

Ex.: A_6^{tree}, $\sim 10^3$ terms!

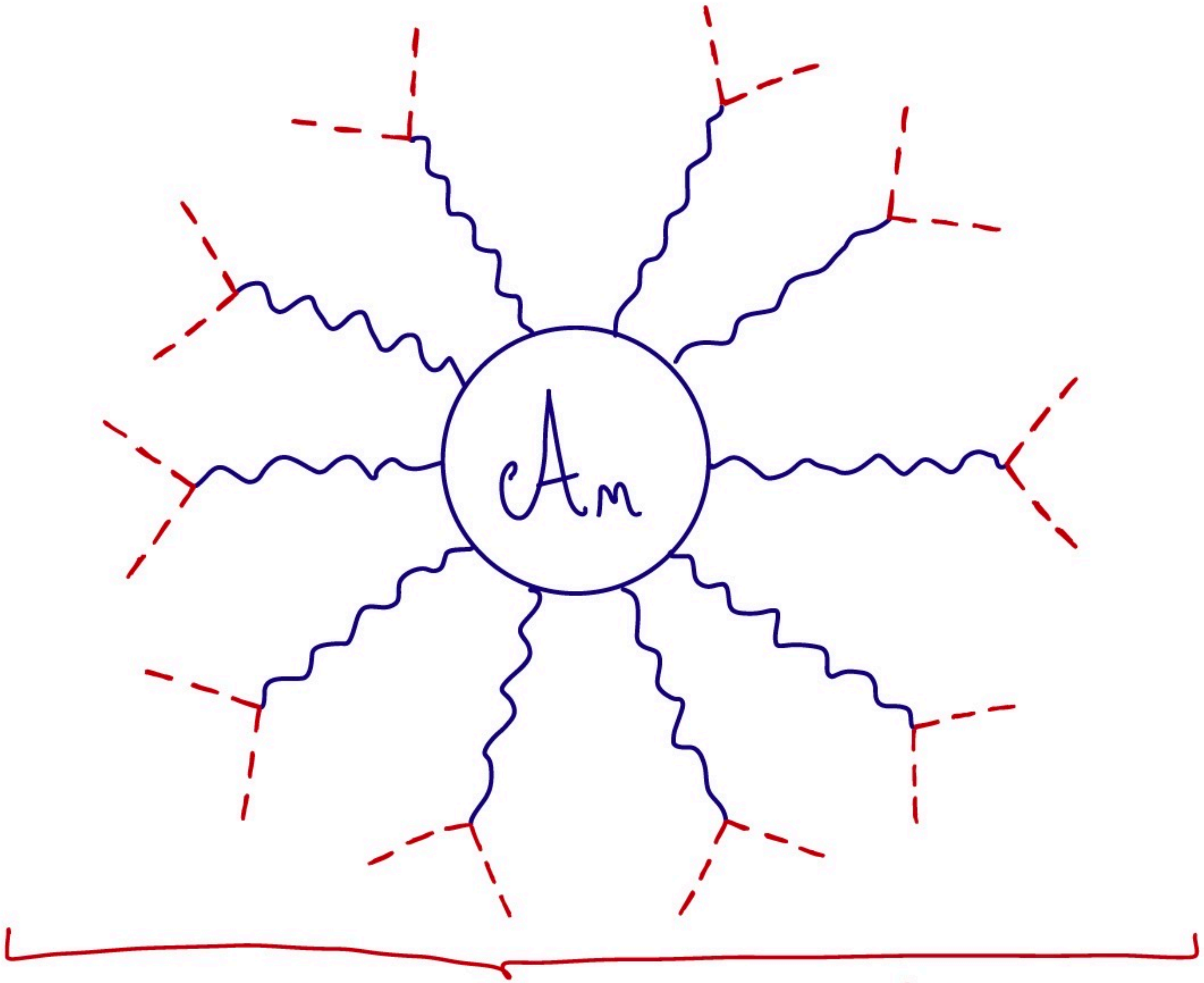
w/ $z_{ij} = z_i - z_j$

Bosonic String.



