PCMI 2025 Research Program Seminars

Monday, July 7, 4:30pm – Noah Kravitz

Tuesday, July 8, 9:40am – Michelle Delcourt

Refined Absorption and Design Theory

The study of combinatorial designs has a rich history spanning nearly two centuries. In a recent breakthrough, the notorious Existence Conjecture for Combinatorial Designs dating back to the 1800s was proved in full by Keevash via the method of randomized algebraic constructions. Subsequently Glock, Kühn, Lo, and Osthus provided an alternate purely combinatorial proof of the Existence Conjecture via the method of iterative absorption. We introduce a novel method of "refined absorption" for designs and use it to provide a new alternate proof of the Existence Conjecture. The method can also be applied in a black-box fashion to many other design theory problems, including proving the High Girth Existence Conjecture and finding sufficiently spread distributions on designs. This is joint work with subsets of Luke Postle and Tom Kelly.

Tuesday, July 8, 3:15pm, Cross Program Lecture – Rob Morris

Ramsey numbers

In the first half of the talk I'll give a gentle introduction to Ramsey numbers, aimed at those who are new to the area; in the second half I'll describe some recent progress on upper bounds for the multicolour Ramsey numbers $R_r(k)$.

Tuesday, July 8, 4:30pm – Bhargav Narayanan

Elementary symmetric polynomials under the fixed point measure

I'll talk about a surprising inequality satisfied by elementary symmetric polynomials under the action of the fixed point measure of a random permutation. Concretely, Ayush Khaitan, Ishan Mata and I recently proved that any n non-negative real numbers $a_1, a_2, \ldots, a_n$ satisfy the inequality

$$(1/n!) \sum_{f \in S_n} \prod_{i:i=f(i)} a_i \geq (1/\binom{n}{2}) \sum_{\{j,k\} \in \binom{[n]}{2}} \sqrt{a_j a_k}.$$

This bound is sharp, and equality is attained if and only if $a_i = 1$ for all $1 \leq i \leq n$.

Where does this inequality come from? Can this inequality be deduced from existing machinery in the literature (vast, primarily algebraic) around symmetric polynomials? These are some of the questions that I will answer in my talk.

Wednesday, July 9, 9:40am – Patrick Morris

A sparse canonical van der Waerden theorem

The canonical van der Waerden theorem asserts that, for sufficiently large n, every colouring of [n] contains either a monochromatic or a rainbow arithmetic progression of length k (k-AP, for short). In this talk, we will determine the threshold at which the binomial random subset $[n]_p$ almost surely inherits this canonical property. As an application, we construct sets $S \subseteq [n]$ that satisfy the canonical van der Waerden property for k-APs and such that the k-APs in S define a k-uniform hypergraph of arbitrarily high girth.

(joint work with J. D. Alvarado, Y. Kohayakawa, G. O. Mota and M. Ortega)

Thursday, July 10, 9:40am – Matthew Kwan

Thursday, July 10, 3:15pm – Julian Sahasrabudhe

The r-colour diagonal Ramsey numbers

In this talk I will discuss some of the ideas behind the recent improvement to the upper bounds on the diagonal multi-colour Ramsey numbers. This is based on joint work with Balister, Bollobás, Campos, Griffiths, Hurley, Morris and Tiba.

Friday, July 11, 9:40am – Rajko Nenadov

The typical structure of sets with bounded sumset

A celebrated Freiman–Ruzsa result states that if a set of integers A is such that |A+A|/|A| < L, then one can find a generalised arithmetic progression of size f(L)|A| and dimension d(L) which contains A. Necessary lower bounds on f(L) and d(L) are of order 2^L and L, respectively. We show that examples which force such a lower bound are rather sporadic in a certain sense. Namely, we show that if L < |A|^{1/4}, then for a typical set A with |A+A|/|A| < L and |A| < k one can take f(L) = (1+o(1))L/2 and d(L) = 1, that is, one can find an arithemtic progression of length (1+o(1))Lk/2 which contains A. This is best possible and constitutes a step towards a qualitative strengthening of a conjecture of Alon, Balogh, Morris, and Samotij on the number of sets with bounded sumsets.

