LEARNING SEMINAR ON NON-ABELIAN p-ADIC HODGE THEORY FALL 2023, IAS

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In this seminar we want to study non-abelian p-adic Hodge theory, i.e. we discuss generalizations of p-adic Hodge theory for coefficients in GL_n. This is inspired by Simpson's famous results in non-abelian Hodge theory in complex geometry.

The search for p-adic analogs of Simpson's correspondence was initiated almost 20 years ago by Gerd Faltings [6], and in less generality independently by Christopher Deninger and Annette Werner [4, 5]. New tools from perfectoid geometry have recently led to vivid progress in the field. In particular, Faltings' category of generalized representations can now be understood through vector bundles for the v-topology on the associated diamond.

In the sequel, we let K denote a non-archimedean field (i.e., a field complete with respect to a nonarchimedean multiplicative norm $|\cdot| : K \to \mathbb{R}_{\geq 0}$), and let X denote a smooth rigid analytic variety over K.

Here is the outline of the talks:

(1) 10/12 **Hodge-Tate sequence.** Compute $R\nu_*\widehat{\mathcal{O}}_X$, where $\nu : X_{\text{proet}} \to X$, and discuss the Hodge-Tate exact sequence

$$0 \to H^1(X, \mathcal{O}_X) \to H^1(X, \widehat{\mathcal{O}}_X) \to H^0(X, \Omega_X(-1)) \to 0, \tag{HT}$$

for K algebraically closed and X proper as in [14, §3] and [3, §13]. Briefly review diamonds and v-sheaves/stacks [15], and compare categories of vector bundles for various topologies as in [7, introduction].

Discuss the multiplicative version of (HT) (under the same assumptions)

$$0 \to \mathrm{Pic}(X) \to \mathrm{Pic}(X_v) \to H^0(X, \Omega_X(-1)) \to 0, \tag{Mul HT}$$

which can be regarded as the p-adic Simpson correspondence for generalized representations in the rank one situation following [7, Theorem 1.3].

(2) 10/19 **p-adic Corlette-Simpson correspondence for $\mathbb{G}_m$.** Explain the p-adic Corlette-Simpson correspondence in rank one following [7, §5]. Give interpretations of pro-finite-étale analytic rank one line bundles as topologically torsion line bundles (and in the case X projective over $\mathbb{C}_p$, as line bundles with vanishing rational Chern class), as in [8, §2, §3]. Geometrize the sequence (Mul HT) as maps of moduli spaces as in [8, §4].

(3) 10/26 **Hitchin maps in non-abelian p-adic Hodge theory.** Review the classical algebraic geometry of Hitchin system, in particular in the curve case review spectral curves, and the action of the Picard stack $\mathcal{P}$ of line bundles on spectral curves on the Hitchin moduli space as in [12] (restricted to the case $G = GL_n$).

Discuss the corresponding moduli spaces in the p-adic situation as in [9, 11] (restricted to $G = GL_n$). Explain the relative pro-étale Picard variety of the spectral variety [10, §2] and explain the relation to the Higgs-Tate torsor of Abbes-Gros [1]. In the curve case, organize these torsors into a family as $\mathcal{P}$-torsor $\mathcal{H}$ over the Hitchin base [11, Theorem 1.3.2].

(4) 11/2 **Local correspondence/geometric Sen theory.** Discuss the correspondence for small objects when X admits a toric chart $X \to \mathbb{T}^n$ according to [10, §4] and [9, Theorem 6.5] (restricted to $G = GL_n$.) Explain how to globalize this construction using the approach to Sen theory by Rogriguez Camargo [13] as in [10, §4].

(5) 11/9 **Global p-adic Simpson for generalized representations.** For X smooth proper over a complete algebraically closed field K, establish the generalized p-adic Simpson correspondence as an equivalence of categories between v-bundles and Higgs bundles as in [10, Theorem 1.1], building on the previous talks. Discuss the correspondence at the geometric level when X is a projective curve as a *completely canonical* isomorphism between the moduli stack of v-bundles and the moduli stack of Higgs bundles twisted by the $\mathcal{P}$-torsor $\mathcal{H}$, following [11].

(6) 11/30 **Locally analytic version of the local correspondences.** Discuss a locally analytic version of the local correspondence (discussed in Talk (4)), see [13]. Study the examples from Shimura varieties and possibly applications to number theory. This talk might also be a place to mention generalizations to reductive groups G.

(7) 12/7 **Non-abelian p-adic Hodge theory via the Hodge-Tate stack.** Explain the statement of [2, Theorem 1.3] and give a sketch of the proof.

References

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