n-point Gluon amplitude

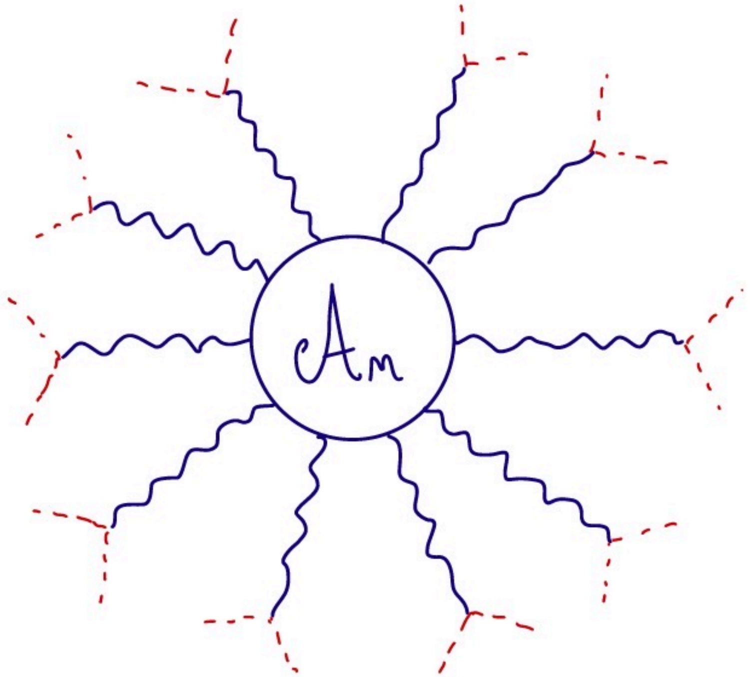
Trick: Gluons coming from scalars



2n Scalar Amplitude

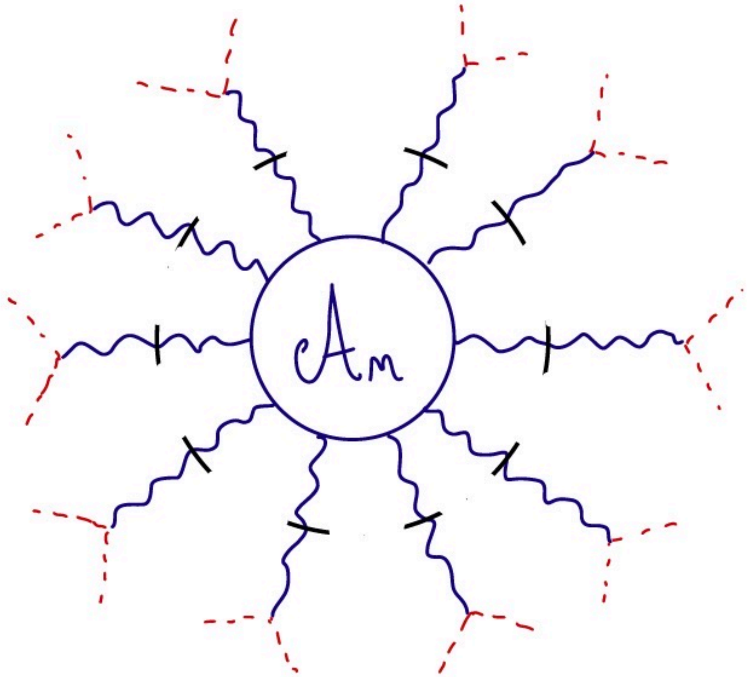
Scattering of 2n-Scalars

$$A^{tree}_{2n\ \text{scalars}}(1,2,\dots,\underline{\underline{2n}}) \propto \int \frac{dz_1 \dots dz_{2n}}{SL(2,\mathbb{R})} \times \prod_{i<j} z_{ij}^{2\alpha' p_i \cdot p_j} \times \underbrace{\frac{1}{z_{12}^2\, z_{34}^2 \cdots z_{2n-1,2n}^2}}_{\text{SINGLE TERM}}$$



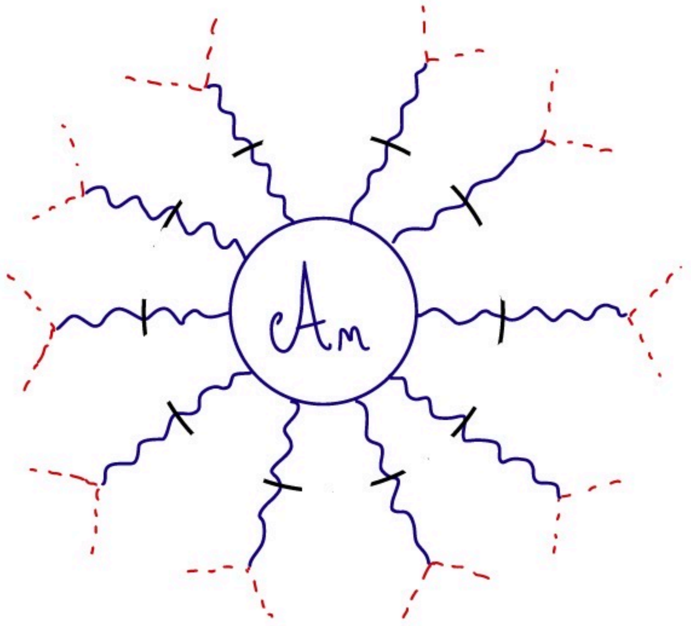
Scattering of 2n-Scalars

$$A^{\text{tree}}_{2n \text{ scalars}}(1,2,\dots,\underline{2n}) \propto \int \frac{dz_1 \dots dz_{2n}}{SL(2,\mathbb{R})} \times \prod_{i<j} z_{ij}^{2\alpha' p_i \cdot p_j} \times \underbrace{\frac{1}{z_{12}^2\, z_{34}^2 \cdots z_{2n-1,2n}^2}}_{\text{SINGLE TERM}}$$



↓ Residues on external gluons

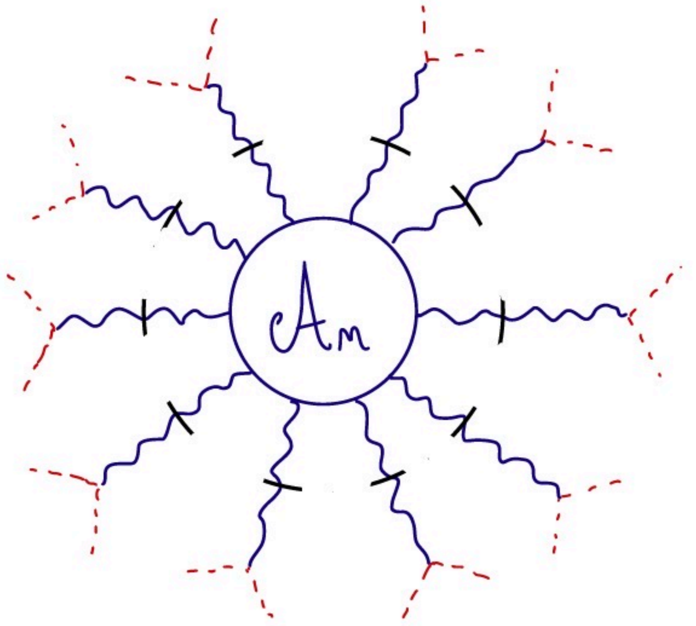
$$A^{\text{tree}}_{\text{gluons}}(1,\dots,n)\,[p_i \cdot p_j]$$