Joint work with Lander Verlinde.

Friday, July 11, 3:15pm, Cross Program Lecture – Janos Pach

How to get from A to B?

János Pach (Rényi Institute, Budapest)

Starting with the famous "14-15 puzzle" attributed to Sam Loyd, many geometric, combinatorial, algebraic, and algorithmic questions can be phrased in the following form. Given two configurations, A and B, how can one transform A to B through a sequence of moves of a given type? Is this possible at all? We discuss several problems of this kind, including a basic problem in motion planning (joint work with Herbert Edelsbrunner), and a more recent puzzle by Kovaldzhi and Brunck with linear algebraic background (joint work with Gábor Tardos). We will also state some challenging open problems.

Friday, July 11, 4:30pm – Adva Mond

Separating Recovering and Cancellative pairs and the Sandglass conjecture

We address a basic extremal problem for pairs (A,B) of families of subsets of [n]. We say that a pair (A,B) is recovering, if for any two sets X, X' in A and any two sets Y, Y' in B, we have that X\Y = X'\Y' implies X=X' and Y\X=Y'\X' implies Y=Y'. We say that a pair (A,B) is cancellative if for any two sets X, X' in A and any two sets Y, Y' in B we have that X\Y = X'\Y implies X=X' and Y\X = Y'\X implies Y=Y'. It is easy to see that a cancellative pair is also recovering.

It was shown by Simonyi and by Nair and Yazdanpanah that 2^n ≤ |A||B| ≤ 2.266^n for any recovering pair (A,B). On the other hand, for cancellative pairs we have 2.25^n ≤ |A||B| ≤ 2.268^n, due to Tolhuizen and B. Janzer. However, it was unclear whether the quantity |A||B| for a recovering pair is smaller than the one for a cancellative pair. We show an exponential separation between the bounds for recovering and cancellative pairs. As a result, we improve the best known bound for recovering pairs.

Based on joint work with Victor Souza and Leo Versteegen.

Monday, July 14, 9:40am – Blair Sullivan

Improving your chances of being heard

Abstract: In a probabilistic graph G, each edge e is “active” with some probability p_e, independent of the other edges. Say the proximity of two vertices u, v is the probability that u can "hear" v in a graph sampled from G - that is, there exists a uv-path using only active edges. For a subset S of V, we can then define its reach to be the minimum proximity between any s in S and v in V. This quantity has been studied in the literature in the context of algorithmic fairness and of multicast in unreliable communication networks. We are interested in the problem of improving the reach in a probabilistic graph via edge augmentation. In other words, what k additional (probabilistic) links would maximally boost the reach of a given S?

In this talk, we describe how the existence of a good edge augmentation implies a cluster structure for the graph in an appropriate metric, and use this structural result to analyze a novel poly(OPT)-approximation algorithm. Our arguments rely on new probabilistic tools for analyzing proximity, inspired by techniques in percolation theory; these tools may be of broader interest. We complement this with hardness results and approximation lower bounds showing there is limited room for improvement. We will conclude with a few open problems and directions.

This is joint work with A. Crane, A. Bhaskara, S. Jain, MMH. Mazumdar, and P. Yalamanchili.

<u>Monday, July 14, 3:15pm, Cross Program Lecture – Noga Alon</u>
Graph-Codes: Problems, Results and Methods
The study of Graph-Codes is motivated by questions in Extremal Combinatorics, Additive Number Theory and Coding Theory. The initial guiding fact is that viewing binary vectors as characteristic vectors of edge-sets of graphs transforms the basic combinatorial questions of Coding Theory into intriguing extremal problems about families of graphs. I will discuss some of these questions and describe several results and open problems. The relevant methods combine Combinatorial and Probabilistic tools with techniques from Information Theory, Number Theory and the theory of Combinatorial Designs.

