

PCMI 2023 XML: MEETING 6

ALBERT ARTILES

PROBLEMS

1. A unimodular lattice in \mathbb{R}^2 is a maximal discrete subgroup of \mathbb{R}^2 with covolume 1. Can you give the set of unimodular lattices in \mathbb{R}^2 a geometric structure? Can you do this for higher dimension?
2. Consider the regular tetrahedron. Can you find a geodesic starting and ending at the same vertex without touching any other vertices?
3. Consider the symmetric group S_n . A derangement is an element $\sigma \in S_n$ without any fix points. Compute the following limit: $\lim_{n \rightarrow \infty} \frac{\#\text{derangements in } S_n}{n!}$. Can you do this for other groups acting on finite sets?