n point gluon amplitude

$$A^{\text{tree}}_{\text{gluons}}(1, \dots, n)$$

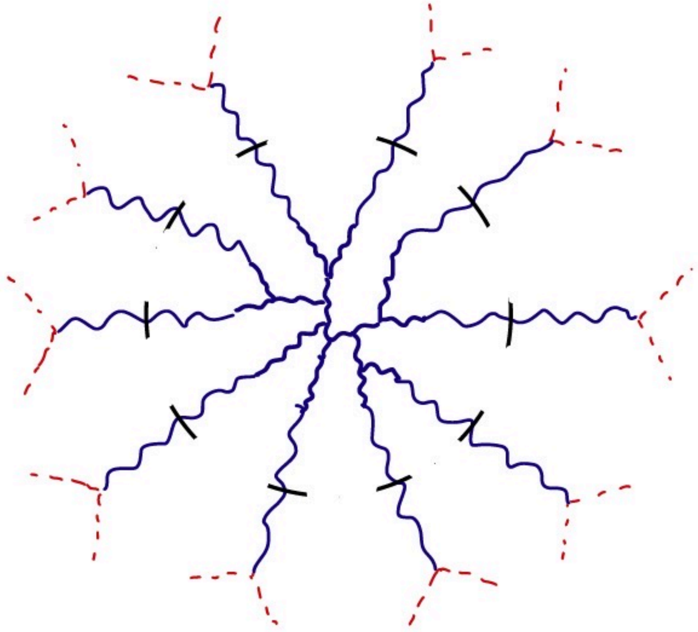
$\downarrow$ Residue on internal gluons



n point gluon amplitude

$$A^{tree}_{gluons}(1,\dots,n)$$

↓ Residue on internal gluons



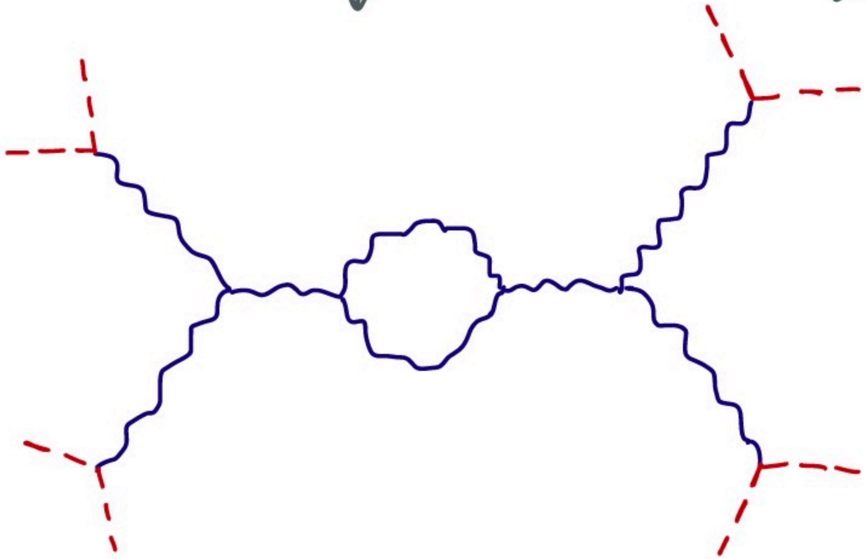
n point gluon Leading Singularity

Loop level

2n-scalar form for loop diagrams [SINGLE TERM!]

↓ Residues

[≠ Bosonic string loop amplitudes]



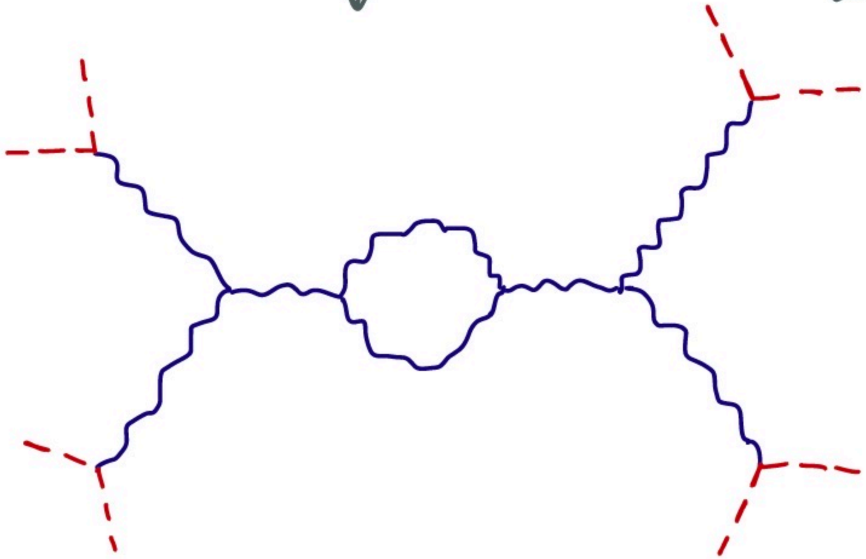
→ Gluon loop leading singularities!

Loop level

2n-scalar form for loop diagrams [SINGLE TERM!]

↓ Residues

[≠ Bosonic string loop amplitudes]



→ Gluon loop leading singularities!

Thank You!

Confinement and Chiral Symmetry Breaking in AMSB QCD

Andrew Gomes
Cornell University

ArXiv: 2106.10288, 2107.02813, 2212.03260

With Csaba Csáki, Hitoshi Murayama, Ofri Telem, Bea Noether, and Digvijay Roy-Varier

The goal

- NON-supersymmetric SU & SO gauge theories with fundamentals
- IR phase? Chiral symmetry breaking (χSB), quark confinement?
  - This "0^{th} order" description is our aim

- Will achieve by augmenting the symmetry then removing it

A supersymmetric approach

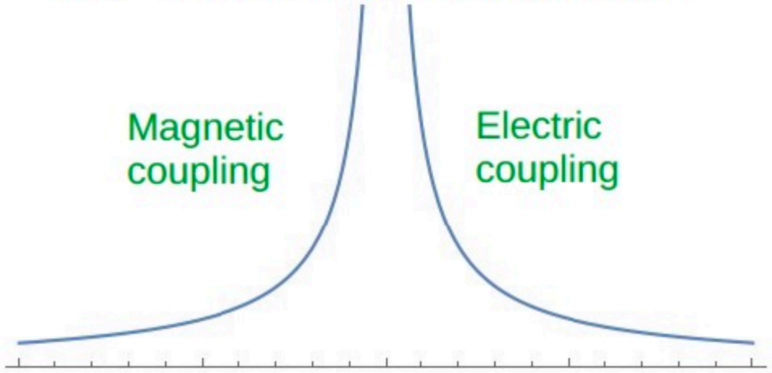
- Augment QCD with gluinos (fermions) and squarks (bosons)
- IR hadrons are polynomials in the quark superfields
  - $M = Q_L Q_R$ and $B_{L/R} = Q_{L/R} \ldots Q_{L/R}$

Superfields! Contain boson and fermion components

- And enough information about potential to fix many exotic IR phases!

- SUSY just a theoretical tool here!

e.g. “Free magnetic” phase:

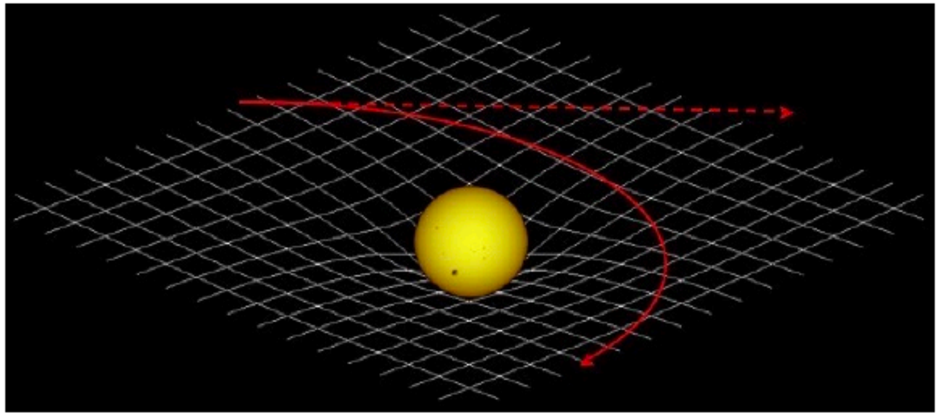


IR: $SU(N_f\text{-}N_c)$ UV: $SU(N_c)$

“Seiberg duality”