<u>Monday, July 14, 4:30pm – Maya Sankar</u>
The Turán Density of Tight Cycles
As introduced in the USS and GSS, the study of extremal numbers is a rich problem, which remains wide open for hypergraphs. I will discuss several recent results on the Turán density (i.e., asymptotic extremal number) of long cycle-like hypergraphs. These results (due to Kamčev–Letzter–Pokrovskiy, Balogh–Luo, and myself) all follow a similar framework, and I will outline a general strategy to prove Turán-type results for tight cycles in larger uniformities or for other "cycle-like" hypergraphs.

One key ingredient in this framework, which I hope to prove in full, is a hypergraph analogue of the statement that a graph has no odd closed walks if and only if it is bipartite. More precisely, for various classes C of "cycle-like" r-uniform hypergraphs — including, for any k, the family of tight cycles of length k modulo r — we equiivalently characterize C-hom-free hypergraphs as those admitting a certain type of coloring of (r-1)-tuples of vertices. This provides a common generalization of results due to Kamčev–Letzter–Pokrovskiy and Balogh–Luo.

<u>Tuesday, July 15, 9:40am – Konstantin Tikhomirov</u>
Combinatorial approach to metric embeddings
The theory of metric embeddings is a very active research area with numerous applications within computer science. The basic problem is, given two metric spaces M1 and M2, either to construct a low-distortion mapping from M1 into M2, or to show that such a mapping does not exist. In the past year, significant progress on several long-standing problems within the field has been achieved, notably

through novel applications of the methods of probabilistic combinatorics. In this talk, I will discuss some of the recent advances. The talk is based on joint works with D.Altschuler, P.Dodos, and K.Tyros.

Tuesday, July 15, 4:30pm – Anita Liebenau

Ramsey with purple edges

On of the most classical problems in Ramsey theory concerns finding the Ramsey numbers R(s,t). Motivated by a question of David Angell, we study a variant of Ramsey numbers where some edges are coloured both red and blue, or: purple. Specifically, we are interested in the largest number g = g(s,t,n), for some s, t and n < R(s,t), such that there exists a red-blue-purple colouring of the edges of K_n with g purple edges, without a red-purple copy of K_s and without a blue-purple copy of K_t. We determine g asymptotically for a large family of parameters, exhibiting strong dependencies with Ramsey-Turán numbers.

Joint work with Thomas Lesgourgues and Nye Taylor.

Wednesday, July 16, 9:40am – Oliver Janzer

Short monochromatic odd cycles

It is easy to see that every k-edge-colouring of the complete graph on 2^k+1 vertices contains a monochromatic odd cycle. In 1973, Erdos and Graham asked to estimate the smallest L(k) such that every k-edge-colouring of K_{2^k+1} contains a monochromatic odd cycle of length at most L(k). Recently, Girao and Hunter obtained the first nontrivial upper bound by showing that L(k)<(2^k)/k^{1-o(1)}, which improves the trivial bound by a polynomial factor. We obtain an exponential improvement by proving that L(k)<2^{k/2+o(1)}. Our proof combines tools from algebraic combinatorics and approximation theory.

Joint work with Fredy Yip.

Thursday, July 17, 9:40am – Liana Yepremyan

Long rainbow paths in graphs and digraphs

We study an old question in combinatorial group theory which can be traced back to a problem of Graham from 1971, restated by Erd\H{o}s and Graham in 1980. Given a group Γ, and some subset $S\subseteq \Gamma$, is it possible to permute S as $s_1,s_2,\ldots, s_d$ so that the partial products $\prod_{1\leq i\leq t} s_i$, $t\in [d]$ are all distinct? Most of the progress towards this problem has been in the case when Γ is a cyclic group. We show that for any group Γ and any $S\subseteq \Gamma$, there is a permutation of S where all but a vanishing proportion of the partial products are distinct, thereby establishing the first asymptotic version of Graham's conjecture under no restrictions on Γ or S.

To do so, we explore a natural connection between Graham's problem and the following very natural question attributed to Schrijver. Given a d-regular graph G properly edge-coloured with d colours, is it always possible to find a rainbow path with $d-1$ edges? We settle this question asymptotically by showing one can find a rainbow path of length $d-o(d)$. A certain natural directed analogue of Schrijver's question gives further implications for Graham's conjecture.

