POSSIBLE PROJECTS

ALBERT ARTILES

Problems

1. Statistics of Reduced Words: Say you have a group with finite presentation $\langle S : R \rangle$. How many reduced word of size n are there?
2. Chromatic number of Cayley Graphs: Say you have group G with generating set S. What is the chromatic number of the associated Cayley graph?
3. Convolution of Measures: Take two probability measures on [0,1] and convolve them. What does distribution look like?
4. Approximation vs Approximation: Look at two ways to approximate numbers in $[0, 1]$, say decimal expansion and continued fraction expansion. make a graph where one axis is the error in decimal expansion and the other axis is the error in the continued fraction expansion.
5. Tree Shifts: Consider the Cayley graph of the free group on two generators. you are to color the vertices of this graph either red or blue in such a way that no two red vertices are adjacent. How many ways are there to color the ball of radius N centered at the origin?
6. Statistics of Interval Breaks: Look at the numbers $\sqrt{n}$ modulo 1 from 1 to N. This breaks the interval [0,1] into subintervals. What is the distribution of these lengths when properly renormalized?
7. Complex Hyperbolic Geometry: Consider the group $PSL(2, \mathbb{C})$ and the subgroup $PSL(2, \mathbb{Z}[i])$. Let μ be the probability measure on $X = PSL(2, \mathbb{C})/PSL(2, \mathbb{Z}[i])$ induced by the Haar measure on $PSL(2, \mathbb{C})$. Can we use unipotent flows to approximate the integral of compactly supported functions on X?
8. Veech Surfaces: Take the double pentagon translation surface. Compute the number of saddle connections in the ball of radius R.
9. Automorphisms of Phylogenetic Trees: Given a n-ary rooted tree with label leaves, what is its automorphism group?
10. Diophantine Approximations: Consider the unit square $I^2 = [0, 1] + i[0, 1] \subset \mathbb{C}$. $Q[i]$ is dense in I^2, but we can ask how fast is it becoming dense. Consider a height function H on $Q[i]$ and pick a $\delta > 0$. For $x \in I^2$, what is the smallest height you can find in the ball of radius δ centered at x?
11. Principal Congruence Subgroups of $SL(2, \mathbb{Z})$ Make a histogram of the orbits of points in the real projective line under the principal congruent subgroups of $SL(2, \mathbb{Z})$. Try and compute a stationary measure for this action.

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