

## PCMI 2023 XML: MEETING 1

ALBERT ARTILES

### PROBLEMS

1. Let  $A$  be a  $2 \times 2$  matrix. What conditions can you come up with so that when  $A$  is applied to  $\mathbb{R}^2$  the lengths of all vectors are preserved?

**Some things to think about first:**

- (1) What is the length of a vector?
- (2) How do matrices act on  $\mathbb{R}^2$ .
- (3) What else do  $I$  need to know in order to solve this problem?

**Some follow up questions**

- (1) What if we change our notion of length? (What does this even mean?)
- (2) What can I say about higher dimensions?
- (3) What other interesting question about this can  $I$  come up with?

2. Consider the map  $T_2(x) = 2x$  modulo 1 having inputs in  $[0, 1)$  and outputting a number in  $[0, 1)$ . Are there any periodic points? What are they?

**Some things to think about first:**

- (1) What does modulo 1 mean?
- (2) What does periodic mean?
- (3) Is  $T_2$  continuous?
- (4) What else do  $I$  need to know in order to answer this question?

**Some follow up questions**

- (1) What can  $I$  say about preperiodic points? (What does preperiodic mean?)
- (2) Is there a point with dense orbit?
- (3) What other questions can  $I$  come up with?