This is based on joint work with Matija Bucic, Bryce Frederickson, Alp Müyesser and Alexey Pokrovskiy.

Thursday, July 17, 3:15pm, Cross Program Lecture – Tadashi Tokieda

title

A world from a sheet of paper

Abstract
Starting from just a sheet of paper, by folding, stacking, crumpling, sometimes tearing, we will explore a variety of phenomena, from magic tricks and geometry to elasticity and the traditional Japanese art of origami. Much of the show consists of table-top demos, which you can try later with friends and family. So, take a sheet of paper . . .

<u>Thursday, July 17, 4:30pm – Maya Stein</u>
Separation systems of subdivisions
A (strong) separating path system of a graph G is a family of paths such that for each ordered pair of edges of G there is a path which contains the first edge but does not contain the second edge. There has been a surge of interest in finding small separating path systems in the last decade. Bonamy et al showed that the smallest size of a separating path system of an n-vertex graph G is O(n) which is best possible by results of Balogh et al.

Recently, Botler and Naia conjectured that we could replace the paths that form the separating system with other families of graphs. Namely, they conjectured that for any fixed graph H with at least one edge, any n-vertex graph G has a separating system of size O(n) consisting of subdivisions of H and single edges. (The single edges are necessary, and we allow depence on H in the term O(n).)

We show Botler and Naia's conjecture is true. This is joint work with George Kontogeorgiou, Matías Pavez-Signé, Taruni Sai Sridhar and Ana Trujillo-Negrete.

<u>Friday, July 18, 9:40pm – Sam Spiro</u>
Generalized Turán Problems for Trees and More
Given a graph H and a family of graphs F, we define the generalized Turán number ex(n,H,F) to be the maximum number of copies of H in an F-free graph on n vertices. We prove a "stability" type result for generalized Turán problems which relates the generalized Tur\'an number ex(n,H,F)$ to the classical Turán number ex(n,F) whenever H is a tree. We discuss some applications of this result, as well as some related work around the rational exponents conjecture for general graphs H.

Joint work with Sean English.

<u>Friday, July 18, 3:15pm – Matthew Jenssen</u>
A new lower bound for the Ramsey numbers R(3,k)
In this talk, I will discuss a new lower bound for the off-diagonal Ramsey numbers R(3,k). The previous best bound is a result of the celebrated analysis of the triangle-free process, due independently to Bohman and Keevash, and Fiz Pontiveros, Griffiths and Morris. We introduce a modified triangle-free process which is simpler to analyse. By running this process from a carefully chosen 'seed graph', we obtain a denser triangle-free graph that remains sufficiently pseudorandom to avoid large independent sets.

<u>Monday, July 21, 9:40pm – Cosmin Pohoata</u>

<u>Monday, July 21, 3:15pm – Sabetta Matsumoto</u>
Title: Twisted topological tangles or: the knot theory of knitting
Abstract: Imagine a 1D curve, then use it to fill a 2D manifold that covers an arbitrary 3D object – this computationally intensive materials challenge has been realized in the ancient technology known as knitting. This process for making functional materials 2D materials from 1D portable cloth dates back to

prehistory, with the oldest known examples dating from the 11th century CE. Knitted textiles are ubiquitous as they are easy and cheap to create, lightweight, portable, flexible and stretchy. As with many functional materials, the key to knitting's extraordinary properties lies in its microstructure. At the 1D level, knits are composed of an interlocking series of slip knots. At the most basic level there is only one manipulation that creates a knitted stitch – pulling a loop of yarn through another loop. However, there exist hundreds of books with thousands of patterns of stitches with seemingly unbounded complexity. The topology of knitted stitches has a profound impact on the geometry and elasticity of the resulting fabric [1]. We have developed a formalization of the topology of two-periodic weft knitted textiles using a construction we call the swatch [2]. Using this construction, we can prove that all two-periodic weft knits form ribbon links [3]. This puts a new spin on additive manufacturing – not only can stitch pattern control the local and global geometry of a textile, but the creation process encodes mechanical properties within the material itself. Unlike standard additive manufacturing techniques, the innate properties of the yarn and the stitch microstructure has a direct effect on the global geometric and mechanical outcome of knitted fabrics.