The road to recovery

- SQCD interesting in its own right, but want real-world QCD → just give masses to superpartners, “theory deformation”
- Deformation must be identical in UV and IR descriptions
- Need something universal… Gravity!
  - Think about Einstein’s equivalence principle ($m_{grav} = m_{inertial}$)



Anomaly-mediated supersymmetry breaking

Randall, Sundrum 9810155
Giudice, Luty, Murayama, Rattazzi 9810442
Arkani-Hamed, Rattazzi 9804068
Katz, Shadmi, Shirman 9906296
Pomarol, Rattazzi 9903448

- In UV, only have loop effects:

Effects in SU:

$$m_Q^2 = m_{\tilde{Q}}^2 = \frac{g^4}{(8\pi^2)^2} 2C_i(3N_c - N_f)m^2,$$

$$m_\lambda = \frac{g^2}{16\pi^2}(3N_c - N_f)m.$$

Pushes scalars to origin – incalculable in THIS description

- Exactly the theory we want when **m** → ∞ (for asymptotically free theories where $3N_c > N_f$)!
- But will always work with **m** < Λ to maintain perturbative control
- Now into the IR…

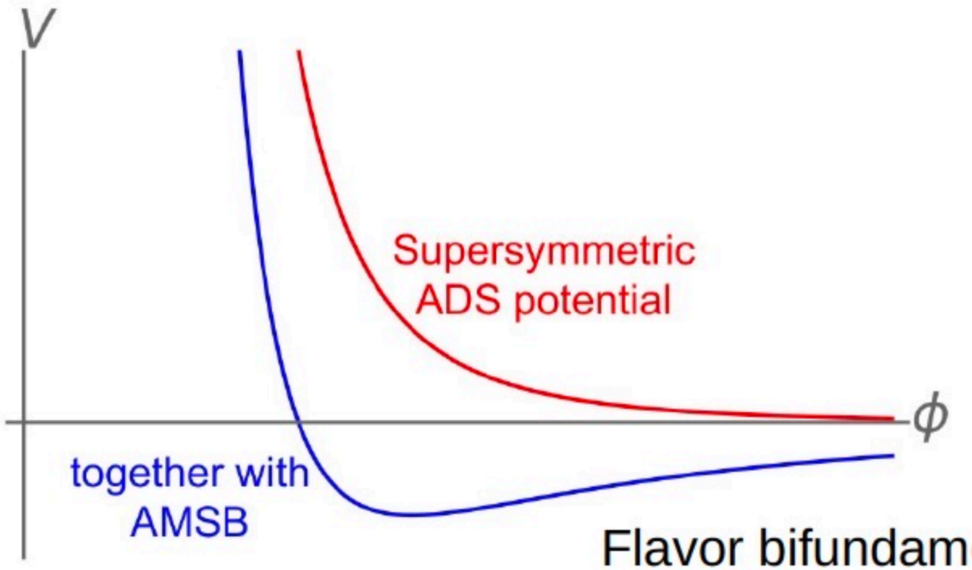
A clean example (SU)

Murayama 2104.01179

- For $N_c > N_f$, have Affleck-Dine-Seiberg (ADS) superpotential

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\det M}\right)^{1/(N_c-N_f)}$$

Very different from non-SUSY QCD!



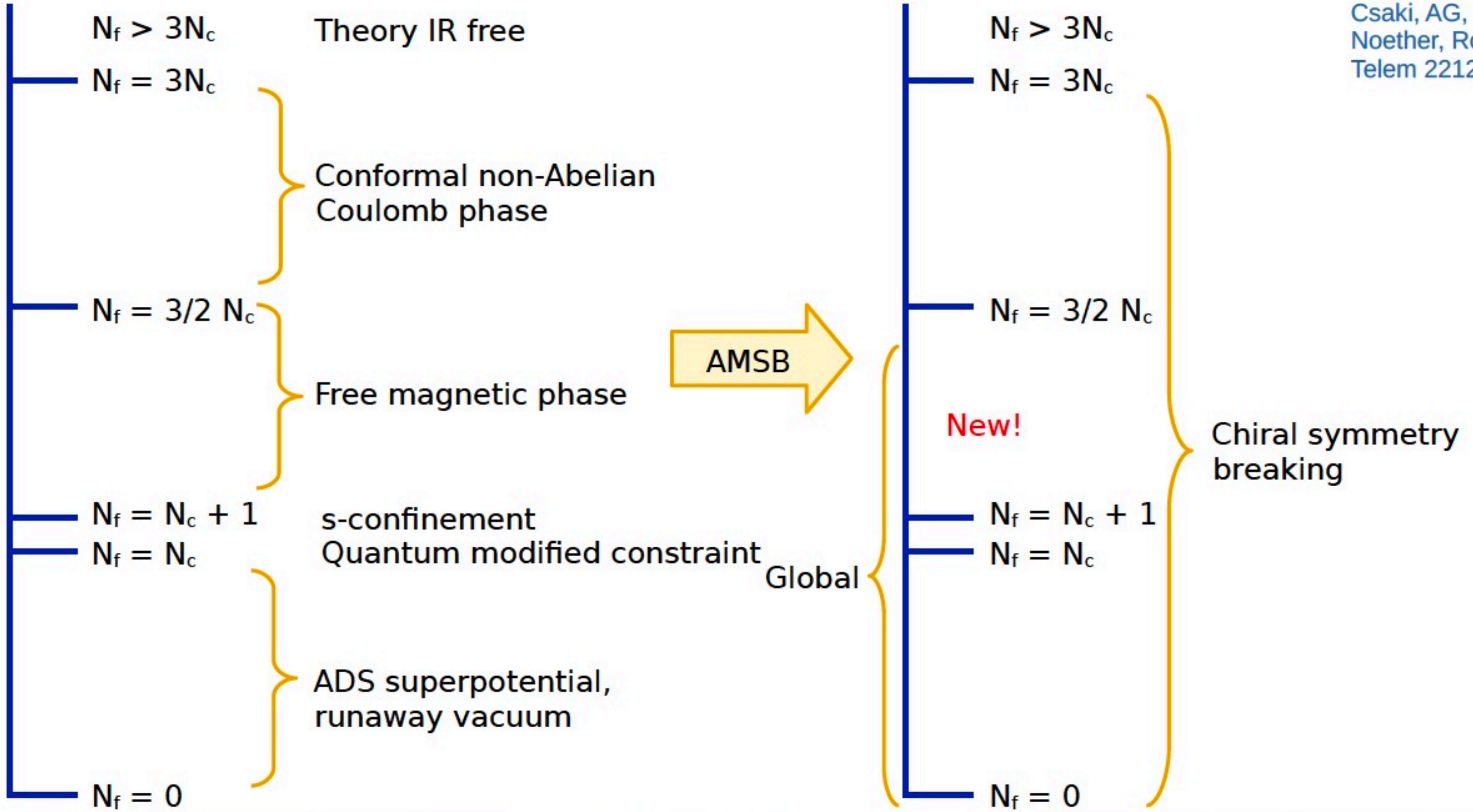
$$Q = \tilde{Q} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \\ \hline 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \phi, \qquad M = \phi^2$$

$$V = \left| 2N_f \frac{1}{\phi}\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)}\right|^2 - (3N_c - N_f)m\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)} + c.c.$$

