

PCMI 2023 XML: MEETING 4

ALBERT ARTILES

PROBLEMS

1. Take a triangle T . A billiard trajectory on T is the following: Pick a point inside T and a direction. Move in a straight line in the direction chosen until you hit a wall. If you hit a wall you will bounce in such a way that the angle of incidence equal the angle of reflection. Then keep going and repeating this pattern. Can you find a periodic trajectory?

2. The names of 100 contestants are placed in 100 wooden boxes, one name per box, and the boxes are lined up on a table in a room. One by one, the contestants are led into the room; each may look in at most 50 boxes, but must leave the room exactly as they found it and is permitted no further communication with the others. The contestants have a chance to plot their strategy in advance, and they are going to need it, because the team of contestants win the prize only if every single contestant finds their own name. Find a strategy for them which has probability of success exceeding 30%.

3. For $n = 1, 2, 3, 4, 5, 6$ place n dots in on a circle in such way that when you draw all the lines connecting all n points each other, no three lines are concurrent. Count the number of regions you broke the circle in.