The authors were partially supported by National Science Foundation grant DMR-1847172, by the Research Corporation for Science Advancement and by the International Center for Sustainability with Chiral Knotted Meta Matter (SKCM²). We would like to thank sarah-marie belcastro, Jen Hom, Jim McCann, Agniva Roy, Saul Schleimer and Henry Segerman for many fruitful conversations. [1] K. Singal, M.S. Dimitriyev, S.E. Gonzalez, et al. Programming mechanics in knitted materials, stitch by stitch. Nat Commun 15, 2622 (2024). [2] S. Markande and S. Matsumoto, in: Proceedings of Bridges 2020: Mathematics, Art, Music, Architecture, Culture, (Tesselations Publishing, 2020), pp. 103–112. [3] M. Kuzbary, S. Markande, S. Matsumoto and S. Pritchard, 2025.

Monday, July 21, 5:30pm – Henry Segerman

Title: Artistic mathematics: truth and beauty

Abstract: I'll talk about my work in mathematical visualization: making accurate, effective, and beautiful pictures, models, and experiences of mathematical concepts. I'll discuss what it is that makes a visualization compelling, and show many examples in the medium of 3D printing, as well as some work in virtual reality and spherical video. I'll also discuss my experiences in teaching a project-based class on 3D printing for mathematics students.

Tuesday, July 22, 9:40pm – Huy Tuan Pham

Selector processes, thresholds and further connections

Positive selector processes are natural stochastic processes driven by sparse Bernoulli random variables. Selector processes play a critical role in the study of suprema of stochastic processes, as reflected in Talagrand's selector process conjecture. Interestingly, Talagrand also drew rich and influential connections between selector processes and the threshold phenomena in probabilistic combinatorics.

I will discuss a quantitative sharp version of Talagrand's selector process conjecture, which strengthens both the selector process conjecture and the Kahn-Kalai conjecture. As an application, I will discuss progress towards another conjecture of Talagrand on the "integrality gap" of a wide class of integer linear programs involving "covers" of general set systems, relating the so-called expectation threshold and its fractional relaxation. In particular, the sharp version of Talagrand's selector process conjecture implies that, for a monotone property over a ground set X, the gap between the expectation threshold and its fractional relaxation is at most (log log |X|), an exponential improvement over the best previously known result.

Time permitting, I will discuss further connections between selector processes, thresholds, and other topics of interest in combinatorics.

Tuesday, July 22, 3:15pm, Cross Program Lecture – Jordan Ellenberg

TITLE: Machine learning and pure math, especially extremal combinatorics

ABSTRACT: I'll talk about some of my own experiences working with both industry and academic collaborators on getting contemporary machine learning devices to produce material of interest to mathematicians. My main focus is going to be the construction of examples in extremal combinatorics (e.g. large capsets, large subsets of the grid with no isosceles triangles, graphs with many edges and no 4-cycles) and I will also talk about the reason why this subject has so far been a particularly fruitful one for ML methods compared to other parts of mathematics.

Some relevant papers:

https://www.nature.com/articles/s41586-023-06924-6

https://arxiv.org/abs/2411.00566

https://arxiv.org/abs/2503.11061

Tuesday, July 22, 4:30pm – Dan Král

Curves on the torus with prescribed intersections

Juvan, Malnič and Mohar [J. Combin. Theory Ser. B 68 (1996), 7-22] proved that for every surface S and every integer k, there exists N(S,k) such that any set of simple closed curves on the surface S, where any two are non-homotopic and intersect at most k times, has a maximum size at most N(S,k). In the case when S is the torus T^2, the problem has an interesting connection to number theory as the Riemann hypothesis yields that N(T^2,k)<=k+O(k^0.5*log k); Aougab and Gaster [Math. Proc. Cambridge Philos. Soc. 174 (2023), 569-584] have recently proven that this bound holds directly. We determine N(T^2,k) exactly for every k. In particular, we show that N(T^2,k)<=k+6 for all k and N(T^2,k)<=k+4 when k is sufficiently large.