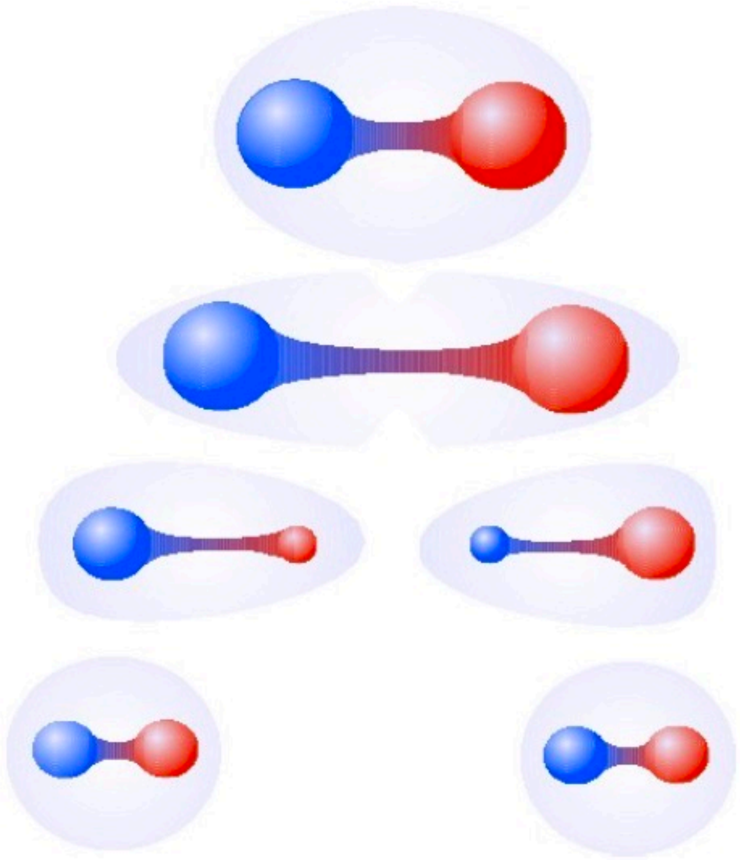
Flavor bifundamental gets a VEV → Chiral symmetry breaking!

Phase collapse

Csaki, AG, Murayama, Noether, Roy-Varier, Telem 2212.03260



The confinement story (SO)



- Why didn't we talk about confinement?
  - Linear potential only until pair production
  - Cannot distinguish from Higgs phase

- Consider $SO(N_c)$ gauge theory with N_f quarks in the vector representation
  - Also has spinor rep (e.g. spin-1/2 of $N_c = 3$)
  - Flux tube in this rep cannot be broken
  - We have a chance here!

The setup

- Consider the specific case $N_f = N_c - 2$ (N=1 family of SW analogs)
- SUSY theory has dual description (Seiberg dual) in terms of a U(1) gauge theory

$$W_{\rm mon} \;=\; \Lambda\left(\frac{\tilde{U}}{\Lambda^{N_F}} - 16\right)\tilde{E}^{+}\,\tilde{E}^{-} \qquad\qquad \tilde{U} \;=\; \det\tilde{M}$$

- Familiar meson M, and U(1) electric charges $E^{\pm}$
  - Though not a true electric/magnetic duality, can think of as magnetic monopoles of original theory

A twin condensation

Csaki, AG, Murayama, Telem 2106.10288, 2107.02813

- As before, add AMSB:

k = O(1)

$$\tilde{M} = 16^{\frac{1}{N_F}} \Lambda \quad , \quad |\tilde{E}^+||\tilde{E}^-| = 16^{\frac{2}{N_F}-1} km\Lambda$$

Usual meson condensate → Chiral symmetry breaking!

Monopole condensate → Confinement!

- First demonstration of confinement and continuous chiral symmetry breaking in a non-supersymmetric gauge theory
- Can extend results to $0 < N_f < 3(N_c - 2)$
  - Same phase collapse seen in SU

Towards a string theory for 2d Yang Mills

Tal Sheaffer

Weizmann Institute of Science

13/07/23

Ongoing work with Ofer Aharony and Suman Kundu (WIS)

Gross-Taylor Series

- 2d YM exactly solvable (Kazakov & Kostov (1980), Rusakov (1990), Witten (1991) & more)
- Exact large N expansion of observables Z, $\langle \mathrm{WL} \rangle$ (Gross-Taylor 1993)
- Organizes into **sums of worldsheet maps**
- **No worldsheet CFT / action**
- Only **particular** maps contribute (supersymmetric localization?)

Worldsheet proposals with supersymmetric localiztation

- **Topological** string theory for 0-coupling YM.
- Localization to **holomorphic** maps Cordes, Moore, Ramgoolam (1994)
  - Only "chiral" contributions
- Localization to **extremal area** maps (soap films!) Hořava (1996)
  - Solutions of **Nambu-Goto** EOM
  - Includes "non-chiral" maps
- vague proposals for finite 't Hooft coupling λ

The devil's in the details

- A term in Hořava's action **vanishes identically** $\Rightarrow$ ill defined moduli-space integral.
  - We found a gauge-invariant non-vanishing replacement $\Rightarrow$ correct measure on moduli space for $\lambda = 0$ (topological Yang Mills).
- Gauge fixing and path integration in Hořava's Nambu-Goto-type theory ill-behaved.
  - We reformulated as a Polyakov-type path-integral
  - Naturally regulates some (hopefully all) the ill-behaved non-chiral maps.
- What about Wilson-loops?
  - We adapted the theory to worldsheets with boundaries.

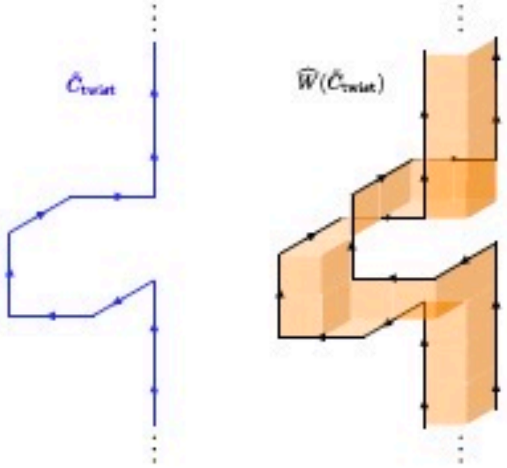
Down the road

- Obtain the correct measure on the **boundary of moduli space**, for maps without boundaries.
- Compute more amplitudes.
- Generalize to nonzero 't Hooft coupling λ - too early for optimism.
- Stringy derivation of 't Hooft spectrum of mesons
- More speculative:
  - Adjoint particles as weights on the string
  - Higher D

Thanks!