The talk is based on joint work with Igor Balla, Marek Filakovský, Bartłomiej Kielak and Niklas Schlomberg.

Wednesday, July 23, 9:40am – Jordan Ellenberg

Smyth's conjecture and a non-deterministic Hasse principle

Smyth's conjecture is a question in algebraic number theory — what kind of linear relations can occur between Galois conjugates? — which (as Smyth observed from the start) is really a question in combinatorics. We can phrase it as a question about non-invertibility of linear combinations of permutation matrices, or as a question about "Diophantine equations in random variables," or as a question about balanced weightings of 3-uniform hypergraphs, or as a question about finite subsets of R^d whose projections in many (i.e. more than d) directions yield the same multiset in R. This latter question in additive combinatorics turns out to be the one that gives us the most purchase on this problem. I'll explain our proof of Smyth's conjecture, which involves finding a finite subset of R^d whose projections are *approximately* the same multiset and then showing that this approximate solution can be perturbed to an on-the-nose solution.

This is joint work with Will Hardt.

Thursday, July 24, 9:40am – Richard Montgomery

Asymptotically-tight packing and covering in Rota's basis conjecture

Rota's basis conjecture from 1989 asserts that, given any n bases B_1,..., B_n of a vector space of dimension n (or, more generally, a matroid of rank n), there is a collection of n disjoint transversal bases. In other words, the elements of B_1,....,B_n can be decomposed into n new bases of the vector space (or matroid), each consisting of exactly one element from each of the original bases B_1,...,B_n.

In this talk, I will discuss some progress towards this conjecture which shows that, in this setting, (1-o(1))n disjoint transversal bases always exist and that the union of the bases B_1,...,B_n can be covered by (1+o(1))n transversal bases.

This is joint work with Lisa Sauermann.

Thursday, July 24, 3:15pm – Bryna Kra

A Mathematician's Challenge

The mathematical landscape is rapidly changing, with uncertain job markets, modified funding priorities, and emerging technologies impacting research and teaching. I will give a personal perspective on what we can do as a community to address the numerous challenges.

Thursday, July 24, 4:30pm – Yuval Wigderson

VC dimensions and regularity

The regularity lemma says that every discrete object can be partitioned into a small number of random-like subobjects. But how small is small? And can we make small smaller if we assume that our given object is simple? And what does it mean for a discrete object to be simple? In this talk, I will answer some of these questions.

Joint with Lior Gishboliner and Asaf Shapira.

Friday, July 25, 9:40pm – Matija Bucić

The spanning tree spectrum: improved bounds and simple proofs

The number of spanning trees of a graph G, denoted t(G), is a well studied graph parameter with numerous connections to other areas of mathematics. In a recent remarkable paper, answering a question of Sedláček from 1969, Chan, Kontorovich, and Pak showed that t(G) takes at least 1.1103^n many different values across simple (and planar) n-vertex graphs G, for large enough n. We give a very short, purely combinatorial proof that at least 1.49^n values are attained. We also prove that exponential growth can be achieved with regular graphs, determining the growth rate in another problem first raised by Sedláček in the late 1960's. We further show that the following modular dual version of the result holds. For any integer N and any u<N there exists a planar graph G on O(log N) vertices with t(G)= u mod N.

Joint work with: Noga Alon and Lior Gishboliner.

Friday, July 25, 3:15pm – Robert Krueger

Weighted clique decompositions

Given a graph G, what is the minimum number of cliques needed to decompose the edge set of G? Erdős, Goodman, and Pósa showed that at most n^2/4 cliques are needed for any n-vertex graph. We study a generalization of this question where the cliques are given a "cost" depending on their size, asymptotically answering a question of Erdős. This is joint work with József Balogh, Jialin He, The Nguyen, and Michael Wigal.