Applications of the modified Villain formulation

Theo Jacobson, University of Minnesota



Based on 2303.06160 with Tin Sulejmanpasic

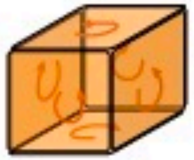
and work in progress

The modified Villain formulation

Remove lattice-scale topological defects (vortices, monopoles)
to endow lattice field theories with features of their continuum limits:

Global symmetries | 't Hooft anomalies | Dualities

Villain: $a_\ell \in \mathbb{R},\ n_p \in \mathbb{Z}, \quad a_\ell \to a_\ell + 2\pi\, m_\ell,\ \ n_p \to n_p + (dm)_p$

monopole:  $(dn)_c \neq 0$

Modified Villain: constrain $(dn)_c = 0$ (Gross, Klebanov '90; Gattringer, Sulejmanpasic '19; Gorantla, Lam, Seiberg, Shao '21)

Abelian Chern-Simons theory

Goal: discretize $U(1)_k$ CS theory on a Euclidean spacetime lattice, with:

- large gauge invariance + level quantization
- $\mathbb{Z}_k$ 1-form symmetry + 't Hooft anomaly
- electric charge of monopoles
- framing

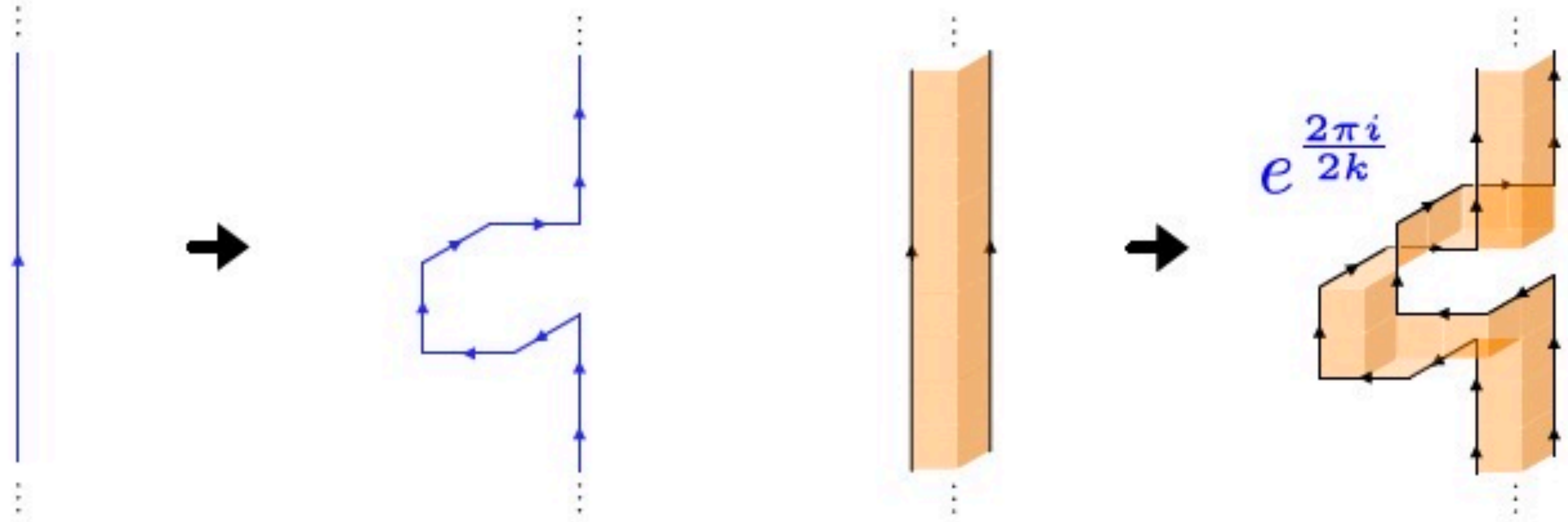
Application: establish boson/fermion dualities nonperturbatively

- extend exact lattice particle-vortex duality

Framing and spin of Wilson lines

Minimal charge Wilson loops are **ribbons:** edges connected by a surface

(ordinary Wilson loops are projected out by a peculiar gauge redundancy)



Compute topological spin $s_q = \frac{q^2}{2k}$ via self-linking

Axion-Maxwell theory

Continuum action: $\mathcal{L} = \frac{iK}{8\pi^2}\,\theta\, F \wedge F, \quad K \in \mathbb{Z}$

Symmetries:

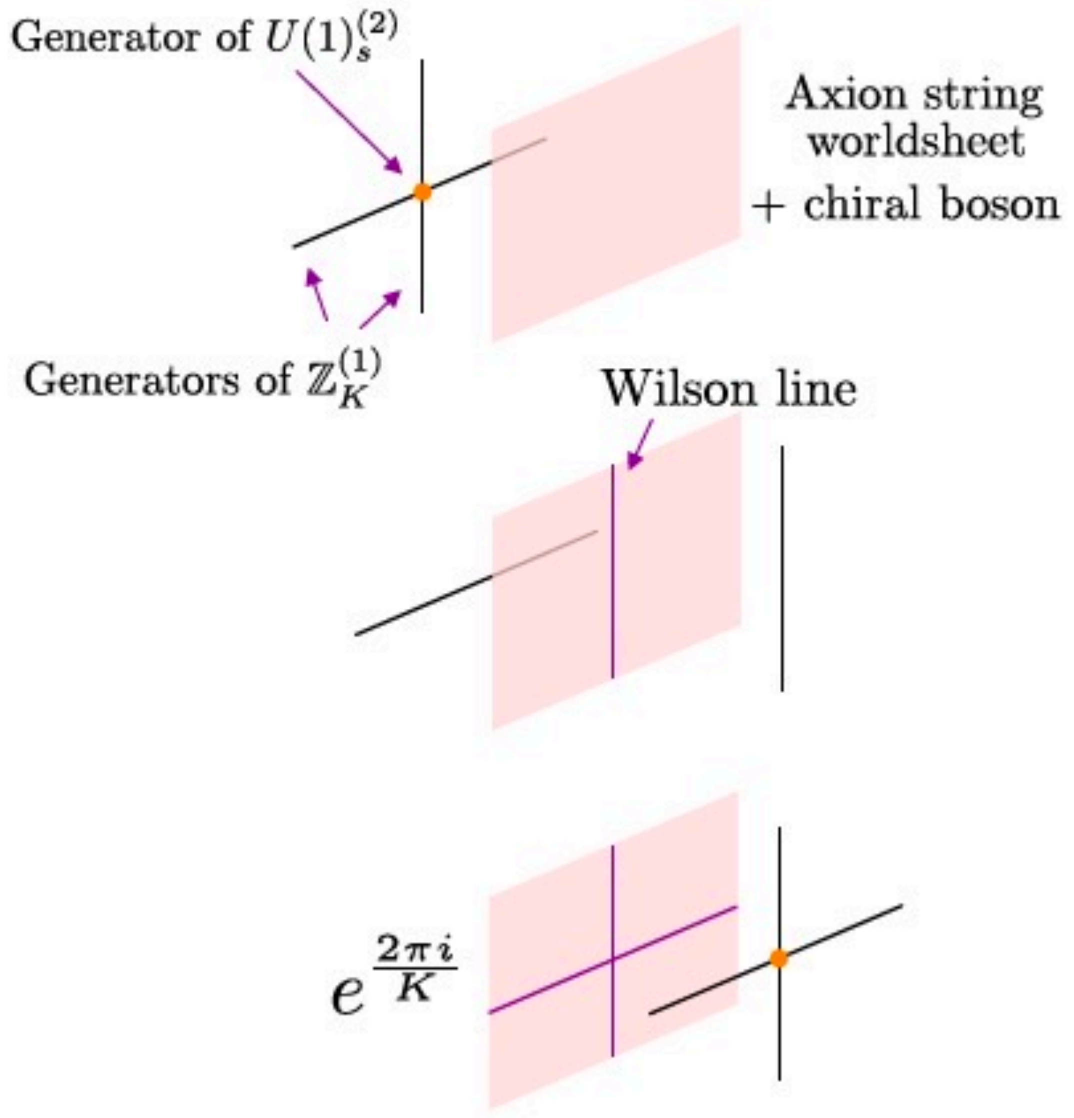
- $\mathbb{Z}_K^{(0)}$ axion shift symmetry
- $\mathbb{Z}_K^{(1)}$ acts on Wilson loops
- $U(1)_m^{(1)}$ acts on 't Hooft loops
- $U(1)_s^{(2)}$ acts on axion strings
- $\cdots$

Higher group structure:

Tanizaki, Ünsal '19
Hidaka, Nitta, Yokokura '20
Brennan, Córdova '20
Choi, Lam, Shao '22

Goal: preserve and study these symmetries on the lattice

Higher-group symmetry and axion strings



Holomorphic Confinement of $\mathcal{N}=1$ SYM

PiTP 2023

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Kasia Budzik, Davide Gaiotto, Brian Williams, Jingxiang Wu, Matt Yu.

Perimeter Institute for Theoretical Physics

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arXiv:2306.01039

Holomorphic Twists and Infinite Symmetries

- Given any SQFT, we obtain the **Holomorphic Twist** by taking cohomology of any nilpotent supercharge, e.g. $Q := Q_-$.
  - Anti-holomorphic translations are Q-exact, so twisted theory is (cohomologically) holomorphic [Johansen], [Witten], [Costello]

$$\{Q, \bar{Q}_{\dot{\alpha}}\} = \partial_{\bar{z}^{\dot{\alpha}}} \tag{1}$$

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- Protected subsector of **semi-chiral operators**, i.e. $[Q, \mathcal{O}] = 0$
  - In SCFTs: includes those counted by **superconformal index**

$$\mathcal{I} = \mathrm{Tr}(-1)^F p^{j_1+j_2-r/2} q^{j_1-j_2-r/2} e^{-\beta\{Q_-, S^-\}} \tag{2}$$

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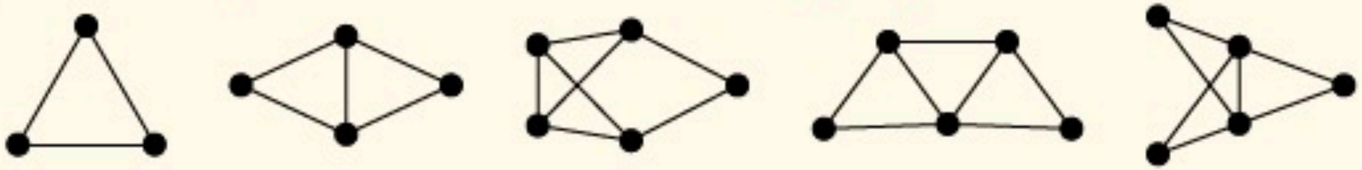
- Theories are equipped with local product called **λ-Bracket**

$$\{\mathcal{O}_1, \mathcal{O}_2\}_\lambda = \oint_{S^3} e^{\lambda\cdot z} d^2z\ \mathcal{O}_1(z)\, \mathcal{O}_2(0) \tag{3}$$

  - **Infinite dimensional symmetry enhancements** analogous to Virasoro and Kac-Moody, but in 4d [Gwilliam, Williams].
  - Higher brackets $\{\mathcal{O}_1, \mathcal{O}_2, \ldots, \mathcal{O}_{n+1}\}_{\lambda_1,\lambda_2,\ldots,\lambda_n}$

Cohomologies and Feynman Diagrams

- Feynman diagrams must be **Laman graphs**.



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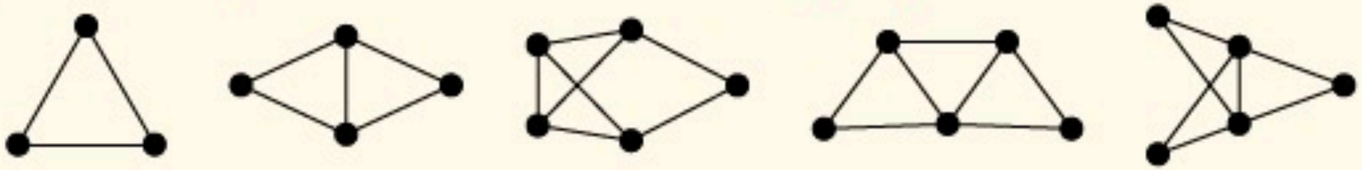

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  - Recursive identities of the operatope used to **bootstrap Feynman integrals** to at least 3-loops (perhaps further!)
- Polynomials in fields and derivatives $\rightsquigarrow$ **Free Cohomology** $\mathcal{V}$
  - **Interacting quantum theory** is obtained from underlying free-classical theory $\mathcal{V}$ as cohomology of a new operator

$$\mathbf{Q} = Q_0 + Q_1 + Q_2 \ldots . \tag{4}$$

    where Q_n is computed by n-loop Feynman diagrams.
  - All perturbative corrections are contained in the brackets!

$$\mathbf{Q}\,\mathcal{O} = \{\mathcal{I}, \mathcal{O}\}_0 + \{\mathcal{I}, \mathcal{I}, \mathcal{O}\}_{0,0} + \{\mathcal{I}, \mathcal{I}, \mathcal{I}, \mathcal{O}\}_{0,0,0} + \ldots . \tag{5}$$

Holomorphic Confinement of $\mathcal{N} = 1$ SYM

- $\mathcal{N} = 1$ **SYM** is $SU(N)$ gauge theory with an adjoint fermion

$$\mathcal{L} = \int d^2\theta \frac{-i}{8\pi} \tau \operatorname{tr} W_\alpha W^\alpha + \text{c.c.} \tag{6}$$

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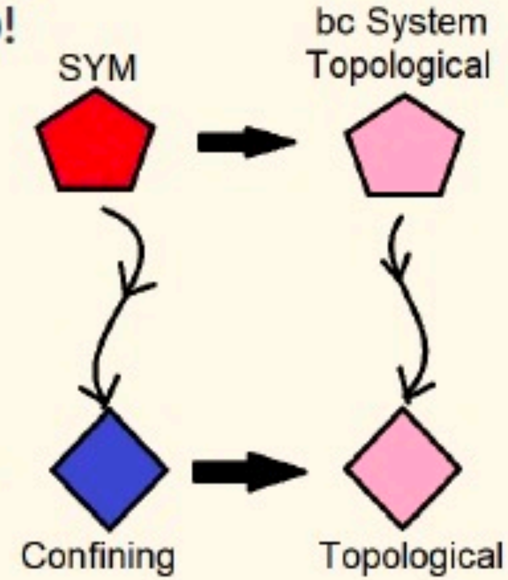
- Being topological is compatible with **confinement**: if topological in the UV, then topological in the IR.
  - Constrains IR physics: the holomorphic twist of the IR must also be topological.



Trace relations and open string vacua

Ji Hoon Lee

2109.02545 D. Gaiotto, JHL
2204.09286 JHL
Work to appear

A recurring theme in our study of holography is the idea that the Feynman diagrams of a large N gauge theory reorganize into a genus expansion of *some* string theory.

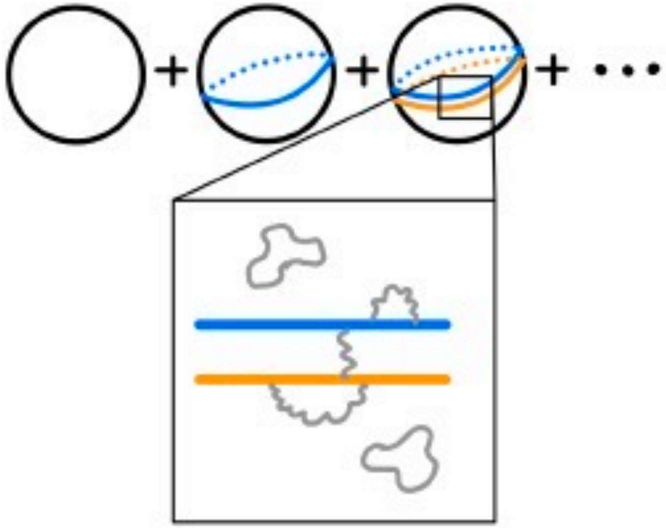
It is natural to ask to what extent the notion of gauge-string duality persists at finite N.

I will explain an intriguing pattern, called the giant graviton expansion, that appears in the spectrum of states of finite N gauge theories on $S^1 \times S^3$ at zero coupling $\lambda = 0$.

Namely, the finite N spectrum of a $U(N)$ gauge theory reorganizes into a systematic set of e^{-kN} corrections to the large N spectrum:

$$Z_{U(N)} = Z_{U(\infty)} \sum_{k=0}^{\infty} (-1)^k x^{kN} \hat{Z}_{U(k)}.$$

These corrections have transparent holographic interpretations in the bulk — they are contributions from k giant graviton branes and open/closed string excitations thereof.



The giant graviton expansion is a formula that seems to suggest rules in the bulk that we did not know before.

$$Z_{U(N)} = Z_{U(\infty)} \sum_{k=0}^{\infty} (-1)^k x^{kN} \hat{Z}_{U(k)}.$$

For example, why is the full bulk partition function in the $\alpha' \to \infty$ limit given by a sum over brane sectors?

Also, why does the sum over brane sectors exhibit huge cancellations to give $Z_{U(N)}$, even when we are computing the free partition function rather than the index?

Recall that giant gravitons were originally discovered to provide a bulk explanation for the presence of finite N trace relations. Thus, let us answer these bulk questions by revisiting how one implements trace relations in a $U(N)$ gauge theory.

The Hilbert space of a free finite N gauge theory is given by the highly-constrained quotient module

$$M_N = M_\infty / \langle \text{ trace relations at } N \rangle$$

of the infinite N Hilbert space M_∞ modulo the ideal of trace relations, over the ring

$$R = \mathbb{C}[\mathrm{Tr}\, X, \mathrm{Tr}\, XY, \mathrm{Tr}\, \psi F_{...}, \mathrm{Tr}\, \partial_{...}\partial_{...}X, \cdots]$$

of all gauge invariant polynomials of fields and their derivatives at infinite N.

A free resolution of M_N

$$\cdots \to V_3 \xrightarrow{\hat{Q}} V_2 \xrightarrow{\hat{Q}} V_1 \xrightarrow{\hat{Q}} M_\infty \to M_N \to 0$$

replaces M_N with an infinite exact sequence of free modules V_k with differential $\hat{Q}$.

V_k is the space of k-th order relations among the generators of M_∞, with the $k = 1$ case V_1 being the space of trace relations.

We can view the free resolution of M_N as the procedure of introducing "heavy" ghosts for a $U(\infty)$ theory that compensate for null states due to trace relations at some N.

I would now like to argue the following (modulo instantons):

$$V_k \simeq \mathcal{H}_\infty^{\text{closed}} \otimes \mathcal{H}_k^{\text{open}}$$

The free module V_k of k-th order trace relations in a $U(N)$ gauge theory should be interpreted in the string dual as the $\alpha' \to \infty$ limit of the space of open string states on k giant graviton branes sharing a closed-string background.

(This is based on matches of the hilbert series of V_k with the k-th term in the giant graviton expansion, for $\frac{1}{2}$ and $\frac{1}{4}$-BPS sectors of $\mathcal{N} = 4$ SYM.)

The giant graviton expansion

$$Z_{U(N)} = Z_{U(\infty)} \sum_{k=0}^{\infty} (-1)^k x^{kN} \hat{Z}_{U(k)}.$$

is then the refined Euler characteristic associated to a free resolution graded by the "heavy ghost" number.

Thank you